The Interaction of Flexible versus Dedicated Technology Choice with Financial Risk Management under Financial Constraints

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Abstract

This paper analyzes the impact of financial constraints in a capacity investment setting. We model a monopolist firm that decides on its technology choice (flexible versus dedicated) and capacity level under demand uncertainty. Differing from the majority of the stochastic capacity investment literature, we assume that the firm is budget-constrained both in the capacity investment and production stages, and that the production stage budget is stochastic. Our analysis contributes to the capacity investment literature by extending the theory of stochastic capacity investment to understand the impact of financial constraints, and by analyzing the impact of budget variability on the profitability of the firm. We demonstrate that budget variability is detrimental for the firm with either technology, thus the firm is better off by hedging the budget uncertainty through proper risk management. One of our main contributions is to analyze the impact of financial constraints on the flexible versus dedicated technology choice. We demonstrate that without production costs, a higher internal budget favors the flexible technology only when the fixed cost of the flexible technology is higher. With production costs in place, and in the absence of fixed cost difference, a higher internal budget favors the dedicated technology. This is because the total investment cost is higher with the dedicated technology.

Contents

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Chapter 1

Introduction and Literature Review

Especially in capital-intensive industries, capacity investment is subject to financing frictions. Firms operate with internal budget constraints, and these constraints impose restrictions on the firm's investment policy both on the capacity investment and on the production stages. As highlighted by Van Mieghem (2003), in the literature, stochastic capacity investment models often ignore these financial constraints. The objective of this paper is to increase our understanding of how these financial constraints affect stochastic capacity investment and technology choice decisions of the firms. A key feature of our paper is that we impose budget restrictions on the firm's operational decisions both in the capacity investment and production stages.

If the internal capital of the firm is not sufficient to finance the desired investment level, then the firm may decide to raise external capital. External capital is more expensive because there exist capital market imperfections such as bankruptcy costs, taxes, financial distress cost or agency costs due to asymmetric information etc. (Froot et al. 1993) that create frictions in the borrowing process of the firm. In this paper, we will assume that these financing frictions are very severe such that the firm cannot borrow from external capital markets, but make investment decisions using their internal capital. This assumption has some practical relevance. Empirical observations document that subsidiaries of firms, in general, operate on pre-determined budget levels that are allocated to them through their headquarters.

Not only firms have internal budget restrictions but also in majority of the cases, the available internal capital may be subject to some risks and may be random. In practice, the capital availability of firms may depend on returns from other investments, or in particular, some financial assets such as treasury bonds. In this case, firms can rely on financial derivatives to engineer these cash flows so as to maximize the return from their operational investments. Another key feature of our paper is that we assume stochastic budget at the production stage and analyze the impact of budget variability as well as risk management decisions of the firm on stochastic capacity investment.

We consider a monopolist firm selling two products in a single selling season under demand and budget uncertainties. The firm chooses the technology (dedicated versus flexible), the capacity investment level and the production level so as to maximize expected profit.

To this end, we model a firm who produces and sells two products under demand uncertainty. The firm chooses between flexible and dedicated technologies that incur fixed and variable investment costs. Differing from the majority of traditional stochastic technology and capacity investment problems, the firm is modeled as being budget constrained. We model the firm's decisions as a two-stage stochastic recourse problem under budget and demand uncertainty. In stage 1,with respect to the stage 2 demand and budget uncertainties, the firm determines the technology choice (flexible or dedicated) and makes its capacity investment using its initial budget. In stage 2, demand and budget uncertainties are realized. This realized budget and the remaining stage 1 budget after capacity investment (if any) determines the available internal capital of the firm at this stage. Subject to this capital availability, the firm then chooses the production quantities for each product within the capacity limits of the chosen technology.

We derive the technology choice, capacity, and production level decisions of the firm. We investigate how the budget uncertainty affects the capacity investment level and the performance of the firm for a given technology. With deterministic budget, we analyze the impact of budget restrictions on the flexible versus dedicated technology choice. To achieve this, we compare our results with the benchmark case of a firm that makes flexible versus dedicated technology choice without budget restrictions. Our results contribute to several streams of research, as detailed below.

The stochastic capacity investment literature analyzes the flexible versus dedicated technology choice under demand uncertainty in a variety of models. As highlighted in Van Mieghem (2003), the operations management literature (often implicitly) assumes that there are no financial constraints on the operational decisions. In practice, there exist capital market imperfections that impose financing restrictions on firms (Harris and Raviv 1991). There is a growing body of work in operations that analyzes the impact of financial constraints on firm's operations decisions. A recent stream of papers (Lederer and Singhal 1988, Buzacott and Zhang 2004, Xu and Birge 2004, Babich and Sobel 2004, Babich et al. 2010, Dada and Hu 2008 and Caldentey and Haugh 2009) analyze the joint financing and operating decisions of the firm and demonstrate the value of integrated decision making. All these papers focus on a single-product setting where technology choice is not relevant. Closest to our paper, Boyabatlı and Toktay (2011) analyzes flexible versus dedicated technology choice in imperfect capital markets. Different from our paper, they assume that the firm can borrow from external capital markets. They analyze the impact of demand uncertainty on the technology choice and capacity investment decision of the firm in a credit-firm equilibrium setting. In our paper, we assume that the firm cannot borrow from external capital markets but we provide a more detailed formalization of operational decisions with fixed cost of technology and variable production costs. Moreover, our focus is to analyze the impact of internal budget on the technology choice. We show that without production costs, a higher internal budget favors flexible technology as flexible technology has a higher fixed cost then the dedicated technology. Interestingly, with symmetric fixed costs, we show that internal budget does not have an impact on the optimal technology choice. With production costs in place, these insights may change. We show that with symmetric fixed costs and production costs, dedicated technology is favored with a higher internal budget level and a lower production costs. This is because the total investment cost with dedicated technology is higher than the flexible technology.

Several finance papers also investigate the interaction of financing and operational decisions. We refer the reader to Boyabatlı and Toktay (2011) for a review of these papers. We highlight Froot et al. (1993) from the finance literature since their modeling of risk management motive is the same as in our paper. In a single-product setting, they show that budget variability is detrimental for the firm's operating profits, and the firm benefits from hedging the budget uncertainty. We extend their result to the two-product setting. We show that the expected optimal profit of the firm decreases in budget variability with either technology. Therefore, hedging budget uncertainty is useful with both technologies. We also demonstrate that optimal capacity investment level with the flexible technology decreases in the budget variability.

The remainder of this paper is organized as follows: In §2, we describe the model and discuss the basis for our assumptions. We solve for the optimal capacity investment decision with uncertain budget and analyze the impact of budget variability with flexible technology and dedicated technology in §3 and §4 respectively. In §5, we analyze the technology choice with deterministic budget and investigate the impact of budget constraint on the technology choice. We conclude in §6 with a discussion of the limitations of our analysis and future research.

Chapter 2

Model Description and Assumptions

We consider a monopolist firm selling two products in a single selling season under demand and budget uncertainties. The firm chooses the technology (dedicated versus flexible), the capacity investment level and the production level so as to maximize expected profit. Differing from the majority of traditional stochastic technology and capacity investment problems, the firm is modeled as being budget constrained. We model the firm's decisions as a two-stage stochastic recourse problem under budget and demand uncertainty. In stage 1,with respect to the stage 2 demand and budget uncertainties, the firm determines the technology choice (flexible or dedicated) and makes its capacity investment using its initial budget. In stage 2, demand and budget uncertainties are realized. This realized budget and the remaining stage 1 budget after capacity investment (if any) determines the available internal capital of the firm at this stage. Subject to this capital availability, the firm then chooses the production quantities for each product within the capacity limits of the chosen technology. The timeline of events is depicted in Figure 2.1.

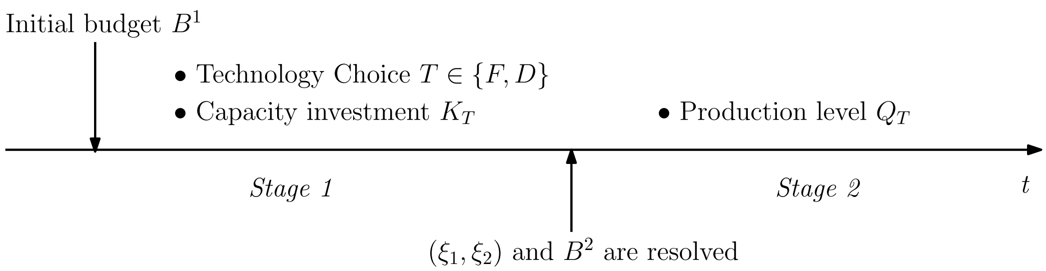


Figure 2.1: Timeline of Events

In stage 1, subject to the initial budget level B^1, the firm determines its technology choice $T \in \{D;F\}$ and the corresponding capacity investment level $\mathbf{K_T}$ with respect to the product market demand uncertainty ξ and stage 2 budget uncertainty B^2. The flexible technology F has a single resource that is capable of producing two products. The dedicated technology D consists of two resources that can each produce a single product. Technology T has fixed (F_T) and variable (c_T) capacity investment costs. The fixed cost of the flexible technology is higher than that of the dedicated technology, i.e. $F_F \geq F_D$. The variable capacity investment cost of the two dedicated resources are identical. The committed technology's fixed and variable capacity investment costs are payable in this stage. Since there's no external financing available, the firm will not be able to invest in capacity if the available stage 1 budget B^1 can not cover the technology's fixed cost F_T.

In stage 2, demand and stage 2 budget uncertainties are realized. We define $R_T \doteq B^2 + B^1 - F_T - c_T 1' K_T \geq 0$ as the firm's total cash level in stage 2 after the firm has paid for the committed technology investment cost F_T and capacity investment cost $c_T 1' K_T$ in stage 1. There exists a marginal production cost of y_T for $T \in \{F;D\}$ at this stage. Since the production is costly, the firm may not be able to fully utilize all of the capacity invested in stage 1. Subject to the available capital R_T and the physical capacity constraints, the firm chooses the production quantities (equivalently, prices) to satisfy demand optimally. With the presence of marginal production cost y_T in stage 2, R_T can be considered as a financial resource with $\frac{R_T}{y_T}$ "units" of capacity. This financial capacity, together with the physical capacity, form a network of resources for production. With flexible technology, one unit of each product should utilize one unit from each resource in production; whereas with dedicated technology, one unit of each product i, for $i = 1,2$, should utilize one unit of financial capacity and one unit of the corresponding dedicated physical capacity (K_D^i) for production. Figure 2.2 illustrates the production network with physical and financial capacity constraints for each technology.

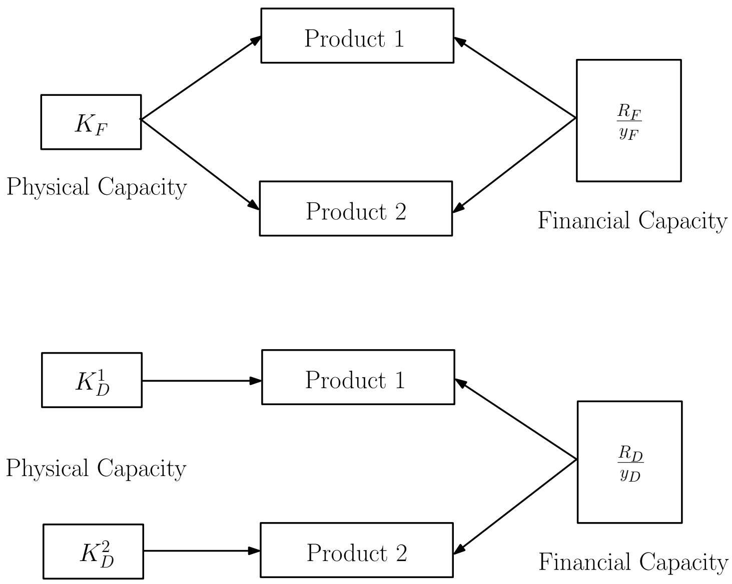


Figure 2.2: Physical and financial resource capacity networks for each technology.

Price-dependent demand for each product is represented by the iso-elastic inverse-demand function $p(q_i;\xi_1) = \xi_i q_i^{1/b}$ for $i = 1,2$. Here, $b \in (-\infty,-1)$ is the constant elasticity of demand, and p and q denote price and quantity, respectively. ξ_i represents the idiosyncratic risk component. (ξ_1,ξ_2) are correlated random variables with continuous distributions that have positive support and bounded expectation (μ_1,μ_2) with covariance matrix Σ, where $\Sigma_{ii} = \sigma_i^2$ and $\Sigma_{ij} = \rho\sigma_1\sigma_2$ for $i \neq j$ and ρ denotes the correlation coefficient. (ξ_1,ξ_2) and B^2 are statistically independent.

Before continuing with the analysis, we first present some of the important notations: A realization of the random variable $\tilde{s}$ is denoted by s and its expectation is denoted by $\bar{s}$. Bold face letters represent vectors of the required size. Vectors are column vectors and $'$ denotes the transpose operator. Vector exponents are taken componentwise. $\mathbf{xy}$ denotes the componentwise product of vectors $\mathbf{x}$ and $\mathbf{y}$ with identical dimensions. Pr denotes probability, $\mathbb{E}$ denotes the expectation operator, $\chi(.)$ denotes the indicator function with $\chi(\varpi) = 1$ if ϖ is true, $(x)^+ \doteq \max(x,0)$ and $\Omega^{01} \doteq \Omega^0 \bigcup \Omega^1$. Monotonic relations (increasing, decreasing) are used in the weak sense otherwise stated. Table 2.1 summarizes the decision variables. Table 6.1 that summarizes other notation and all proofs are provided in Appendix A.

Stage	Name	Meaning
Stage 1	$T \in \{D, F\}$	Technology choice, dedicated or flexible
	$\mathbf{K_T}$	Capacity investment level with technology T
Stage 2	$\mathbf{Q_T}$	Production quantity with technology T

Table 2.1: Decision variables by stage

Chapter 3

Flexible Technology

In this chapter, we analyze the firm's decision problem with flexible technology. We will first describe the optimal solution for the firm's capacity investment K_F^* and production decisions Q_F^* with stochastic stage 2 budget B^2 in §3.1. In §3.2, we analyze the same with marketing clearing price assumption. Finally, in §3.3, we focus on the firm's decision problem with deterministic stage 2 budget under the clearance price assumption.

3.1 Stochastic Stage 2 Budget

In this section, we describe the optimal solution for the firm's capacity investment and production decisions with flexible technology using backward induction starting from stage 2.

3.1.1 Stage 2: Production Decision

In stage 1, the firm with initial budget B^1 invested in capacity level K_F and has $B^1 - c_F K_F - F_F$ amount of internal capital left. In this stage, the firm observes the demand ξ and stage 2 budget B^2. This stage 2 budget, together with the internal capital left from stage 1 investment determines the internal capital R_F of the firm at this stage. The firm determines the production quantities $Q_F' = (Q_F^1, Q_F^2)$ within the existing flexible physical capacity K_F and financial capacity $\frac{R_F}{y_F}$ to maximize the stage 2 profit Ψ_F with the flexible technology.

Proposition 1 *For a given B^2, K_F and ξ, the optimal production quantity vector in stage 2 with flexible technology $Q_F^* = (Q_F^1, Q_F^2)$ is characterized by*

$$\mathbf{Q}_\mathbf{F}^*\left(B^2,\xi\right)' = \begin{cases} \left(\left[\frac{\left(1+\frac{1}{b}\right)\xi_1}{y_F}\right]^{-b}, \left[\frac{\left(1+\frac{1}{b}\right)\xi_2}{y_F}\right]^{-b}\right) & \text{if } \xi \in \Omega_F^1 \\ \left(\min\left(\frac{R_F}{y_F}, K_F\right)\frac{\xi_1{}^{-b}}{\xi_1{}^{-b}+\xi_2{}^{-b}},\ \min\left(\frac{R_F}{y_F}, K_F\right)\frac{\xi_2{}^{-b}}{\xi_1{}^{-b}+\xi_2{}^{-b}}\right) & \text{if } \xi \in \Omega_F^2 \end{cases}$$

where

$$\begin{aligned} \Omega_F^1 &\doteq \left\{\xi : 0 \leq \xi_1^{-b} + \xi_2^{-b} < \left[\frac{y_F}{1+\frac{1}{b}}\right]^{-b} \min\left[\frac{R_F}{y_F}, K_F\right]\right\} \\ \Omega_F^2 &\doteq \left\{\xi : \xi_1^{-b} + \xi_2^{-b} \geq \left[\frac{y_F}{1+\frac{1}{b}}\right]^{-b} \min\left[\frac{R_F}{y_F}, K_F\right]\right\}. \end{aligned}$$

As illustrated in Figure 2.2, the resource network for the flexible technology is composed of two resources in series, the financial and the physical resources; hence the total production quantity is bounded by the minimum capacity of these two resources, i.e. $\left(\min\left[\frac{R_F}{y_F}, K_F\right]\right)$. When the demand realization is low, this capacity limit is not binding in optimality, as the firm optimally chooses a production plan such that the marginal profit for each product are identical and equal to the marginal production cost y_F (and does not fully utilize the available capacity). However, when the demand realization is sufficiently high, the firm always optimally uses up to its available net capacity and allocates the resource in such a way that the marginal profits for each product are equal. Ω_F^1 (Ω_F^2) represents the demand region where the firm is not constrained (constrained) by the net capacity. Figure 3.1 illustrates these two regions.

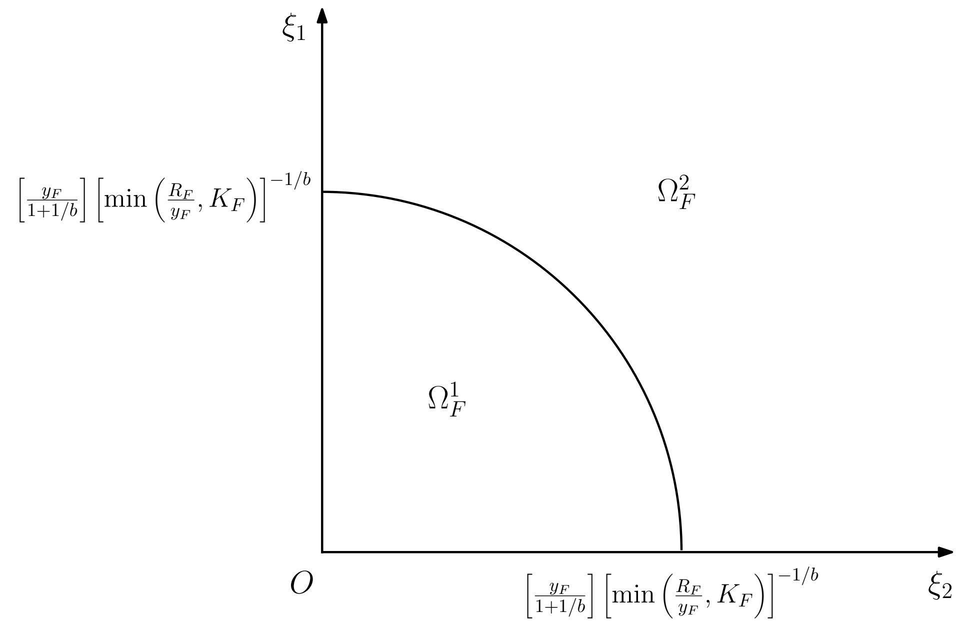


Figure 3.1: (ξ_1, ξ_2) space for stage 2 optimal production with flexible technology.

The corresponding optimal stage 2 profit is given as

$$\Psi_F(K_F, B^2, \xi) = \begin{cases} R_F + \Gamma_F^1(Q_F, B^2, \xi) & \text{if} \quad \xi \in \Omega_F^1 \\ R_F + \Gamma_F^2(Q_F, B^2, \xi) & \text{if} \quad \xi \in \Omega_F^2 \end{cases} \tag{3.1}$$

with

$$\begin{cases} \Gamma_F^1(Q_F, B^2, \xi) \doteq y_F\left(\xi_1^{-b} + \xi_2^{-b}\right)\left[\frac{1+\frac{1}{b}}{y_F}\right]^{-b}\left(\frac{-1}{b+1}\right) \\ \Gamma_F^2(Q_F, B^2, \xi) \doteq \min\left(\frac{R_F}{y_F}, K_F\right)^{1+1/b}\left[\xi_1^{-b} + \xi_2^{-b}\right]^{\frac{-1}{b}} - y_F \min\left(\frac{R_F}{y_F}, K_F\right) \end{cases}$$

3.1.2 Stage 1: Capacity Investment

In this stage, the firm decides the capacity investment level K_F to maximize the expected profit:

$$\begin{aligned} \Pi_F^* \quad &= \quad \max_{K_F} \mathbb{E}\left[\Psi_F(K_F, B^2, \xi)\right] \\ \text{s.t.} \quad & \quad 0 \le K_F \le \frac{B^1 - F_F}{c_F} \end{aligned} \tag{3.2}$$

where the expectation is taken with respect to the demand ξ and stage 2 budget B^2 uncertainties. Let $\Pi_F(K_F)$ denote the expected profit for a given K_F. As B^2 and ξ are assumed to be independent, the expected profit can be rewritten as $\mathbb{E}_{B^2}\left[\mathbb{E}_{\xi}\left[\Psi_F(K_F,B^2,\xi)\right]\right]$. The constraint ensures that the firm can not invest more than the stage 1 initial budget B^1. As illustrated in Proposition 1, for high demand realization ($\xi \in \Omega_F^2$), the firm's production quantities depends on the net capacity constraint $\min\left(\frac{R_F}{y_F},K_F\right)$. We define $\overline{B}_F \doteq (y_F+c_F)K_F-(B^1-F_F)$ as the threshold of the firm's stage 2 internal budget B^2, such that for $B^2>\overline{B}_F$, the firm's net capacity is the physical capacity K_F, i.e. $\min\left(\frac{R_F}{y_F},K_F\right)=K_F$; and for $B^2<\overline{B}_F$, the net capacity is the financial capacity, i.e. $\min\left(\frac{R_F}{y_F},K_F\right)=\frac{R_F}{y_F}$. For analytical convenience, we denote the corresponding Ω_F^i region for $B^2\in[0,\overline{B}_F)$ and $B^2\in[\overline{B}_F,\frac{B^1-F_F}{c_F}]$ as $\Omega^i_{F\,B^2<\overline{B}_F}$ and $\Omega^i_{F\,B^2\geq\overline{B}_F}$ respectively, where

$$\begin{cases} \Omega^1_{F\,B^2<\overline{B}_F} \doteq \left\{\xi : 0\leq \xi_1^{-b}+\xi_2^{-b} < \left[\frac{y_F}{1+\frac{1}{b}}\right]^{-b}\frac{R_F}{y_F}\right\} & \text{if} \quad B^2<\overline{B}_F, \\ \Omega^2_{F\,B^2<\overline{B}_F} \doteq \left\{\xi : \xi_1^{-b}+\xi_2^{-b} \geq \left[\frac{y_F}{1+\frac{1}{b}}\right]^{-b}\frac{R_F}{y_F}\right\} & \end{cases}$$

$$\begin{cases} \Omega^1_{F\,B^2\geq\overline{B}_F} \doteq \left\{\xi : 0\leq \xi_1^{-b}+\xi_2^{-b} < \left[\frac{y_F}{1+\frac{1}{b}}\right]^{-b}K_F\right\} & \text{if} \quad B^2\geq\overline{B}_F. \\ \Omega^2_{F\,B^2\geq\overline{B}_F} \doteq \left\{\xi : \xi_1^{-b}+\xi_2^{-b} \geq \left[\frac{y_F}{1+\frac{1}{b}}\right]^{-b}K_F\right\} & \end{cases}$$

From (3.1) and (3.2), we directly obtain

$$\begin{aligned} \Pi_F(K_F) \quad = \quad & \overline{B}^2+B^1-F_F-c_FK_F \\ & +\int_{B^l}^{\min(\max(B^l,\overline{B}_F),B^u)} G_F^1(K_F,B^2)\,dF(B^2) \\ & +\int_{\min(\max(B^l,\overline{B}_F),B^u)}^{B^u} G_F^2(K_F,B^2)\,dF(B^2). \end{aligned} \tag{3.3}$$

where

$$\begin{aligned} G_F^1(K_F,B^2) \quad = \quad & \iint_{\Omega^1_{F\,B^2<\overline{B}_F}} \left[\left(\xi_1^{-b}+\xi_2^{-b}\right)\left[\frac{1+\frac{1}{b}}{y_F}\right]^{-b}\left(\frac{y_F}{-b-1}\right)\right] \mathrm{d}\Phi(\xi_1,\xi_2) \\ & +\iint_{\Omega^2_{F\,B^2<\overline{B}_F}} \left[\left(\frac{R_F}{y_F}\right)^{(1+1/b)}\left[\xi_1^{-b}+\xi_2^{-b}\right]^{\frac{-1}{b}}-R_F\right] \mathrm{d}\Phi(\xi_1,\xi_2) \\ G_F^2(K_F,B^2) \quad = \quad & \iint_{\Omega^1_{F\,B^2\geq\overline{B}_F}} \left[\left(\xi_1^{-b}+\xi_2^{-b}\right)\left[\frac{1+\frac{1}{b}}{y_F}\right]^{-b}\left(\frac{y_F}{-b-1}\right)\right] \mathrm{d}\Phi(\xi_1,\xi_2) \end{aligned}$$

$$+\iint_{\Omega^2_{F\,B^2\geq \overline{B}_F}} \left[(K_F)^{(1+1/b)} \left[\xi_1^{-b} + \xi_2^{-b} \right]^{\frac{-1}{b}} - y_F K_F \right] \mathrm{d}\Phi(\xi_1, \xi_2)$$

$F(.)$ and $\Phi(.)$ are the the cumulative distribution functions of the stage 2 budget $B^2 \in [B^l, B^u]$ and the demand ξ respectively. Taking the first-order derivative w.r.t. K_F and after some algebra, we obtain

$$\begin{aligned}\frac{\partial \Pi_F}{\partial K_F} &= -c_F + \int_{B^l}^{\min(\max(B^l,\overline{B}_F),B^u)} H_F^1\left(K_F, B^2\right) \mathrm{d}F(B^2) \\ &\quad + \int_{\min(\max(B^l,\overline{B}_F),B^u)}^{B^u} H_F^2\left(K_F, B^2\right) \mathrm{d}F(B^2).\end{aligned} \tag{3.4}$$

where

$$\begin{aligned}H_F^1(K_F,B^2) &= \iint_{\Omega^2_{F\,B^2<\overline{B}_F}} \left[\left(\frac{-c_F}{y_F} \right) (1+1/b) \left(\frac{R_F}{y_F} \right)^{(1/b)} \left[\xi_1^{-b} + \xi_2^{-b} \right]^{\frac{-1}{b}} + c_F \right] \mathrm{d}\Phi(\xi), \\ H_F^2(K_F,B^2) &= \iint_{\Omega^2_{F\,B^2\geq\overline{B}_F}} \left[(1+1/b)(K_F)^{(1/b)} \left[\xi_1^{-b} + \xi_2^{-b} \right]^{\frac{-1}{b}} - y_F \right] \mathrm{d}\Phi(\xi).\end{aligned}$$

Note that $H_F^1(K_F,B^2)$ is the marginal revenue at the states in which the firm is financially constrained at stage 2, i.e. $\frac{R_F}{y_F} \leq K_F$. Therefore, adding one more unit of capacity decreases the net capacity the firm can utilize, and as a result, reduces the marginal revenue at these states. On the other hand, $H_F^2(K_F,B^2)$ represents the marginal revenue at states in which the firm has enough budget to produce all the invested stage 1 physical capacity, i.e. $\frac{R_F}{y_F} > K_F$. Therefore, adding one more unit of capacity increases marginal revenue at these states. The following proposition characterizes the firm's optimal stage 1 capacity investment level K_F^*.

Proposition 2 *With flexible technology, the optimal capacity investment level $K_F^*(B^1)$ for a given stage 1 internal budget level B^1 is characterized by*

$$K_F^*(B^1) = \begin{cases} 0 & \text{if } \left.\frac{\partial \Pi_F}{\partial K_F}\right|_{K_F=0} \leq 0 \\ \frac{B^1 - F_F}{c_F} & \text{if } \left.\frac{\partial \Pi_F}{\partial K_F}\right|_{K_F=\frac{B^1-F_F}{c_F}} \geq 0 \\ \widehat{K}_F & \text{otherwise.} \end{cases}$$

where $\widehat{K}_F \in \left(0, \frac{B^1}{c_F}\right)$ is the unique solution to $\frac{\partial \Pi_F}{\partial K_F} = 0$ as defined in (3.4).

Note that $\widehat{K}_F$ is the optimal capacity investment in the absence of a stage 1 budget constraint (the "stage 1 budget-unconstrained optimal capacity"). If the stage 1 budget B^1 is high enough to cover the corresponding capacity investment cost $c_F\hat{K}_F + F_F$, the firm invests in $K_F^* = \widehat{K}_F$, otherwise, it uses up all the available stage 1 budget for the capacity investment, i.e. $K_F^* = \frac{B^1 - F_F}{c_F}$. In such a case, the production cost at stage 2 is covered by the stage 2 budget B^2 alone. As a special case of this model, when there is no production cost at stage 2, i.e. $y_F = 0$, it is easy to show that the firm optimally invests in $K_F^* = \left(\mathbb{E}\left[\left(\xi_1^{-b} + \xi_2^{-b}\right)^{\frac{-1}{b}}\right]\left(1+\frac{1}{b}\right)\frac{1}{c_F}\right)^{-b}$.

3.2 Stochastic Stage 2 Budget with Clearance Strategy

In this section, we analyze the firm's decision problem under stochastic stage 2 budget and clearance strategy in order to simplify our analysis. With the clearance strategy, the firm always produces up to its available capacity, i.e. $Q_F^1 + Q_F^2 = \min\left(\frac{R_F}{y_F}, K_F\right)$. This assumption is in the spirit of Van Mieghem and Dada (1999) where they have a budget-unconstrained capacity investment problem. With budget constraints, the available capacity is given by the minimum of the financial and physical capacity.

Corollary 1 *With flexible technology, the optimal stage 2 production quantity vector under clearance strategy is given by*

$$\mathbf{Q_F^*}\left(B^2, \xi\right)' = \left[\min\left(\frac{R_F}{y_F}, K_F\right)\frac{{\xi_1}^{-b}}{{\xi_1}^{-b} + {\xi_2}^{-b}},\ \min\left(\frac{R_F}{y_F}, K_F\right)\frac{{\xi_2}^{-b}}{{\xi_1}^{-b} + {\xi_2}^{-b}}\right]$$

The firm optimally allocates its available net capacity $\min\left(\frac{R_F}{y_F}, K_F\right)$ between the two products. For analytical convenience, we define $M_F \doteq \mathbb{E}[(\xi_1^{-b} + \xi_2^{-b})^{-\frac{1}{b}}]$. It follows directly from (3.3) that under clearance strategy, the firm's expected profit Π_F for a given K_F is given by

$$\begin{aligned}\Pi_F(K_F) \quad = \quad & \int_{B^l}^{\min(\max(B^l,\overline{B}_F),B^u)} \left[M_F\left(\frac{R_F}{y_F}\right)^{1+\frac{1}{b}}\right] \mathrm{d}F(B^2) \\ & + \int_{\min(\max(B^l,\overline{B}_F),B^u)}^{B^u} \left[R_F + M_F K_F^{1+\frac{1}{b}} - y_F K_F\right] \mathrm{d}F(B^2).\end{aligned} \tag{3.5}$$

Since clearance strategy is suboptimal, to preserve the concavity of the problem in K_F, we make the following assumption:

Assumption 1 *The stage 1 budget B^1 satisfies $B^1 \leq c_F \left[\frac{(1+\frac{1}{b})M_F}{y_F}\right]^{-b} + F_F$.*

This assumption states that the initial budget B^1 of the firm is not very large. It is easy to show that in the general model, the firm optimally invests in $K_F^* \leq \left[\frac{(1+\frac{1}{b})M_F}{c_F+y_F}\right]^{-b}$ where the equality holds with ample initial stage 1 budget B^1 and the clearance assumption. Since we are interested in analyzing financially constrained firms, imposing Assumption 1 does not create any distortion. With this assumption, the internal stage 1 budget B^1 can still be large enough to support the first-best investment level.

Lemma 1 *The expected profit Π_F is concave in capacity investment level K_F.*

Lemma 1 follows directly from Proposition 2 and Assumption 1. We now obtain the optimality condition for the capacity investment level.

Corollary 2 *Under clearance assumption, the optimality condition in (3.4) is given by*

$$\begin{aligned}\frac{\partial \Pi_F}{\partial K_F} &= -c_F + \int_{B^l}^{\min(\max(B^l,\overline{B}_F),B^u)} \left[M_F\left(\frac{-c_F}{y_F}\right)(1+\frac{1}{b})\left(\frac{R_F}{y_F}\right)^{\frac{1}{b}} + c_F\right] \mathrm{d}F(B^2) \qquad (3.6)\\ &\quad + \int_{\min(\max(B^l,\overline{B}_F),B^u)}^{B^u} \left[M_F(1+\frac{1}{b})(K_F)^{\frac{1}{b}} - y_F\right] \mathrm{d}F(B^2).\end{aligned}$$

where $M_F \doteq \mathbb{E}[(\xi_1^{-b}+\xi_2^{-b})^{-\frac{1}{b}}]$.

We next analyze the impact of stage 2 budget variability on the firm's optimal expected profit and the capacity investment level with the flexible technology

Proposition 3 *Let B^2 and $\underline{B}^2$ be two random variables such that B^2 is stochastically more variable than $\underline{B}^2$. With flexible technology, the firm's optimal expected profit Π_F^* and the optimal capacity investment level K_F^* is larger if the firm is exposed to $\underline{B}^2$ than with B^2.*

It follows that higher stage 2 budget variability is detrimental to the firm's performance. Therefore, the firm is better off by completely removing the stage 2 budget uncertainty. If stage 2 budget depends on a tradable asset that is linked to a financial index, this can be achieved by fully hedging the exposure to this index. It also follows that higher stage 2 budget variability is also detrimental to the firm's optimal capacity investment level with the flexible technology. If the firm is allowed to use financial instruments to engineer its stage 2 budget, then the firm optimally fully hedges and this increases the optimal capacity investment level.

3.3 Deterministic Stage 2 Budget with Clearance Strategy

In this section, we investigate the firm's capacity investment and production quantity decisions under deterministic stage 2 budget with clearance strategy. We continue to assume that Assumption 1 holds. The firm's decision problem can be depicted as follows. In stage 1, in the presence of demand uncertainty (ξ_1, ξ_2), the firm makes its capacity investment K_F with stage 1 budget B^1 and knows that the stage 2 available budget would be B^2. In stage 2, demand uncertainty (ξ_1, ξ_2) is resolved and the firm chooses the production plan Q_F for each product within the net capacity limits.

With clearance strategy, the firm's production quantity Q_F is still characterized by Corollary 1. We now obtain the closed-form expressions for stage 1 optimal capacity investment level K_F^* and the associated expected profit function Π_F^*, which are depicted as in Proposition 4 and Corollary 3 respectively.

Proposition 4 *With deterministic stage 2 budget B^2 and clearance assumption, the firm's optimal stage 1 capacity investment level K_F^* is*

$$K_F^* = K_F^0 = \left[\frac{\left(1+\frac{1}{b}\right)M_F}{c_F + y_F}\right]^{-b} \qquad if\ B^2 \in [0, +\infty) \tag{3.7}$$

for $B^1 \in \left((c_F + y_F)K_F^0 + F_F, +\infty\right)$; and

$$K_F^* = \begin{cases} \overline{K}_F = \frac{B^2 + B^1 - F_F}{c_F + y_F} & if\ B^2 \in [0, \widehat{B}_F^2] \\ K_F^0 = \left[\frac{\left(1+\frac{1}{b}\right)M_F}{c_F + y_F}\right]^{-b} & if\ B^2 \in (\widehat{B}_F^2, +\infty) \end{cases} \tag{3.8}$$

for $B^1 \in \left(c_F K_F^0 + F_F, (c_F + y_F)K_F^0 + F_F\right]$; *and*

$$K_F^* = \begin{cases} \overline{K}_F = \frac{B^2+B^1-F_F}{c_F+y_F} & \text{if } B^2 \in \left[0, y_F\left(\frac{B^1-F_F}{c_F}\right)\right] \\ \frac{B^1-F_F}{c_F} & \text{if } B^2 \in \left(y_F\left(\frac{B^1-F_F}{c_F}\right), +\infty\right) \end{cases} \tag{3.9}$$

for $B^1 \in \left[0, c_F K_F^0 + F_F\right]$, *where* $\widehat{B}_F^2 \doteq (c_F + y_F)K_F^0 - (B^1 - F_F)$.

In Proposition 4, K_F^0 denotes the firm's optimal capacity investment in the absence of a stage 1 budget constraint (i.e. the "stage 1 budget-unconstrained optimal capacity") whereas $\overline{K}_F = \frac{B^2+B^1-F_F}{c_F+y_F}$ is the capacity investment level such that the firm's physical capacity investment $\overline{K}_F$ matches exactly the financial capacity $\frac{R_F}{y_F}$ at the production stage. $\frac{B^1-F_F}{c_F}$ represents the firm's stage 1 capacity investment limit.

The intuition behind Proposition 4 is straightforward: If the stage 1 budget B^1 is higher than the threshold $(c_F + y_F)K_F^0 + F_F$, then the firm invests in K_F^0 regardless of stage 2 budget B^2. Because the stage 1 budget alone can cover both the stage 1 capacity investment cost $c_F K_F^0 + F_F$ and the stage 2 production cost $y_F K_F^0$. For $B^1 \leq (c_F + y_F)K_F^0 + F_F$, the firm is stage 1 budget-constrained and the optimal capacity investment decision also depends on the stage 2 budget B^2. For a moderate stage 1 budget such that $B^1 \in \left(c_F K_F^0 + F_F, (c_F + y_F)K_F^0 + F_F\right]$, if the stage 2 budget B^2 is lower than $\widehat{B}_F^2$ (i.e. $B^1 + B^2 < (c_F + y_F)K_F^0 + F_F$), the firm cannot invest in K_F^0 and can only invest in $\overline{K}_F$ to maximize the available net capacity $\left(\frac{R_F}{y_F}, K_F\right)$ for stage 2 production. If the stage 2 budget is higher than $\widehat{B}_F^2$, the firm optimally invest in K_F^0 because the total available budget $B^1 + B^2$ can cover the capacity investment and the associated production cost $(c_F + y_F)K_F^0 + F_F$ that cannot be covered by stage 1 budget B^1 alone. For stage 1 budget less than $c_F K_F^0 + F_F$, the firm can never invest in K_F^0 regardless of stage 2 budget B^2. In such a case, if $B^2 \leq y_F\left(\frac{B^1-F_F}{c_F}\right)$, both stage 1 and stage 2 budgets are limited, the firm optimally invests in $\overline{K}_F$ to maximize the stage 2 net capacity for production. If $B^2 > y_F\left(\frac{B^1-F_F}{c_F}\right)$, then the firm uses up all of its stage 1 budget for capacity investment because the associated production cost $y_F\left(\frac{B^1-F_F}{c_F}\right)$ can be covered by stage 2 budget alone.

It follows from Proposition 4 that with deterministic stage 2 budget B^2 and clearance assumption, the firm's optimal expected profit Π_F^* is given by Corollary 3.

Corollary 3 *With deterministic stage 2 budget B^2 and clearance assumption, the firm's optimal expected profit Π_F is*

$$\Pi_F^* = B^2 + B^1 - F_F + \frac{c_F + y_F}{-(b+1)} K_F^0 \qquad if\ B^2 \in [0, +\infty) \tag{3.10}$$

for $B^1 \in \big((c_F + y_F)K_F^0 + F_F, +\infty\big)$; and

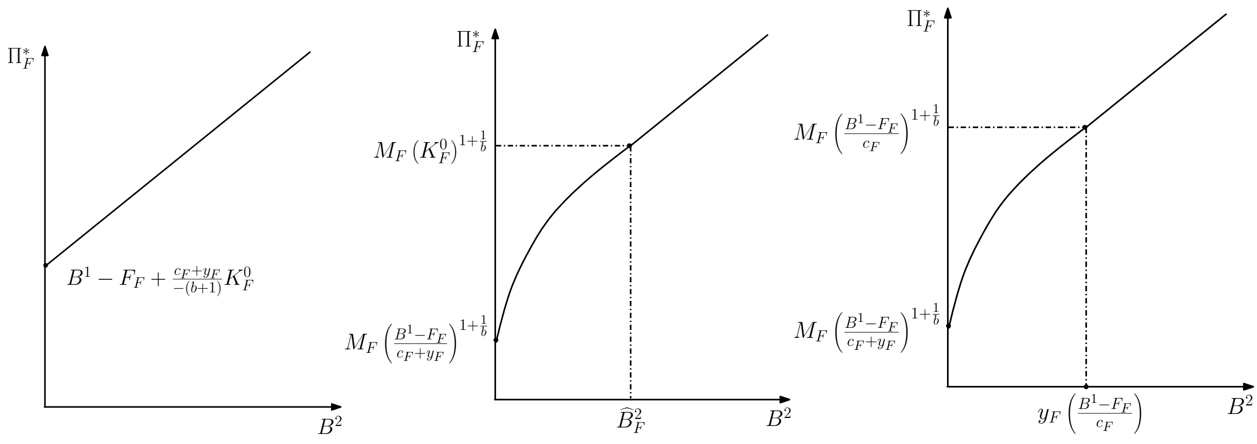
$$\Pi_F^* = \begin{cases} M_F \overline{K}_F^{1+\frac{1}{b}} & \text{if } B^2 \in [0, \widehat{B}_F^2] \\ B^2 + B^1 - F_F + \frac{(c_F+y_F)}{-(b+1)} K_F^0 & \text{if } B^2 \in (\widehat{B}_F^2, +\infty) \end{cases} \tag{3.11}$$

for $B^1 \in \big(c_F K_F^0 + F_F, (c_F + y_F)K_F^0 + F_F\big]$; and

$$\Pi_F^* = \begin{cases} M_F \overline{K}_F^{1+\frac{1}{b}} & \text{if } B^2 \in \left[0, y_F\left(\frac{B^1 - F_F}{c_F}\right)\right] \\ B^2 + M_F\left(\frac{B^1 - F_F}{c_F}\right)^{1+\frac{1}{b}} - y_F\left(\frac{B^1 - F_F}{c_F}\right) & \text{if } B^2 \in \left(y_F\left(\frac{B^1 - F_F}{c_F}\right), +\infty\right) \end{cases} \tag{3.12}$$

for $B^1 \in \big[0, c_F K_F^0 + F_F\big]$, where $K_F^0 = \left[\frac{(1+\frac{1}{b})M_F}{c_F+y_F}\right]^{-b}$, $\overline{K}_F = \frac{B^2+B^1-F_F}{c_F+y_F}$ and $M_F = \mathbb{E}[(\xi_1^{-b} + \xi_2^{-b})^{-\frac{1}{b}}]$.

The following three Figures illustrates Π_F^* of Corollary 3.



(a) Π_F in (3.10) (b) Π_F in (3.11) (c) Π_F in (3.12)

Figure 3.2: Optimal expected profit Π_F^*

Chapter 4

Dedicated Technology

In this chapter, we turn our attention to analyze the firm's decision problem with dedicated technology. We will first provide the analysis for the firm's capacity investment and production decisions with stochastic stage 2 budget B^2 in §4.1. In §4.2, we investigate the same with marketing clearing price assumption of dedicated technology. Finally, in §4.3, we analyze the firm's decision problem with deterministic stage 2 budget under the clearance price assumption.

We first make an important remark that we will use throughout this section. Since ξ has a symmetric bivariate normal distribution, i.e. $\overline{\xi}_1 = \overline{\xi}_2$ and $\sigma_1 = \sigma_2$, the firm optimally invests the same amount for each product capacity, i.e. $K_D^{1^*} = K_D^{2^*}$. Thus we can use a single capacity K_D^* to characterize $K_D^* = (K_D^*; K_D^*)$. We also note that R_D yields $(B^2 + B^1 - F_D - 2c_D K_D)$. In the rest of the analysis, we drop the vector notation and use K_D^* instead.

4.1 Stochastic Stage 2 Budget

In this section, we solve for the firm's optimal capacity investment and production decisions with dedicated technology.

4.1.1 Stage 2: Production Decision

In stage 1, the firm with initial budget B^1 invested in capacity level K_F and has $B^1 - c_F K_F - F_F$ amount of internal capital left. In this stage, the firm observes the demand ξ and stage 2 budget B^2. This stage 2 budget, together with the internal capital left from stage 1 investment determines the internal capital R_D of the firm at this stage. The firm determines the production quantities $Q'_D = (Q_D^1, Q_D^2)$ within the existing flexible physical capacity K_F and financial capacity $\frac{R_D}{y_D}$ to maximize the stage 2 profit Ψ_D with the dedicated technology.

Proposition 5 *For a given B^2, K_D and ξ, the optimal production quantity vector in stage 2 with dedicated technology $Q_D^* = \left(Q_D^1, Q_D^2\right)$ is given by*

$$\mathbf{Q_D^*}\left(B^2,\xi\right)' = \begin{cases} \left(\left[\frac{\left(1+\frac{1}{b}\right)\xi_1}{y_D}\right]^{-b}, \left[\frac{\left(1+\frac{1}{b}\right)\xi_2}{y_D}\right]^{-b}\right) & if \;\; \xi \in \Omega_D^1 \\ \left(K_D, \left[\frac{\left(1+\frac{1}{b}\right)\xi_2}{y_D}\right]^{-b}\right) & if \;\; \xi \in \Omega_D^2 \\ \left(\left[\frac{\left(1+\frac{1}{b}\right)\xi_1}{y_D}\right]^{-b}, K_D\right) & if \;\; \xi \in \Omega_D^3 \\ \left(K_D, \min\left[\left(\frac{R_D}{y_D} - K_D\right)^+, K_D\right]\right) & if \;\; \xi \in \Omega_D^4 \\ \left(\min\left[\left(\frac{R_D}{y_D} - K_D\right)^+, K_D\right], K_D\right) & if \;\; \xi \in \Omega_D^5 \\ \left(\frac{R_D}{y_D}\left[\frac{\xi_1{}^{-b}}{\xi_1{}^{-b}+\xi_2{}^{-b}}\right], \frac{R_D}{y_D}\left[\frac{\xi_2{}^{-b}}{\xi_1{}^{-b}+\xi_2{}^{-b}}\right]\right) & if \;\; \xi \in \Omega_D^6 \end{cases} \tag{4.1}$$

where

$$\Omega_D^1 \doteq \xi : \begin{cases} \xi_1^{-b} + \xi_2^{-b} < \frac{R_D}{y_D}\left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \\ \xi_1 \le K_D{}^{-1/b}\left[\frac{y_D}{1+\frac{1}{b}}\right] \\ \xi_2 \le K_D{}^{-1/b}\left[\frac{y_D}{1+\frac{1}{b}}\right] \end{cases}$$

$$\Omega_D^2 \doteq \xi : \begin{cases} \xi_1 \ge K_D^1{}^{-1/b}\left[\frac{y_D}{1+\frac{1}{b}}\right] \\ \xi_2 \le \min\left(\left(\frac{R_D}{y_D} - K_D\right)^+, K_D\right)^{-1/b}\left[\frac{y_D}{1+\frac{1}{b}}\right] \end{cases}$$

$$\Omega_D^3 \doteq \xi : \begin{cases} \xi_1 \leq \min\left(\left(\frac{R_D}{y_D} - K_D\right)^+, K_D\right)^{-1/b} \left[\frac{y_D}{1+\frac{1}{b}}\right] \\ \xi_2 \geq K_D{}^{-1/b} \left[\frac{y_D}{1+\frac{1}{b}}\right] \end{cases}$$

$$\Omega_D^4 \doteq \xi : \begin{cases} \xi_2 \geq \min\left(\left(\frac{R_D}{y_D} - K_D\right)^+, K_D\right)^{-1/b} \left[\frac{y_D}{1+\frac{1}{b}}\right] \\ \xi_1 \min\left(\left(\frac{R_D}{y_D} - K_D\right)^+, K_D\right)^{-1/b} \geq \xi_2 K_D{}^{-1/b} \end{cases}$$

$$\Omega_D^5 \doteq \xi : \begin{cases} \xi_1 \geq \min\left(\left(\frac{R_D}{y_D} - K_D\right)^+, K_D\right)^{-1/b} \left[\frac{y_D}{1+\frac{1}{b}}\right] \\ \xi_2 \min\left(\left(\frac{R_D}{y_D} - K_D\right)^+, K_D^1\right)^{-1/b} \geq \xi_1 K_D{}^{-1/b} \end{cases}$$

$$\Omega_D^6 \doteq \xi : \begin{cases} \xi_1 \min\left(\left(\frac{R_D}{y_D} - K_D\right)^+, K_D\right)^{-1/b} \leq \xi_2 K_D^{-1/b} \\ \xi_2 \min\left(\left(\frac{R_D}{y_D} - K_D\right)^+, K_D\right)^{-1/b} \leq \xi_1 K_D^{-1/b} \\ \xi_1^{-b} + \xi_2^{-b} \geq \frac{R_D}{y_D} \left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \end{cases}$$

As illustrated in Figure 2.2, the resource network for dedicated technology is composed of two physical resources, one for each technology, and a financial resource that is connected to both in series. Therefore, when the stage 2 total capital level R_D, i.e. the financial resource, is sufficiently large, the financial capacity is not binding in optimality and the production quantity for each product is only limited by its physical capacity K_D. On the other hand, for a low capital level R_D, apart from the physical capacity constraint on each product, the firm's production decision is also constrained by the financial capacity $\frac{R_D}{y_D}$.

Ω_D^1 represents the demand region where the demand realization is too low that it is not optimal for the firm to use up any of its available resources up to their capacities for production. Thus, the firm chooses a production plan such that the marginal profit is identical for both products and equals the marginal production cost y_D. Ω_D^{24} (Ω_D^{35}) is the demand region where product 1 (product 2) is more profitable than the other product. Hence, the firm optimally prioritizes its production until the physical capacity is used up. In Ω_D^2 (Ω_D^3), the demand for product 2 (product 1) is too low that after prioritizing production for the other product, it is never optimal for the firm to use up all the remaining resources for its production. On the other hand, in Ω_D^4 (Ω_D^5), the firm optimally fully utilizes all its remaining resources for production of product 2 (product 1), as its demand is sufficiently high. Ω_D^6 is the demand region where both demands are sufficiently high and the firm is budget constrained. Hence, the firm fully utilized its financial capacity and allocates this capacity in such a way that the marginal profit is identical for both products. Figure 4.1, 4.2 and 4.3 illustrate these demand regions with respect to different stage 2 budget realizations.

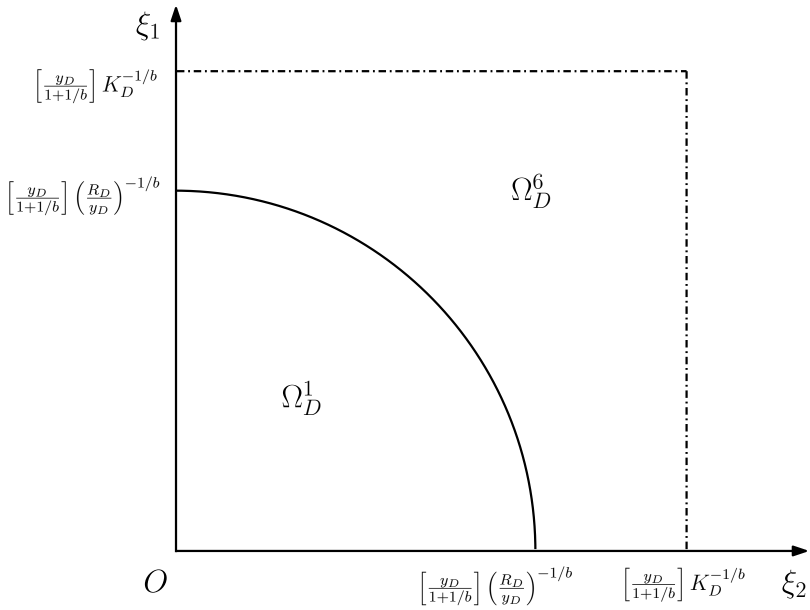


Figure 4.1: (ξ_1, ξ_2) Space for stage 2 production with dedicated technology for $\frac{R_D}{y_D} < K_D$.

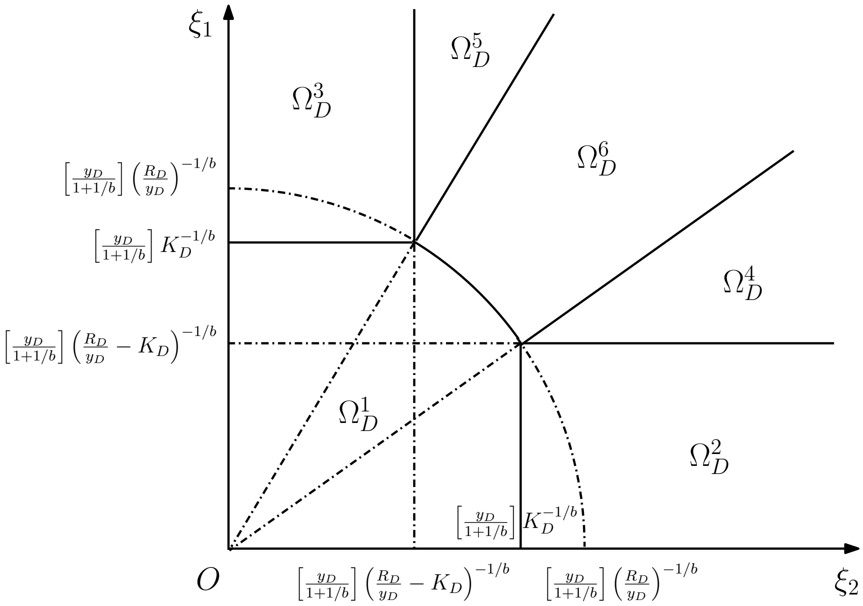


Figure 4.2: (ξ_1, ξ_2) Space for stage 2 production with dedicated technology for $K_D \leq \frac{R_D}{y_D} < 2K_D$.

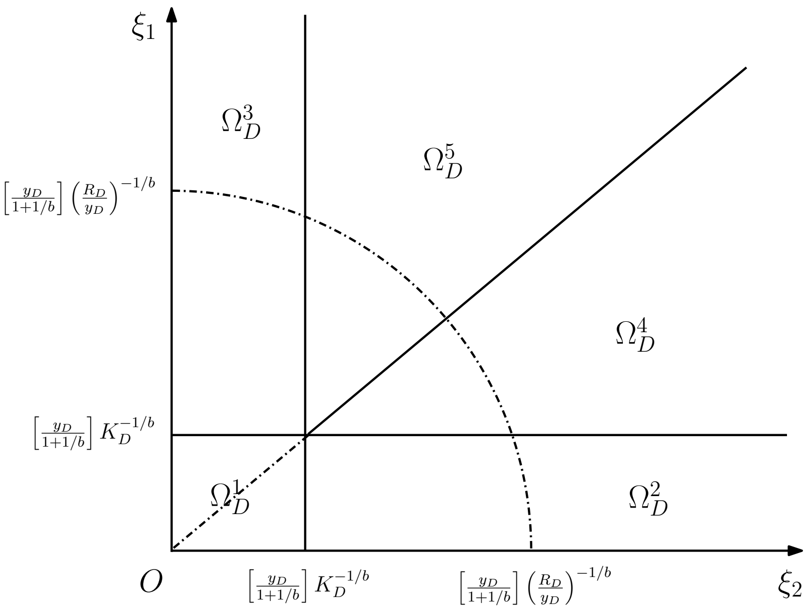


Figure 4.3: (ξ_1, ξ_2) Space for stage 2 production with dedicated technology for $\frac{R_D}{y_D} \geq 2K_D$.

4.1.2 Stage 1: Capacity Investment

In this stage, the firm decides the capacity investment level K_D to maximize the expected profit:

$$\begin{aligned} \Pi_D^* &= \max_{K_D} \mathbb{E}\left[\Psi_D(K_D, B^2, \xi)\right] \\ \text{s.t.} &\quad 0 \leq K_D \leq \frac{B^1 - F_D}{2c_D} \end{aligned} \tag{4.2}$$

where the expectation is taken over demand ξ and stage 2 budget B^2 uncertainties. Let $\Pi_D(K_D)$ denote the expected profit for a given K_D. The constraint ensures that the firm does invest more than the available stage 1 budget B^1. As illustrated in Proposition 5, the optimal stage 2 profit Ψ_D take different forms with respect to the ordering of K_D, the total capacity investment $2K_D$ for both products and the available financial capacity $\frac{R_D}{y_D}$. Similar to the flexible technology case, we define $\underline{B}_D \doteq (2c_D + y_D)K_D - (B^1 - F_D)$ and $\overline{B}_D \doteq (2c_D + 2y_D)K_D - (B^1 - F_D)$ as the two critical thresholds of stage 2 budget realization respectively.

For a sufficiently low stage 2 budget such that $B^2 \leq \underline{B}_D$ we have $\frac{R_D}{y_D} < K_D$, the firm can not afford full capacity K_D production for any product and only Ω_D^1 and Ω_D^6 regions exist. On the other hand, for a sufficiently high stage 2 budget level such that $B^2 \geq \overline{B}_D$, the financial constraint is no more an issue for stage 2 production, i.e. $\frac{R_D}{y_D} \geq 2K_D$, then Ω_D^6 region will vanish and Ω_D^4 and Ω_D^5 regions merge together. While for a moderate stage 2 budget level $\underline{B}_D \leq B^2 \leq \overline{B}_D$ such that $K_D \leq \frac{R_D}{y_D} \leq 2K_D$, the firm is able to produce full capacity K_D for either of the products, but not full capacity $2K_D$ for both products. For analytical convenience, we denote Ω_D^i regions for $B^2 < \underline{B}_D$, $\underline{B}_D \leq B^2 \leq \overline{B}_D$ and $B^2 > \overline{B}_D$ as $\Omega^i_{DB^2<\underline{B}_D}$, $\Omega^i_{D\underline{B}_D\leq B^2\leq\overline{B}_D}$ and $\Omega^i_{DB^2>\overline{B}_D}$ respectively.

In this stage 1 optimization problem, the firm faces the trade-off between the risk of over-investing when the stage 2 budget realization is sufficiently low ($B^2 < \underline{B}_D$) versus the possibility of revenue loss when the stage 2 budget realization is sufficiently high ($B^2 \geq \overline{B}_D$). When stage 2 budget is sufficiently high, the expected marginal profit is always positive, as the financial constraint is no longer a factor and the firm operates in a budget-unconstrained setting. When stage 2 budget is sufficiently low, the production is constrained by the financial capacity, and adding additional units of physical capacity in stage 1 will further limit the budget the firm can utilize for production in stage 2, and thus marginal revenue is negative at these states. The effect, however, is not clear with moderate stage 2 budget. When one product demand dominates (and thus is prioritized for full capacity production), the firm benefits from an additional unit of physical capacity for that product. On the other hand, adding physical capacity in stage 1 may reduce the available budget for production of the dominated product in stage 2, at states where the dominated product demand is sufficiently high (but is still dominated) so that it is optimal for the firm to use up all the available budget for its production (after producing full capacity of the dominating product). In such cases, the marginal loss in the dominated product market will always be higher than the marginal profit in the dominating product market, and the marginal expected revenue is negative. This argument does not continue to hold in states in which the dominated product demand is low, as its production is not constrained by financial capacity, and thus, the firm benefits from an additional physical capacity, and the expected marginal revenue is positive.

Proposition 6 *With dedicated technology, the optimal capacity investment level $K_D^*(B^1)$ for a given stage 1 budge level B^1 is characterized by*

$$K_D^*(B^1) = \begin{cases} 0 & \text{if } \left.\frac{\partial \Pi_D}{\partial K_D}\right|_{K_D=0} \leq 0, \\ \frac{B^1 - F_D}{2c_D} & \text{if } \left.\frac{\partial \Pi_D}{\partial K_D}\right|_{K_D=\frac{B^1-F_D}{2c_D}} \geq 0, \\ \widehat{K}_D & \text{otherwise} \end{cases} \tag{4.3}$$

where $\widehat{K}_D \in \left(0, \frac{B^1-F_D}{2c_D}\right)$ is the unique solution to $\frac{\partial \Pi_D}{\partial K_D} = 0$ as defined in (6.55).

The explicit expressions for the optimality condition is given in (6.55) in the proof of Proposition 6. Note that $\widehat{K}_D$ is the optimal capacity investment in the absence of a stage 1 budget constraint (the “stage 1 budget-unconstrained optimal capacity”). If the stage 1 budget B^1 is high enough to cover the capacity investment cost $2c_D\widehat{K}_D + F_D$, the firm optimally invests in $K_D^* = \widehat{K}_D$, otherwise, it uses up all the available budget for the capacity investment, i.e. $\left(K_D^* = \frac{B^1-F_D}{2c_D}\right)$. In such a case, $R_D = B^2$ and the production cost is covered by the stage 2 budget alone. When the stage 2 production is costless, i.e. $y_D = 0$, it is easily obtained that the firm's optimal capacity investment level is $K_D^* = \left(\frac{\overline{\xi}\left(1+\frac{1}{b}\right)}{c_D}\right)^{-b}$.

4.2 Stochastic Stage 2 Budget with Clearance Strategy

In this section, we focus on the firm's decision problem under stochastic stage 2 budget and clearance price assumption. With the clearance strategy, *i*) when the financial capacity is not sufficient to finance one full capacity investment, i.e. $\frac{R_D}{y_D} < K_D$, the firm uses its full financial capacity $\frac{R_D}{y_D}$ and optimally allocates this capacity between the two products; *ii*) when the financial capacity is sufficient to finance both products up to full capacity, i.e. $\frac{R_D}{y_D} \geq 2K_D$, the firm optimally produces up to full capacity K_D for each product; and *iii*) otherwise, i.e. $K_D \leq \frac{R_D}{y_D} < 2K_D$, the firm either allocates K_D to one product and $\frac{R_D}{y_D} - K_D$ to the other, or optimally allocates $\frac{R_D}{y_D}$ between the two products. It follows directly from Proposition 5 and the clearance price assumption that under dedicated technology the firm's stage 2 optimal production plan $\mathbf{Q}^*_\mathbf{D}(B^2, \xi)$ is characterized by Corollary 4.

Corollary 4 *With dedicated technology, the optimal stage 2 production quantity vector under clearance strategy is given by*

$$\mathbf{Q}^*_\mathbf{D}\left(B^2,\xi\right)' = \begin{cases} \left(\frac{R_D}{y_D}\left[\frac{\xi_1{}^{-b}}{\xi_1{}^{-b}+\xi_2{}^{-b}}\right], \frac{R_D}{y_D}\left[\frac{\xi_2{}^{-b}}{\xi_1{}^{-b}+\xi_2{}^{-b}}\right]\right) & \text{for } \frac{R_D}{y_D} < K_D \\ (K_D, K_D) & \text{for } \frac{R_D}{y_D} \geq 2K_D \end{cases} \tag{4.4}$$

and for $K_D \leq \frac{R_D}{y_D} < 2K_D$,

$$\mathbf{Q}^*_\mathbf{D}\left(B^2,\xi\right)' = \begin{cases} \left(K_D, \frac{R_D}{y_D} - K_D\right) & \text{if } (\xi_1,\xi_2) \in \Omega_D^{1c} \\ \left(\frac{R_D}{y_D} - K_D, K_D\right) & \text{if } (\xi_1,\xi_2) \in \Omega_D^{2c} \\ \left(\frac{R_D}{y_D}\left[\frac{\xi_1{}^{-b}}{\xi_1{}^{-b}+\xi_2{}^{-b}}\right], \frac{R_D}{y_D}\left[\frac{\xi_2{}^{-b}}{\xi_1{}^{-b}+\xi_2{}^{-b}}\right]\right) & \text{if } (\xi_1,\xi_2) \in \Omega_D^{3c} \end{cases} \tag{4.5}$$

where

$$\begin{aligned} \Omega_D^{1c} &\doteq \left\{\xi : \xi_1^{-b} \geq \left[\frac{K_D}{\frac{R_D}{y_D} - K_D}\right]\xi_2^{-b}\right\} \\ \Omega_D^{2c} &\doteq \left\{\xi : \xi_1^{-b} < \left[\frac{\frac{R_D}{y_D} - K_D}{K_D}\right]\xi_2^{-b}\right\} \\ \Omega_D^{3c} &\doteq \left\{\xi : \left[\frac{\frac{R_D}{y_D} - K_D}{K_D}\right]\xi_2^{-b} \leq \xi_1^{-b} < \left[\frac{K_D}{\frac{R_D}{y_D} - K_D}\right]\xi_2^{-b}\right\}. \end{aligned}$$

Figure 4.4 illustrates the Ω_D^{ic} regions for $i = 1,2,3$ under clearance assumption.

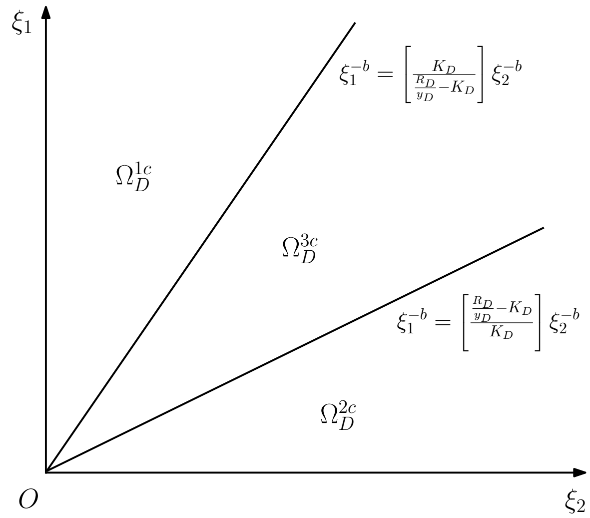


Figure 4.4: (ξ_1, ξ_2) space for stage 2 optimal production under clearance assumption.

In Ω_D^{1c} (Ω_D^{2c}) region, product 1 (product 2) is more profitable than the other product, thus, it is optimal for the firm to prioritize its financial capacity $\frac{R_D}{y_D}$ for product 1 (product 2) until the physical capacity K_D is used up. In Ω_D^{3c} region, the firm optimally partitions its financial capacity $\frac{R_D}{y_D}$ between the two products depending on the demand realization. It follows from Corollary 4 that the firm's expected profit Π_D for a given K_D is characterized by

$$\begin{aligned}\Pi_D(K_D) &= \int_{B^l}^{\min(\max(B^l,\underline{B}_D),B^u)} \left[G_D^{1c}\right] \mathrm{d}F(B^2) \\ &+ \int_{\min(\max(B^l,\underline{B}_D),B^u)}^{\min(\max(B^l,\overline{B}_D),B^u)} \left[G_D^{2c}\right] \mathrm{d}F(B^2) \\ &+ \int_{\min(\max(B^l,\overline{B}_D),B^u)}^{B^u} \left[G_D^{3c}\right] \mathrm{d}F(B^2)\end{aligned} \tag{4.6}$$

where

$$\begin{aligned}G_D^{1c} &= \left(\frac{R_D}{y_D}\right)^{1+\frac{1}{b}} \mathbb{E}\left[(\xi_1^{-b}+\xi_2^{-b})^{-\frac{1}{b}}\right] \\ G_D^{2c} &= \iint_{\Omega_D^{1c}} \left[\xi_1 K_D^{1+\frac{1}{b}} + \xi_2\left(\frac{R_D}{y_D}-K_D\right)^{1+\frac{1}{b}}\right] \mathrm{d}\Phi(\xi_1,\xi_2) \\ &+ \iint_{\Omega_D^{2c}} \left[\xi_1\left(\frac{R_D}{y_D}-K_D\right)^{1+\frac{1}{b}} + \xi_2 K_D^{1+\frac{1}{b}}\right] \mathrm{d}\Phi(\xi_1,\xi_2)\end{aligned}$$

$$
\begin{aligned}
& + \iint_{\Omega_D^{3c}} \left[\left(\frac{R_D}{y_D} \right)^{1+\frac{1}{b}} (\xi_1^{-b} + \xi_2^{-b})^{-\frac{1}{b}} \right] \mathrm{d}\Phi(\xi_1, \xi_2) \\
G_D^{3c} &= R_D + 2\overline{\xi} K_D^{1+\frac{1}{b}} - 2 y_D K_D
\end{aligned}
$$

Since clearance strategy is suboptimal, to preserve the concavity of the problem in K_D, paralleling Assumption 1 with the flexible technology, we make the following assumption:

Assumption 2 *The stage 1 budget B^1 satisfies $B^1 \leq 2c_D \left[\frac{(1+\frac{1}{b})\mathbb{E}[\min(\xi_1,\xi_2)]}{y_D} \right]^{-b} + F_D$.*

Lemma 2 *The expected profit Π_D is concave in capacity investment level K_D.*

Lemma 2 follows directly from Proposition 6 and Assumption 2. We now obtain the optimality condition for the capacity investment level.

Corollary 5 *Under clearance assumption, the optimality condition in (4.3) is given by*

$$
\begin{aligned}
\frac{\partial \Pi_D}{\partial K_D} &= \int_{B^l}^{\min(\max(B^l, \underline{B}_D), B^u)} \left[\frac{\partial G_D^{1c}}{\partial K_D} \right] \mathrm{d}F(B^2) \\
&+ \int_{\min(\max(B^l, \underline{B}_D), B^u)}^{\min(\max(B^l, \overline{B}_D), B^u)} \left[\frac{\partial G_D^{2c}}{\partial K_D} \right] \mathrm{d}F(B^2) \\
&+ \int_{\min(\max(B^l, \overline{B}_D), B^u)}^{B^u} \left[\frac{\partial G_D^{3c}}{\partial K_D} \right] \mathrm{d}F(B^2)
\end{aligned} \tag{4.7}
$$

where

$$
\begin{aligned}
\frac{\partial G_D^{1c}}{\partial K_D} &= \left(1 + \frac{1}{b}\right) \left(\frac{R_D}{y_D} \right)^{\frac{1}{b}} \left(\frac{-2c_D}{y_D} \right) \mathbb{E} \left[(\xi_1^{-b} + \xi_2^{-b})^{-\frac{1}{b}} \right] \\
\frac{\partial G_D^{2c}}{\partial K_D} &= \iint_{\Omega_D^{1c}} \left[\xi_1 \left(1 + \frac{1}{b}\right) K_D^{\frac{1}{b}} - \xi_2 \left(1 + \frac{1}{b}\right) \left(\frac{R_D}{y_D} - K_D \right)^{\frac{1}{b}} \left(\frac{2c_D}{y_D} + 1 \right) \right] \mathrm{d}\Phi(\xi_1, \xi_2) \\
&+ \iint_{\Omega_D^{2c}} \left[-\xi_1 \left(1 + \frac{1}{b}\right) \left(\frac{R_D}{y_D} - K_D \right)^{\frac{1}{b}} \left(\frac{2c_D}{y_D} + 1 \right) + \xi_2 \left(1 + \frac{1}{b}\right) K_D^{\frac{1}{b}} \right] \mathrm{d}\Phi(\xi_1, \xi_2) \\
&+ \iint_{\Omega_D^{3c}} \left[\left(1 + \frac{1}{b}\right) \left(\frac{R_D}{y_D} \right)^{\frac{1}{b}} \left(\frac{-2c_D}{y_D} \right) (\xi_1^{-b} + \xi_2^{-b})^{-\frac{1}{b}} \right] \mathrm{d}\Phi(\xi_1, \xi_2) \\
\frac{\partial G_D^{3c}}{\partial K_D} &= -2c_D + 2\overline{\xi} \left(1 + \frac{1}{b}\right) K_D^{\frac{1}{b}} - 2y_D
\end{aligned}
$$

We next analyze the impact of stage 2 budget variability on the firm's optimal expected profit with dedicated technology.

Proposition 7 *Let B^2 and $\underline{B}^2$ be two random variables such that B^2 is stochastically more variable than $\underline{B}^2$. With dedicated technology, the firm's optimal expected profit Π_D^* is larger if the firm is exposed to $\underline{B}^2$ than with B^2.*

It follows that higher stage 2 budget variability is detrimental to the firm's performance with dedicated technology. Therefore, the firm is better off by completely removing the stage 2 budget uncertainty. If the firm is allowed to use financial instruments to engineer its stage 2 budget, then the firm optimally fully hedges and this increases the firm's expected profit. Unfortunately, same result cannot be proven to hold for the capacity investment level. Nevertheless, in general, we expect the optimal capacity investment level with the dedicated technology to be decreasing with stage 2 budget variability.

4.3 Deterministic Stage 2 Budget with Clearance Strategy

In this section, we investigate the firm's capacity investment and production quantity decisions under deterministic stage 2 budget with clearance strategy. We continue to assume that Assumption 2 holds. The firm's decision problem can be depicted as follows. In stage 1, in the presence of demand uncertainty (ξ_1, ξ_2), the firm makes its capacity investment K_D with stage 1 budget B^1 and knows that the stage 2 available budget would be B^2. In stage 2, demand uncertainty (ξ_1, ξ_2) is resolved and the firm chooses the production plan Q_D for each product within the financial capacity $\frac{R_D}{y_D}$ and physical capacity K_D for each product.

With clearance strategy, the firm's optimal stage 2 production plan Q_D^* is given by Corollary 4. We now characterize the firm's stage 1 capacity investment decision K_D^*. We impose the following assumption to simplify our analysis.

Assumption 3 $\mathbb{E}[\max(\xi_1, \xi_2)] \leq \mathbb{E}[\min(\xi_1, \xi_2)] \left(\frac{2c_D}{y_D} + 1\right)$

Under Assumption 3, we restrict the firm's optimal capacity investment level such that it is never optimal for the firm to invest in a stage 1 capacity that is more than $\underline{K}_D = \frac{B^2+B^1-F_D}{2c_D+2y_D}$, where $\underline{K}_D$ makes the firm's total physical capacity investment $2\underline{K}_D$ identical to the financial capacity $\frac{R_D}{y_D}$.

Proposition 8 *With deterministic stage 2 budget B^2 and clearance assumption, with Assumption 3, the firm's optimal stage 1 capacity investment level K_D^* is*

$$K_D^* = K_D^0 \quad if\ B^2 \in [0, +\infty) \tag{4.8}$$

for $B^1 \in \big(2(c_D+y_D)K_D^0 + F_D, +\infty\big)$; and

$$K_D^* = \begin{cases} \underline{K}_D & if\ B^2 \in [0, \widehat{B}_D^2] \\ K_D^0 & if\ B^2 \in (\widehat{B}_D^2, +\infty) \end{cases} \tag{4.9}$$

for $B^1 \in \big(2c_D K_D^0 + F_D, 2(c_D+y_D)K_D^0 + F_D\big]$; and

$$K_D^* = \begin{cases} \underline{K}_D & if\ B^2 \in \left[0, y_D\left(\frac{B^1-F_D}{c_D}\right)\right] \\ \frac{B^1-F_D}{2c_D} & if\ B^2 \in \left(y_D\left(\frac{B^1-F_D}{c_D}\right), +\infty\right) \end{cases} \tag{4.10}$$

for $B^1 \in \big[0, 2c_D K_D^0 + F_D\big]$, where $\widehat{B}_D^2 \doteq 2(c_D+y_D)K_D^0 - (B^1 - F_D)$.

Here, $K_D^0 = \left[\frac{\bar{\xi}\left(1+\frac{1}{b}\right)}{c_D+y_D}\right]^{-b}$ is the firm's optimal capacity investment in the absence of a stage 1 budget constraint (i.e. the "stage 1 budget-unconstrained optimal capacity"). $\underline{K}_D = \frac{B^2+B^1-F_D}{2c_D+2y_D}$ is the capacity investment level such that the firm's total physical capacity investment for both products $2\underline{K}_D$ matches exactly the financial capacity $\frac{R_D}{y_D}$ (i.e. $2\underline{K}_D = \frac{R_D}{y_D}$). $\frac{B^1-F_D}{2c_D}$ represents the firm's physical capacity ivestment limit for each product.

The explanation for Proposition 8 is very similar to Proposition 4 of flexible technology case (here, K_D^0 and $\underline{K}_D$ correspond to K_F^0 and $\overline{K}_F$ respectively) and thus, is omitted. It follows from Proposition 8 that with deterministic stage 2 budget B^2 and clearance assumption, the firm's optimal expected profit Π_D^* is characterized by Corollary 6.

Corollary 6 *With deterministic stage 2 budget B^2 and clearance assumption, the firm's optimal expected profit Π_D is*

$$\Pi_D^* = B^2 + B^1 - F_D + \frac{c_D + y_D}{-(b+1)}(2K_D^0) \quad if\ B^2 \in [0, +\infty) \tag{4.11}$$

for $B^1 \in \big(2(c_D + y_D)K_D^0 + F_D, +\infty\big)$; and

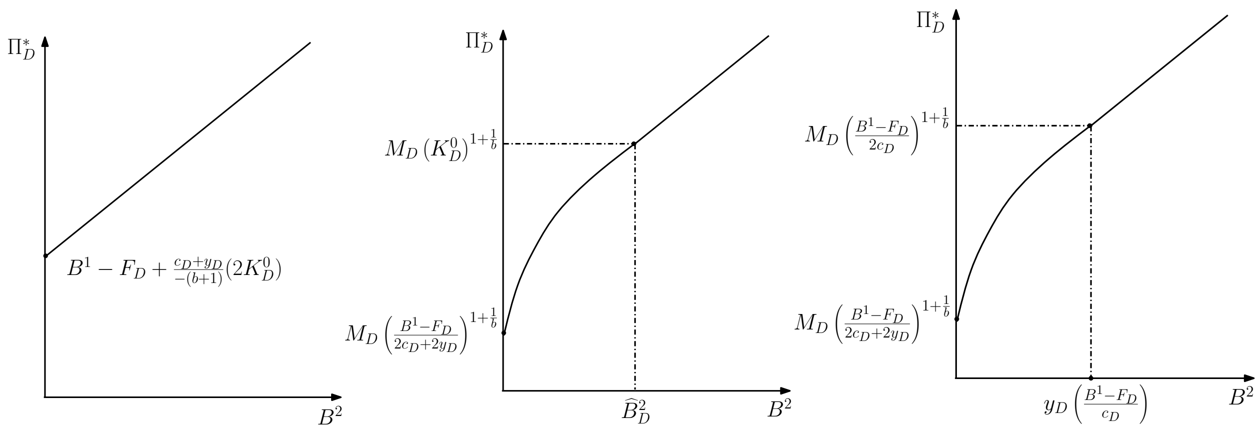
$$\Pi_D^* = \begin{cases} M_D \underline{K}_D^{1+\frac{1}{b}} & if\ B^2 \in [0, \widehat{B}_D^2] \\ B^2 + B^1 - F_D + \frac{c_D + y_D}{-(b+1)}(2K_D^0) & if\ B^2 \in (\widehat{B}_D^2, +\infty) \end{cases} \tag{4.12}$$

for $B^1 \in \big(2c_D K_D^0 + F_D, 2(c_D + y_D)K_D^0 + F_D\big]$; and

$$\Pi_D^* = \begin{cases} M_D \underline{K}_D^{1+\frac{1}{b}} & if\ B^2 \in \left[0, y_D\left(\frac{B^1 - F_D}{c_D}\right)\right] \\ B^2 + M_D\left(\frac{B^1 - F_D}{2c_D}\right)^{1+\frac{1}{b}} - 2y_D\left(\frac{B^1 - F_D}{2c_D}\right) & if\ B^2 \in \left(y_D\left(\frac{B^1 - F_D}{c_D}\right), +\infty\right) \end{cases} \tag{4.13}$$

for $B^1 \in \big[0, 2c_D K_D^0 + F_D\big]$, where $M_D \doteq 2\overline{\xi}$.

Figure 4.5 summarizes Π_D^* in Corollary 6.



(a) Π_D in (4.11) (b) Π_D in (4.12) (c) Π_D in (4.13)

Figure 4.5: Optimal expected profit Π_D^*

Chapter 5

Technology Choice

In this section, we characterize the firm's optimal technology choice under deterministic stage 2 budget. We note there that our focus is the choice between flexible and dedicated technology. Since there is a fixed cost associated with each technology, it may be optimal for the firm not to invest any technology. We will assume that fixed cost is not very large such that it is optimal for the firm to invest in the chosen technology. We will also assume that the stage 1 budget B^1 is larger than F_F, i.e. the highest fixed cost. Otherwise, the firm does not have any budget to invest in capacity at stage 1. In this stage, for a given stage 1 and stage 2 budget levels B^1 and B^2, the firm optimally chooses technology $T^* \in \{F, D\}$ to maximize the expected profit. The firm's optimal expected profit with each technology, Π_F^* and Π_D^*, are characterized by Corollary 3 and Corollary 6 respectively. In §5.1, we analyze the firm's technology selection problem without production cost, i.e. $y_T = 0$. §5.2 analyzes the same with production cost $y_T \geq 0$.

5.1 No Production Cost

In this section, as there is no production cost, the firm's stage 2 production will never be financially constrained. Therefore, the firm optimally follows the clearance price strategy.

Proposition 9 *With production cost $y_F = y_D = 0$ and non-symmetric fixed cost $F_F \geq F_D$, for a given stage 1 budget B^1, there exists a unique variable cost threshold $\bar{c}_F$ such that when $c_F \leq \bar{c}_F$*

($c_F > \overline{c}_F$), it is optimal to invest in flexible (dedicated) technology. This threshold is given by

$$\overline{c}_F = \begin{cases} \overline{c}_F^1 = \left(\frac{M_F}{2^{-\frac{1}{b}}\overline{\xi}}\right)^{\frac{b}{b+1}} \left[\frac{B^1-F_F}{B^1-F_D}\right] c_D & \text{if } F_F \le B^1 < \max\left(F_F, F_D + 2c_D K_D^0\right) \\ \overline{c}_F^2 = \left[\frac{M_F (B^1-F_F)^{1+\frac{1}{b}}}{B^1-F_D+\frac{2c_D K_D^0}{-(b+1)}}\right]^{\frac{b}{b+1}} & \text{if } \max\left(F_F, F_D + 2c_D K_D^0\right) \le B^1 < F_D + 2c_D K_D^0 + b(F_D - F_F) \\ \overline{c}_F^3 = \left[\frac{\left[(1+\frac{1}{b})M_F\right]^{-b}}{(F_D-F_F)(b+1)+2c_D K_D^0}\right]^{\frac{-1}{b+1}} & \text{if } B^1 \ge F_D + 2c_D K_D^0 + b(F_D - F_F) \end{cases} \tag{5.1}$$

where $K_D^0 = \left[\frac{\overline{\xi}(1+\frac{1}{b})}{c_D}\right]^{-b}$ and $M_F = \mathbb{E}\left[\left(\xi_1^{-b} + \xi_2^{-b}\right)^{-\frac{1}{b}}\right]$. Both $\overline{c}_F^1$ and $\overline{c}_F^2$ increase in stage 1 budget B^1.

In Proposition 9, $\overline{c}_F^1$ and $\overline{c}_F^2$ increase in stage 1 budget B^1, i.e. higher internal budget favors flexible technology investment. This is because flexible technology has a higher fixed cost. With symmetric fixed cost, i.e. $F_F = F_D$, the threshold in Proposition 9 yields,

$$\overline{c}_F = c_D \left(\frac{\mathbb{E}\left[\left(\xi_1^{-b} + \xi_2^{-b}\right)^{-\frac{1}{b}}\right]}{2^{-\frac{1}{b}}\overline{\xi}} \right)^{\frac{b}{b+1}} \ge c_D \tag{5.2}$$

Interestingly, the stage 1 budget B^1 has no effect on the variable cost threshold $\overline{c}_F$. The inequality in (5.2) only holds at equality if the product markets are deterministic ($\sigma = 0$), or the product markets are perfectly positively correlated ($\rho = 1$). We note that $\overline{c}_F \ge c_D$, since flexible technology has the capacity pooling benefit, which is captured by the term $\mathbb{E}\left[\left(\xi_1^{-b} + \xi_2^{-b}\right)^{-\frac{1}{b}}\right]$ in (5.2). In addition, there is no capacity pooling benefit ($\overline{c}_F = c_D$) only if the demands are deterministic or are random but perfectly positively correlated.

5.2 Positive Production Cost

In this section, we focus on technology choice when there is positive production cost $y_T > 0$ for each technology. We assume clearance strategy for both technologies. We also assume that Assumption 3 holds for dedicated technology. To analyze the technology choice problem, we use the results presented in §3.3 and §4.3.

Proposition 10 *If the stage 1 budget is unconstrained, i.e. B^1 is sufficiently high, there exists a unique flexible technology variable cost threshold $\widehat{c}_F^L$, which is given by*

$$\widehat{c}_F^L = \left(\frac{\mathbb{E}^{-b}\left[\left(\xi_1^{-b}+\xi_2^{-b}\right)^{-\frac{1}{b}}\right]\left(1+\frac{1}{b}\right)^{-b}}{(b+1)(F_D-F_F)+2(c_D+y_D)\left[\frac{\overline{\xi}\left(1+\frac{1}{b}\right)}{c_D+y_D}\right]^{-b}} \right)^{\frac{-1}{1+b}} - y_F \tag{5.3}$$

such that the firm optimally invests in flexible (dedicated) technology for $c_F < \widehat{c}_F^L$ ($c_F \geq \widehat{c}_F^L$).

In the absence of the stage 1 budget constraint, the firm always invests in the first-best capacity level K_T^0 under each technology, regardless of the stage 2 budget B^2. We also note that without production cost, i.e. $y_T = 0$, the threshold $\widehat{c}_F^L$ given by (5.3) yields exactly $\overline{c}_F^3$ in Proposition 9.

Corollary 7 *With symmetric production cost and fixed cost, i.e. $y_F = y_D = y$ and $F_D = F_F$, the threshold $\widehat{c}_F^L$ in Proposition 10 yields*

$$\widehat{c}_F^L = \left(\frac{M_F}{2^{-\frac{1}{b}}\overline{\xi}}\right)^{\frac{b}{b+1}} c_D + \left[\left(\frac{M_F}{2^{-\frac{1}{b}}\overline{\xi}}\right)^{\frac{b}{b+1}} - 1\right] y \tag{5.4}$$

As can be observed from (5.4), the threshold $\widehat{c}_F^L$ increases in production cost y. It follows that an increase in production cost y favors flexible technology. This is because the total investment cost $2(c_D+y_D)K_D^0 + F_D$ is higher with dedicated technology than the same with flexible technology, i.e. $c_F K_F^0 + F_F$, for $c_F = \widehat{c}_F^L$. Therefore, an increase in y is more detrimental for the dedicated technology.

Proposition 11 *With symmetric production cost $y_F = y_D = y$ and no fixed cost $F_F = F_D = 0$, if $B^1 \leq 2c_D K_D^0$, there exist three variable cost thresholds $\widehat{c}_F^i$ for $i = \{1,2,3\}$ such that $\widehat{c}_F^1 < \widehat{c}_F^2 < \widehat{c}_F^3$, which are characterized by*

i) For $B^2 \in [y\frac{B^1}{c_D}, +\infty)$, the threshold $\widehat{c}_F^1$ uniquely solves

$$M_F\left(\frac{B^1}{c_F}\right)^{1+\frac{1}{b}} - y\left(\frac{B^1}{c_F}\right) - 2\overline{\xi}\left(\frac{B^1}{2c_D}\right)^{1+\frac{1}{b}} + 2y\left(\frac{B^1}{2c_D}\right) = 0 \tag{5.5}$$

ii) For $B^2 \in [y\frac{B^1}{\widehat{c}_F^3}, y\frac{B^1}{c_D})$, the threshold $\widehat{c}_F^2$ uniquely solves

$$B^2 + M_F \left(\frac{B^1}{c_F}\right)^{1+\frac{1}{b}} - y\left(\frac{B^1}{c_F}\right) - 2\overline{\xi}\left(\frac{B^2+B^1}{2c_D+2y}\right)^{1+\frac{1}{b}} = 0 \tag{5.6}$$

iii) For $B^2 \in [0, y\frac{B^1}{\widehat{c}_F^3})$, the threshold $\widehat{c}_F^3$ is characterized by

$$\widehat{c}_F^3 = \left(\frac{M_F}{2^{-\frac{1}{b}}\overline{\xi}}\right)^{\frac{b}{b+1}} c_D + \left[\left(\frac{M_F}{2^{-\frac{1}{b}}\overline{\xi}}\right)^{\frac{b}{b+1}} - 1\right] y \tag{5.7}$$

where $M_F = \mathbb{E}\left[\left(\xi_1^{-b} + \xi_2^{-b}\right)^{-\frac{1}{b}}\right]$. In addition, the first threshold $\widehat{c}_F^1$ decreases in B^1 and increases in y. The second threshold $\widehat{c}_F^2$ decreases in B^2 and the third threshold $\widehat{c}_F^3$ increases in y.

Since the total investment cost is higher with the dedicated technology, a higher stage 1 budget B^1 or stage 2 budget B^2 favors dedicated technology. Similarly, an increase in production cost y is more detrimental for dedicated technology and thus, favors flexible technology.

Chapter 6

Conclusion

This paper contributes to the stochastic capacity investment literature by analyzing the impact of financial constraints. A joint operational and financial perspective is adopted to develop theory and insights into capacity management and technology choice. In a two-product setting, we analyze the impact of internal budget level at the capacity investment and production stages as well as the impact of production stage budget variability on the capacity investment decision and the expected profit of the firm with dedicated and flexible technology investments. With deterministic budget, we also analyze the impact of financial constraints on the choice between flexible and dedicated technology.

We demonstrate that the available capital at the production stage forms a network of resources with the physical capacity investment with each technology. We show that budget variability decreases the expected optimal profit with either technology, therefore the firm is better off by fully hedging the budget uncertainty. For flexible technology, the optimal capacity investment level also decreases in budget variability. With dedicated technology, we expect the same result continue to hold. On the technology choice with deterministic budget levels, we show that without production cost, fixed cost difference is the main determinant of the impact of financial constraints: A higher internal budget always favors the flexible technology due to its higher fixed cost. With production costs in place, the opposite holds true. We show that if there is no fixed cost difference, a higher internal budget favors the dedicated technology. This is because the total investment cost is higher with the dedicated technology. The key insight is that the impact of financial constraints on technology choice critically depends on the production costs.

Other interesting research directions remain. For the dedicated technology, with stochastic budget, for specific distributions of uncertainties, it is interesting to analyze the impact of budget variability. For tractability purposes, we impose Assumption 3 for the dedicated technology analysis. It is important to relax this assumption to generalize the analysis. For the technology choice problem, we only provide partial results with deterministic budget levels. Future work needs to generalize this analysis, and analyze the technology choice with stochastic budget.

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Technical Appendix

In this technical appendix, we provide the proofs of our technical statements in our paper.

Name	Meaning
(F_T, c_T)	fixed and variable capacity costs of technology T
y_T	unit production cost with technology T
K_T	capacity investment level with technology T
B^1	stage 1 budget
B^2	stage 2 budget
B^l	lower bound of stage 2 budget
B^u	upper bound of stage 2 budget
$\xi = (\xi_1, \xi_2)$	multiplicative demand intercept in product markets
Σ	covariance matrix of ξ
ρ	coefficient of correlation in ξ
σ	standard deviation of ξ_1 and ξ_2
$Q_T^* = (Q_T^1, Q_T^2)$	optimal production plan with Technology T
Γ_T	optimal stage 2 operating profit with technology T
Ψ_T	optimal stage 2 profit with technology T
Π_T	optimal expected stage 1 profit with technology T

Table 6.1: Summary of notations

Proof of Proposition 1: Note that, with flexible technology and for given capacity investment level K_F, demand realization ξ and budget level B^2, the firm's optimal profit in stage 2 is $\Psi_F(B^2,\xi) = R_F + \Gamma_F(Q_F,B^2,\xi)$ where

$$
\begin{aligned}
\Gamma_F(Q_F,B^2,\xi) = \max_{Q_F} \quad & \xi_1(Q_F^1)^{1+\frac{1}{b}} + \xi_2(Q_F^2)^{1+\frac{1}{b}} - y_F(Q_F^1+Q_F^2) \\
\text{s.t.} \quad & 1'Q_F \leq \min\left(\frac{R_F}{y_F}, K_F\right) \\
& Q_F \geq 0
\end{aligned}
\tag{6.1}
$$

The first two terms in the objective function are the operating revenues in each product market, the third term is the production cost. The first constraint ensures that production quantities are within the net capacity limit and the second constraint captures the non-negativity. Let $g_F(Q_F) \doteq \xi_1(Q_F^1)^{1+\frac{1}{b}} + \xi_2(Q_F^2)^{1+\frac{1}{b}} - y_F(Q_F^1+Q_F^2)$ define the objective function in (6.1), we will solve the problem by first proving $g_F(Q_F)$ is strictly concave in Q_F and solving the KKT conditions as follows:

1. Proof of concavity

We directly obtain

$$
\begin{cases}
\frac{\partial^2\Gamma}{\partial(Q_F^i)^2} = \frac{1}{b}\left(1+\frac{1}{b}\right)\xi_i {Q_F^i}^{\left(\frac{1}{b}-1\right)} < 0 \text{ (with } b<-1\text{)} \\
\frac{\partial^2\Gamma}{\partial(Q_F^1)^2}\frac{\partial^2\Gamma}{\partial(Q_F^2)^2} - \left[\frac{\partial^2\Gamma}{\partial(Q_F^1)\partial(Q_F^2)}\right]^2 = \prod_i \frac{1}{b}\left(1+1/b\right)\ \xi_i\ (Q_F^i)^{\left(\frac{1}{b}-1\right)} - 0 > 0
\end{cases}
$$

for $i=0,1$. Therefore, the Hessian matrix $D^2\Gamma_F(Q_F,B^2,\xi)$ is negative definite for $\mathbf{Q_F} \geq \mathbf{0}$ and $b<-1$ and $g_F(Q^F)$ is strictly concave in Q_F. Since the constraints in (6.1) are linear, first-order KKT conditions are necessary and sufficient for optimality and ${Q_F}^*$ is unique. We can then proceed to solve the KKT conditions

2. Solution for KKT conditions

If ${Q_F}^*$ is an optimal solution to (6.1), then there exist λ and $\mu=(\mu_1,\mu_2)$ that satisfy:

$$
\begin{aligned}
\mathbf{1}'{Q_F}^* &\leq \min\left(\frac{R_F}{y_F}, K_F\right), && (6.2)\\
{Q_F}^* &\geq \mathbf{0}, && (6.3)\\
\left(1+\frac{1}{b}\right)\xi_1({Q_F^1}^*)^{\frac{1}{b}} - y_F - \lambda + \mu_1 &= 0, && (6.4)
\end{aligned}
$$

$$\left(1+\frac{1}{b}\right)\xi_2({Q_F^2}^*)^{\frac{1}{b}} - y_F - \lambda + \mu_2 \quad = \quad 0, \tag{6.5}$$

$$\lambda\left(\min\left(\frac{R_F}{y_F}, K_F\right) - \mathbf{1}'{Q_F}^*\right) \quad = \quad 0, \tag{6.6}$$

$$\mu {Q_F}^* \quad = \quad \mathbf{0} \tag{6.7}$$

with $\lambda \geq 0$ and $\mu \geq \mathbf{0}$. Observe that $\lim_{Q_F^i \to 0^+}\left(1+\frac{1}{b}\right)\xi_i(Q_F^i)^{\frac{1}{b}} \to \infty$ for $i = 1,2$, so the nonnegativity constraints are never binding, which means $\mu = \mathbf{0}$. This leaves us only 2 cases to analyze.

Case 1: $\mathbf{1}'{Q_F}^* < \min\left(\frac{R_F}{y_F}, K_F\right)$

It follows from (6.6) that $\lambda = 0$. From (6.4) and (6.5), we obtain:

$$\begin{aligned}\left(1+\frac{1}{b}\right)\xi_i({Q_F^1}^*)^{\frac{1}{b}} \quad &= \quad y_F \\ \Rightarrow {Q_F^i}^* \quad &= \quad \xi_i^{-b}\left[\frac{1+\frac{1}{b}}{y_F}\right]^{-b}\end{aligned} \tag{6.8}$$

This solution is valid only if it satisfies all the other constraints. Note that ${Q_F^i}^* = \xi_i^{-b}\left[\frac{1+\frac{1}{b}}{y_F}\right]^{-b} > 0$ for $i = 1,2$, we need to check only for (6.2) . Substitute the values of ${Q_F^i}^*$ in (6.8) to (6.2), we have:

$$\begin{aligned}\left(\xi_1^{-b}+\xi_2^{-b}\right)\left[\frac{1+\frac{1}{b}}{y_F}\right]^{-b} \quad &< \quad \min\left(\frac{R_F}{y_F}, K_F\right) \\ \Rightarrow \xi_1^{-b}+\xi_2^{-b} \quad &< \quad \min\left(\frac{R_F}{y_F}, K_F\right)\left[\frac{y_F}{1+\frac{1}{b}}\right]^{-b}\end{aligned} \tag{6.9}$$

Case 2: $\mathbf{1}'{Q_F}^* = \min\left(\frac{R_F}{y_F}, K_F\right)$, which means $\lambda \geq 0$

From (6.4) and (6.5):

$$\begin{aligned}\frac{{Q_F^1}^*}{{Q_F^2}^*} \quad &= \quad \frac{\xi_1^{-b}}{\xi_2^{-b}} \\ \Rightarrow {Q_F^1}^* \quad &= \quad {Q_F^2}^*\frac{\xi_1^{-b}}{\xi_2^{-b}}\end{aligned}$$

Using that relationship and the binding constraints of $\mathbf{1}'{Q_F}^* = \min\left(\frac{R_F}{y_F}, K_F\right)$, we achieve

$${Q_F^i}^* = \min\left(\frac{R_F}{y_F}, K_F\right)\left(\frac{\xi_i^{-b}}{\xi_1^{-b}+\xi_2^{-b}}\right) \tag{6.10}$$

This solution is also valid only if $\lambda \geq 0$, which implies: $\lambda = \xi^i (Q_F^{i\,*})^{1+\frac{1}{b}} - y_F \geq 0$ for $i = 1,2$. After some algebra, the condition for (6.10) to be valid is:

$$\xi_1^{-b} + \xi_2^{-b} \geq \min\left(\frac{R_F}{y_F}, K_F\right) \left[\frac{y_F}{1+\frac{1}{b}}\right]^{-b} \tag{6.11}$$

Combine the solutions and the respective valid conditions from the 2 cases, we have the optimal stage 1 production plan Q_F^* is described as follows:

$$Q_F^*\left(B^2,\xi\right)' = \begin{cases} \left(\left[\frac{\left(1+\frac{1}{b}\right)\xi^1}{y_F}\right]^{-b}, \left[\frac{\left(1+\frac{1}{b}\right)\xi_2}{y_F}\right]^{-b}\right) & \text{if} \quad \xi \in \Omega_F^1 \\ \left(\min\left(\frac{R_F}{y_F}, K_F\right) \frac{\xi^{1-b}}{\xi^{1-b}+\xi_2{}^{-b}}, \min\left(\frac{R_F}{y_F}, K_F\right) \frac{\xi_2{}^{-b}}{\xi^{1-b}+\xi_2{}^{-b}}\right) & \text{if} \quad \xi \in \Omega_F^2 \end{cases}$$

where

$$\begin{aligned} \Omega_F^1 &\doteq \left\{\xi : 0 \leq \xi_1^{-b} + \xi_2^{-b} < \left[\frac{y_F}{1+\frac{1}{b}}\right]^{-b} \min\left(\frac{R_F}{y_F}, K_F\right)\right\} \\ \Omega_F^2 &\doteq \left\{\xi : \xi_1^{-b} + \xi_2^{-b} \geq \left[\frac{y_F}{1+\frac{1}{b}}\right]^{-b} \min\left(\frac{R_F}{y_F}, K_F\right)\right\}. \end{aligned}$$

with that optimal production plan, it follows that the optimal operating profit from the production would be

$$\Gamma_F\left(Q_F, B^2, \xi\right) = \begin{cases} \Gamma_F^1(Q_F, B^2, \xi) \doteq y_F \left(\xi_1^{-b} + \xi_2^{-b}\right) \left[\frac{1+\frac{1}{b}}{y_F}\right]^{-b} \left(\frac{1}{-b-1}\right) & \text{if} \quad \xi \in \Omega_F^1 \\ \Gamma_F^2(Q_F, B^2, \xi) \doteq \min\left(\frac{R_F}{y_F}, K_F\right) \left[\left[\frac{\min\left(\frac{R_F}{y_F}, K_F\right)}{\xi_1^{-b}+\xi_2^{-b}}\right]^{\frac{1}{b}} - y_F\right] & \text{if} \quad \xi \in \Omega_F^2 \end{cases} \tag{6.12}$$

This completes the proof of proposition 1. ■

Proof of Proposition 2: We will first establish that Π_F is concave in K_F. Recall that the firm's optimal expected stage 1 profit is given by

$$\begin{aligned} \Pi_F(K_F) &= \overline{B}^2 + B^1 - F_F - c_F K_F \\ &\quad + \int_{B^l}^{\min(\max(B^l, \overline{B}_F), B^u)} G_F^1(K_F, B^2)\, dF(B^2) \\ &\quad + \int_{\min(\max(B^l, \overline{B}_F), B^u)}^{B^u} G_F^2(K_F, B^2)\, dF(B^2). \end{aligned}$$

where

$$
\begin{aligned}
G_F^1(K_F, B^2) &= \int\int_{\Omega^1_{F B^2 < \overline{B}_F}} \left[\left(\xi_1^{-b} + \xi_2^{-b} \right) \left[\frac{1+\frac{1}{b}}{y_F} \right]^{-b} \left(\frac{y_F}{-b-1} \right) \right] \mathrm{d}\Phi(\xi_1, \xi_2) \\
&\quad + \int\int_{\Omega^2_{F B^2 < \overline{B}_F}} \left[\left(\frac{R_F}{y_F} \right)^{(1+1/b)} \left[\xi_1^{-b} + \xi_2^{-b} \right]^{\frac{-1}{b}} - R_F \right] \mathrm{d}\Phi(\xi_1, \xi_2) \\
G_F^2(K_F, B^2) &= \int\int_{\Omega^1_{F B^2 \geq \overline{B}_F}} \left[\left(\xi_1^{-b} + \xi_2^{-b} \right) \left[\frac{1+\frac{1}{b}}{y_F} \right]^{-b} \left(\frac{y_F}{-b-1} \right) \right] \mathrm{d}\Phi(\xi_1, \xi_2) \\
&\quad + \int\int_{\Omega^2_{F B^2 \geq \overline{B}_F}} \left[(K_F)^{(1+1/b)} \left[\xi_1^{-b} + \xi_2^{-b} \right]^{\frac{-1}{b}} - y_F K_F \right] \mathrm{d}\Phi(\xi_1, \xi_2)
\end{aligned}
$$

Taking the first-order derivative w.r.t. K_F and after some algebra, we obtain

$$
\begin{aligned}
\frac{\partial \Pi_F}{\partial K_F} &= -c_F + \int_{B^l}^{\min(\max(B^l, \overline{B}_F), B^u)} H_F^1 \left(K_F, B^2 \right) \mathrm{d}F(B^2) \\
&\quad + \int_{\min(\max(B^l, \overline{B}_F), B^u)}^{B^u} H_F^2 \left(K_F, B^2 \right) \mathrm{d}F(B^2).
\end{aligned}
$$

where

$$
\begin{aligned}
H_F^1(K_F, B^2) &= \int\int_{\Omega^2_{F B^2 < \overline{B}_F}} \left[\left(\frac{-c_F}{y_F} \right) (1+1/b) \left(\frac{R_F}{y_F} \right)^{(1/b)} \left[\xi_1^{-b} + \xi_2^{-b} \right]^{\frac{-1}{b}} + c_F \right] \mathrm{d}\Phi(\xi), \\
H_F^2(K_F, B^2) &= \int\int_{\Omega^2_{F B^2 \geq \overline{B}_F}} \left[(1+1/b)(K_F)^{(1/b)} \left[\xi_1^{-b} + \xi_2^{-b} \right]^{\frac{-1}{b}} - y_F \right] \mathrm{d}\Phi(\xi).
\end{aligned}
$$

Note that $\frac{\partial \Pi_F}{\partial K_F}$ corresponds term by term to the expected stage 1 revenue $\Pi_F(K_F)$. The reason that $\frac{\partial \Pi_F}{\partial K_F}$ does not contain any terms from differentiating the limits of integration is the continuity of the realized profit in ξ. Even though an infinitesimal change in K_F affects the boundary between the regions, and hence the probability of ξ falling into any of the regions, the corresponding terms cancel out because the realized profit is continuous on the boundaries.

It is easy to verify that $\frac{\partial^2 \Pi_F(K_F)}{\partial {K_F}^2} < 0$ for the cases $\min(\max(B^l, \overline{B}_F), B^u) = B^l$ (i.e. B^l is sufficiently high) or $\min(\max(B^l, \overline{B}_F), B^u) = B^u$ (i.e. B^u is sufficiently low), and thus, $\Pi_F(K_F)$ is concave in K_F. We now turn to prove the concavity of Π_F in K_F for moderate B^l and B^u (i.e. $\min(\max(B^l, \overline{B}_F), B^u) = \overline{B}_F$). The second order condition is given by

$$
\frac{\partial^2 \Pi_F(K_F)}{\partial {K_F}^2} = \int_{B^l}^{\overline{B}_F} \frac{\partial H_F^1(K_F, B^2)}{\partial K^F} \, dF(B_1) + \int_{\overline{B}_F}^{B^u} \frac{\partial H_F^2(K_F, B^2)}{\partial K_F} \, dF(B_1)
$$

$$
\begin{aligned}
& +\frac{\partial \overline{B}_F}{\partial K_F} f(\overline{B}_F)\left[H_F^1(K_F,\overline{B}_F)-H_2(K_F^2,\overline{B}_F)\right] \\
= \quad & \int_0^{\overline{B}_F}\left[\iint_{\Omega^2_{F\,B^2\leq\overline{B}_F}}\left[\frac{c_F^2}{y_F^2}\frac{1}{b}(1+\frac{1}{b})\left(\frac{R_F}{y_F}\right)^{(\frac{1}{b}-1)}\left[\xi_1^{-b}+\xi_2^{-b}\right]^{\frac{-1}{b}}+c_F\right]d\Phi(\xi)\right]dF(B_1) \\
& +\int_{\overline{B}_F}^{\infty}\left[\iint_{\Omega^2_{F\,B^2\geq\overline{B}_F}}\left[\frac{1}{b}(1+\frac{1}{b})(K_F)^{(\frac{1}{b}-1)}\left[\xi_1^{-b}+\xi_2^{-b}\right]^{\frac{-1}{b}}-y_F\right]d\Phi(\xi)\right]dF(B_1) \\
& +(c_F+y_F)f(\overline{B}_F)\left[H_F^1(K_F,\overline{B}_F)-H_F^2(K_F,\overline{B}_F)\right]
\end{aligned}
$$

where $f(.)$ is the pdf of the stage 2 internal budget B^2. Note that the first two terms of the second order condition are non-positive because the assumption of $b<-1$. We will now investigate the sign of the third term. Because both (c_F+y_F) and $f(\overline{B}_F)$ are positive, the third term has the same sign as $\left[H_F^1(K_F,\overline{B}_F)-H_F^2(K_F,\overline{B}_F)\right]$. Recall that we have $\frac{R_F}{y_F}=K_F$ at $B^2=\overline{B}_F$ form the definition of $\overline{B}_F$. It follows that at $B^2=\overline{B}_F$, we obtain

$$
\Omega^2_{F\,B^2<\overline{B}_F}=\Omega^2_{F\,B^2\geq\overline{B}_F}=\widehat{\Omega}^2_F \doteq \left\{\xi:\xi_1{}^{-b}+\xi_1{}^{-b}\geq\left[\frac{y_F}{1+\frac{1}{b}}\right]^{-b}K_F\right\}
$$

. It is easy to verify that $H_F^1(K_F,\overline{B}_F)\leq 0$ and $H_F^2(K_F,\overline{B}_F)\geq 0$, i.e.

$$
H_F^1(K_F,\overline{B}_F)=\iint_{\widehat{\Omega}^2_F}\left[\left(\frac{-c^F}{y_F}\right)(1+1/b)(K_F)^{(1/b)}\left[\xi_1^{-b}+\xi_2^{-b}\right]^{\frac{-1}{b}}+c^F\right]d\Phi(\xi)\leq 0 \quad (6.13)
$$

and

$$
H_F^2(K_F,\overline{B}_F)=\iint_{\widehat{\Omega}^2_F}\left[(1+1/b)(K_F)^{(1/b)}\left[\xi_1^{-b}+\xi_2^{-b}\right]^{\frac{-1}{b}}-y_F\right]d\Phi(\xi)\geq 0 \quad (6.14)
$$

From (6.13) and (6.14), we have $\left[H_F^1(K_F,\overline{B}_F)-H_F^2(K_F,\overline{B}_F)\right]\leq 0$ and thus $\frac{\partial^2\Pi_F(K_F)}{\partial {K_F}^2}\leq 0$. It follows that $\Pi_F(K_F)$ is concave in K_F and the optimal investment capacity K_F^* is characterized as in proposition 2. ■

Proof of Proposition 3: We use the following result from Ross (1996):

Lemma 3 *Let B^2 and $\underline{B}^2$ be two random variables. We say that B^2 is more variable than $\underline{B}^2$, if $\mathbb{E}[h(B^2)]\leq\mathbb{E}[h(\underline{B}^2)]$ for all increasing, concave h.*

Following from Lemma 3, in order to prove Proposition 3, it is sufficient to show that both the expected profit Π_F and the first-order derivative $\frac{\partial \Pi_F}{\partial K_F}$ are concave in stage 2 budget B^2. We first demonstrate the concavity of Π_F in B^2. Recall that Π_F is characterized by 3.5, i.e.

$$\begin{aligned}\Pi_F(K_F) &= \overline{B}^2 + B^1 - F_F - c_F K_F \\ &\quad + \int_{B^l}^{\min(\max(B^l,\overline{B}_F),B^u)} \left[M_F\left(\frac{R_F}{y_F}\right)^{1+\frac{1}{b}} - R_F\right] \mathrm{d}F(B^2) \\ &\quad + \int_{\min(\max(B^l,\overline{B}_F),B^u)}^{B^u} \left[M_F K_F^{1+\frac{1}{b}} - y_F K_F\right] \mathrm{d}F(B^2).\end{aligned}$$

For analytical convenience, we define

$$\begin{aligned}H_F^{1c} &= M_F\left(\frac{R_F}{y_F}\right)^{1+\frac{1}{b}} - R_F \\ H_F^{2c} &= M_F K_F^{1+\frac{1}{b}} - y_F K_F\end{aligned}$$

The proof of concavity of Π_F has the following structure:

1. Prove the concavity of H_F^{1c} and H_F^{2c} respectively.

2. Show that at the boundary $B^2 = \overline{B}_F$: $\frac{\partial H_F^{1c}}{\partial B^2} \geq \frac{\partial H_F^{2c}}{\partial B^2}$.

It is easily obtained that both H_F^{1c} and H_F^{2c} are concave in B^2, and at the boundary $B^2 = \overline{B}_F$, we have $\frac{\partial H_F^{1c}}{\partial B^2} \geq \frac{\partial H_F^{2c}}{\partial B^2}$ under Assumption 1. Hence, Π_F is concave in B^2. Following from Corollary 2, we can establish the concavity of $\frac{\partial \Pi_F}{\partial K_F}$ in B^2 by using a similar fashion and thus, the proof is omitted. This completes the proof for Proposition 3. ■

Proof of Proposition 4: It follows from (3.5) that under deterministic stage 2 budget and clearance assumption, the firm's expected profit is given by

$$\Pi_F = \begin{cases} R_F + M_F K_F^{1+\frac{1}{b}} - y_F K_F & \text{if } K_F \in \left[0, \min\left(\overline{K}_F, \frac{B^1-F_F}{c_F}\right)\right] \\ M_F\left(\frac{R_F}{y_F}\right)^{1+\frac{1}{b}} & \text{if } K_F \in \left(\min\left(\overline{K}_F, \frac{B^1-F_F}{c_F}\right), \frac{B^1-F_F}{c_F}\right] \end{cases} \tag{6.15}$$

where $\overline{K}_F = \frac{B^2+B^1-F_F}{c_F+y_F}$ such that the firm's physical capacity matches exactly the firm's financial capacity, i.e. $\overline{K}_F = \frac{R_F}{y_F}$. And $\frac{B^1-F_F}{c_F}$ represents the firm's physical capacity limit. We note that Π_F is continuous at the kink $K_F = \min\left(\overline{K}_F, \frac{B^1-F_F}{c_F}\right)$. Taking the first-order derivative w.r.t. K_F, we obtain,

$$\frac{\partial \Pi_F}{\partial K_F} = \begin{cases} -c_F - y_F + M_F\left(1+\frac{1}{b}\right) K_F^{\frac{1}{b}} & \text{if } K_F \in \left[0, \min\left(\overline{K}_F, \frac{B^1-F_F}{c_F}\right)\right] \\ -M_F\left(1+\frac{1}{b}\right)\left(\frac{R_F}{y_F}\right)^{\frac{1}{b}}\left(\frac{c_F}{y_F}\right) & \text{if } K_F \in \left(\min\left(\overline{K}_F, \frac{B^1-F_F}{c_F}\right), \frac{B^1-F_F}{c_F}\right] \end{cases} \tag{6.16}$$

Since Π_F is continuous and $\frac{\partial \Pi_F}{\partial K_F} \leq 0$ for $K_F \in \left(\min\left(\overline{K}_F, \frac{B^1 - F_F}{c_F}\right), \frac{B^1 - F_F}{c_F}\right]$, the optimal capacity level K_F^* can not exceed the limit $\min\left(\overline{K}_F, \frac{B^1 - F_F}{c_F}\right)$. It is easy to demonstrate that Π_F is concave in K_F for $K_F \in \left[0, \min\left(\overline{K}_F, \frac{B^1 - F_F}{c_F}\right)\right]$ and the first-best capacity investment level $K_F^0 = \left[\frac{(1+\frac{1}{b})M_F}{c_F + y_F}\right]^{-b}$. In order to explicitly characterize the optimal capacity investment level K_F^*, we discuss the following two cases:

Case 1: $\frac{B^1 - F_F}{c_F} \leq \overline{K}_F \Longleftrightarrow B^2 \geq y_F\left(\frac{B^1 - F_F}{c_F}\right)$

We note that Π_F is concave in K_F and the firm can not purchase physical capacity more than the capacity limit $\frac{B^1 - F_F}{c_F}$, therefore we obtain

$$K_F^* = \begin{cases} K_F^0 & \text{if } K_F^0 \leq \frac{B^1 - F_F}{c_F} \\ \frac{B^1 - F_F}{c_F} & \text{if } K_F^0 > \frac{B^1 - F_F}{c_F} \end{cases} \tag{6.17}$$

Case 2: $\frac{B^1 - F_F}{c_F} > \overline{K}_F \Longleftrightarrow B^2 < y_F\left(\frac{B^1 - F_F}{c_F}\right)$

In this case, it is never optimal for the firm to invest in physical capacity more than $\overline{K}_F$ as we have discussed above. The firm's optimal capacity investment level is given by

$$K_F^* = \begin{cases} K_F^0 & \text{if } K_F^0 \leq \overline{K}_F \\ \overline{K}_F & \text{if } K_F^0 > \overline{K}_F \end{cases} \tag{6.18}$$

We note that if $B^1 \geq (c_F + y_F)K_F^0 + F_F$, (6.18) yields,

$$K_F^* = K_F^0 \tag{6.19}$$

After combining the two cases and simplifying, we obtain the results. ■

Proof of Proposition 5: We will solve the production problem of dedicated technology by solving for the firm's optimal production plan, which is formulated as follows.

$$\begin{aligned} \Gamma_D(K_D, Q_D, B^2, \xi) = \max_{Q_D} \quad & \xi_1 (Q_D^1)^{1+\frac{1}{b}} + \xi_2 (Q_D^2)^{1+\frac{1}{b}} - y_D(Q_D^1 + Q_D^2) \\ \text{s.t.} \quad & Q_D^1 \leq K_D \\ & Q_D^2 \leq K_D \\ & y_D(Q_D^1 + Q_D^2) \leq R_D \end{aligned} \tag{6.20}$$

$$Q_D^1 \geq 0$$

$$Q_D^2 \geq 0$$

The first two terms in the objective function are the operating revenues in each product market, the third term is the production cost. The first two constraints ensure that production quantities are within the respective capacity limits and the third constraint ensure that total production costs do not exceed budget limit, respectively. The last two constraints capture the nonnegativity of the production quantities.

Let $Q_D^* = ({Q_D^1}^*, {Q_D^2}^*)$ and $g_D(Q_D)$ denote the optimal solution vector and objective function for (6.20), respectively. First, note that as $R_D > 0$, there always exists a feasible solution. Similar to the production optimization problem with flexible technology, we can solve the problem by first proving $g_D(Q_D)$ is strictly concave in Q_D and solving the KKT conditions as follows.

1. Proof of concavity

We directly obtain

$$\begin{cases} \frac{\partial^2\Gamma}{\partial(Q_D^i)^2} = \frac{1}{b}\left(1+\frac{1}{b}\right)\xi_i {Q_D^i}^{\left(\frac{1}{b}-1\right)} < 0 \text{ (with } b < -1) \\ \frac{\partial^2\Gamma}{\partial(Q_D^1)^2}\frac{\partial^2\Gamma}{\partial(Q_D^2)^2} - \left[\frac{\partial^2\Gamma}{\partial(Q_D^1)\partial(Q_D^2)}\right]^2 = \prod_i \frac{1}{b}\left(1+1/b\right)\ \xi_i\ (Q_D^i)^{\left(\frac{1}{b}-1\right)} - 0 > 0 \end{cases}$$

for $i = 0, 1$. Therefore, the Hessian matrix $D^2\Gamma_D(K_D, Q_D, B^2, \xi)$ is negative definite for $\mathbf{Q_D} \geq \mathbf{0}$ and $b < -1$ and $g_D(Q_D)$ is strictly concave in Q_D. Since the constraints in (6.20) are linear, first-order KKT conditions are necessary and sufficient for optimality and ${Q_D}^*$ is unique. We can then proceed to solve the KKT conditions.

2. Solution for KKT conditions

If ${Q_D}^*$ is an optimal solution to (6.20), then there exist λ, $\nu = (\nu_1, \nu_2)$ and $\mu = (\mu_1, \mu_2)$ that satisfy:

$$\mathbf{1}'{Q_D}^* \leq \frac{R_D}{y_D}, \tag{6.21}$$

$${Q_D}^* \leq K_D, \tag{6.22}$$

$${Q_D}^* \geq \mathbf{0}, \tag{6.23}$$

$$\left(1+\frac{1}{b}\right)\xi_1({Q_D^1}^*)^{\frac{1}{b}} - y_D - \lambda + \mu_1 - \nu_1 = 0, \tag{6.24}$$

$$\begin{aligned}
\left(1+\frac{1}{b}\right)\xi_2({Q_D^2}^*)^{\frac{1}{b}} - y_D - \lambda + \mu_2 - \nu_2 &= 0, && (6.25)\\
\lambda\left(\frac{R_D}{y_D} - \mathbf{1}'{Q_D}^*\right) &= 0, && (6.26)\\
\mu {Q_D}^* &= \mathbf{0}, && (6.27)\\
\nu\left(K_D - {Q_D}^*\right) &= \mathbf{0} && (6.28)
\end{aligned}$$

with $\lambda \geq 0$, $\nu \geq \mathbf{0}$ and $\mu \geq \mathbf{0}$. Observe that $\lim_{Q_D^i \to 0^+}\left(1+\frac{1}{b}\right)\xi_i(Q_D^i)^{\frac{1}{b}} \to \infty$ for $i = 1,2$, so the nonnegativity constraints are never binding, which means $\mu = \mathbf{0}$. Also note that different realization of stage 2 budget B^2 may further impose certain restrictions on the production plan, and thus, making some solutions of the KKT's infeasible. With that consideration, we will have to solve the problem with the following different stage 2 budget B^2 realizations.

Case 1: $B^2 < \underline{B}_D$

It follows that $\frac{R_D}{y_D} < K_D$. This case means the available budget is not enough to cover full production for either of the capacity investment. Thus, it follows that the physical capacity constraints ${Q_D}^* \leq K_D$ are never binding, which leads to $\nu = \mathbf{0}$.

Subcase 1.1: $Q_D^1 + Q_D^2 < \frac{R_D}{y_D}$

It follows that $\lambda = 0$. This case means the firm does not use up all its available budget for production. From (6.24) and (6.25), we obtain:

$$\begin{aligned}
\left(1+\frac{1}{b}\right)\xi_i({Q_D^i}^*)^{\frac{1}{b}} &= y_D\\
\Rightarrow {Q_D^i}^* &= \xi_i^{-b}\left[\frac{1+\frac{1}{b}}{y_D}\right]^{-b}
\end{aligned}$$

This solution is valid if it satisfies all other constraints, observe that, as ${Q_D^i}^* = \xi_i^{-b}\left[\frac{1+\frac{1}{b}}{y_D}\right]^{-b} > 0$ for $i = 1,2$, we need to check only for (6.21) and (6.22). Also, as $\frac{R_D}{y_D} < K_D$ in this case, if the solution satisfies (6.21), (6.22) will also hold. It follows from (6.21) and (6.29):

$$\left(\xi_1^{-b} + \xi_2^{-b}\right)\left[\frac{1+\frac{1}{b}}{y_D}\right]^{-b} < \frac{R_D}{y_D}$$

$$\Rightarrow \xi_1^{-b} + \xi_2^{-b} \quad < \quad \frac{R_D}{y_D}\left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \tag{6.29}$$

Subcase 1.2: $Q_D^1 + Q_D^2 = \frac{R_D}{y_D}$

It follows that $\lambda \geq 0$. This case means the firm uses up all its available budget for production. From (6.24) and (6.25) and $Q_D^1 + Q_D^2 = \frac{R_D}{y_D}$, we obtain:

$$Q_D^{i\,*} = \frac{R_D}{y_D}\left(\frac{\xi_i^{-b}}{\xi_1^{-b}+\xi_2^{-b}}\right) \tag{6.30}$$

This solution is valid if it satisfies all other constraints and $\lambda \geq 0$, observe that, as $Q_D^{i\,*} = \frac{R_D}{y_D}\left(\frac{\xi_i^{-b}}{\xi_1^{-b}+\xi_2^{-b}}\right) > 0$ for $i = 1,2$. As $\frac{R_D}{y_D} < K_D$ in this case, the solution in (6.30) satisfies (6.21). The condition for $\lambda \geq 0$ is $\lambda = \xi_i (Q_D^{i\,*})^{\frac{1}{b}} - y_D \geq 0$ for $i = 1,2$. After some algebra, the condition for (6.30) to be valid is:

$$\xi_1^{-b} + \xi_2^{-b} \geq \frac{R_D}{y_D}\left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \tag{6.31}$$

Combining the two subcases, we obtain the optimal production plan for $\frac{R_D}{y_D} < K_D$ (i.e. $B^2 < \underline{B}_D$) as follows:

$$Q_D^*\left(B^2,\xi\right)' = \begin{cases} \left(\left[\frac{(1+\frac{1}{b})\xi_1}{y_D}\right]^{-b}, \left[\frac{(1+\frac{1}{b})\xi_2}{y_D}\right]^{-b}\right) & \text{if} \quad \xi \in \Omega^1_{DB^2<\underline{B}_D} \\ \left(\frac{R_D}{y_D}\frac{\xi_1^{-b}}{\xi_1^{-b}+\xi_2^{-b}}, \frac{R_D}{y_D}\frac{\xi_2^{-b}}{\xi_1^{-b}+\xi_2^{-b}}\right) & \text{if} \quad \xi \in \Omega^6_{DB^2<\underline{B}_D} \end{cases}$$

where

$$\begin{aligned} \Omega^1_{DB^2<\underline{B}_D} &\doteq \left\{\xi : 0 \leq \xi_1^{-b} + \xi_2^{-b} < \left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b}\frac{R_D}{y_D}\right\} \\ \Omega^6_{DB^2<\underline{B}_D} &\doteq \left\{\xi : \xi_1^{-b} + \xi_2^{-b} \geq \left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b}\frac{R_D}{y_D}\right\}. \end{aligned}$$

Note that, for $B^2 < \underline{B}_D$:

$$\begin{aligned} \min\left(\left(\frac{R_D}{y_D} - K_D\right)^+, K_D\right) &= 0 \\ \min\left(\left(\frac{R_D}{y_D} - K_D\right)^+, K_D\right) &= 0 \end{aligned}$$

Thus, from (4.1), Ω_D^2, Ω_D^3, Ω_D^4 and Ω_D^5 regions vanish. $\Omega_D^1 = \Omega^1_{DB^2<\underline{B}_D}$ and $\Omega_D^6 = \Omega^6_{DB^2<\underline{B}_D}$, the solution in (4.1) reduces to the above solution.

Case 2: $\underline{B}_D \leq B^2 < \overline{B}_D$

This leads to $K_D \leq \frac{R_D}{y_D} < 2K_D$. This case means the available budget is sufficient to cover full production for either one of the capacity investment but not both. Thus, it follows that $\nu_1\nu_2 = 0$.

Subcase 2.1: ${Q_D^1}^* + {Q_D^2}^* < \frac{R_D}{y_D}$, ${Q_D^1}^* < K_D$ and ${Q_D^2}^* < K_D$

It follows that $\lambda = \nu_1 = \nu_2 = 0$. This case means the firm does not use up all its available budget and capacity investment K_D for production. From (6.24) and (6.25), we obtain:

$$\begin{aligned} \left(1+\frac{1}{b}\right)\xi_i({Q_D^i}^*)^{\frac{1}{b}} &= y_D \\ \Rightarrow {Q_D^i}^* &= \xi_i^{-b}\left[\frac{1+\frac{1}{b}}{y_D}\right]^{-b} \end{aligned} \tag{6.32}$$

This solution is valid if it satisfies all other constraints, observe that, as ${Q_D^i}^* = \xi_i^{-b}\left[\frac{1+\frac{1}{b}}{y_D}\right]^{-b} > 0$ for $i = 1,2$, we need to check only for (6.21) and (6.22). Thus, the validity condition for (6.21) is:

$$\begin{aligned} \left(\xi_1^{-b}+\xi_2^{-b}\right)\left[\frac{1+\frac{1}{b}}{y_D}\right]^{-b} &< \frac{R_D}{y_D} \\ \Rightarrow \xi_1^{-b}+\xi_2^{-b} &< \frac{R_D}{y_D}\left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \end{aligned} \tag{6.33}$$

The condition for (6.22) is:

$$\begin{aligned} \xi_i^{-b}\left[\frac{1+\frac{1}{b}}{y_D}\right]^{-b} &\leq K_D \\ \Rightarrow \xi_i^{-b} &\leq K_D\left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \end{aligned} \tag{6.34}$$

Subcase 2.2: ${Q_D^1}^* + {Q_D^2}^* < \frac{R_D}{y_D}$, ${Q_D^2}^* < K_D$ and ${Q_D^1}^* = K_D$

It follows that $\lambda = \nu_2 = 0$ and $\nu_1 \geq 0$. This case means the firm uses up all the invested capacity for product 1 K_D, but not all its capacity for product 2. From (6.25), we obtain:

$$\left(1+\frac{1}{b}\right)\xi_2({Q_D^2}^*)^{\frac{1}{b}} = y_D$$

$$
\begin{aligned}
\Rightarrow {Q_D^2}^* &= \xi_2^{-b}\left[\frac{1+\frac{1}{b}}{y_D}\right]^{-b} \\
{Q_D^1}^* &= K_D
\end{aligned}
\tag{6.35}
$$

This solution is valid if it satisfies all other constraints and $\nu_1 \geq 0$, observe that, as ${Q_D^2}^* = \xi_2^{-b}\left[\frac{1+\frac{1}{b}}{y_D}\right]^{-b} > 0$, we need to check only for (6.21) and (6.22). Note that, as $\max(K_D, K_D) \leq \frac{R_D}{y_D} < K_D + K_D$ and $Q_D^1 = K_D$, any solution that satisfies (6.21) will also satisfies (6.22). It follows that:

$$
\begin{aligned}
{Q_D^1}^* + {Q_D^2}^* &= K_D + \xi_2^{-b}\left[\frac{1+\frac{1}{b}}{y_D}\right]^{-b} \\
&\leq \frac{R_D}{y_D} \\
\Rightarrow \xi_2^{-b} &\leq \left(\frac{R_D}{y_D} - K_D\right)\left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b}
\end{aligned}
\tag{6.36}
$$

The condition for $\nu_1 \geq 0$ is:

$$
\begin{aligned}
\nu_1 &= \left(1+\frac{1}{b}\right)\xi_1({Q_D^1}^*)^{\frac{1}{b}} - y_D \\
&\geq 0 \\
\Rightarrow \xi_1^{-b} &\geq K_D\left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b}
\end{aligned}
\tag{6.37}
$$

Subcase 2.3: ${Q_D^1}^* + {Q_D^2}^* < \frac{R_D}{y_D}$, ${Q_D^1}^* < K_D$ and ${Q_D^2}^* = K_D$

This case is symmetric to the subcase 3.2 and thus, the optimal production plan is achieved as follows:

$$
\begin{aligned}
{Q_D^1}^* &= \xi_1^{-b}\left[\frac{1+\frac{1}{b}}{y_D}\right]^{-b} \\
{Q_D^2}^* &= K_D
\end{aligned}
\tag{6.38}
$$

and the condition for it to be valid is:

$$
\begin{aligned}
\xi_1^{-b} &\leq \left(\frac{R_D}{y_D} - K_D\right)\left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \\
\xi_2^{-b} &\geq K_D\left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b}
\end{aligned}
$$

Subcase 2.4: ${Q_D^1}^* + {Q_D^2}^* = \frac{R_D}{y_D}$, ${Q_D^2}^* \leq K_D$ and ${Q_D^1}^* = K_D$

It follows that $\lambda \geq 0$, $\nu_2 = 0$ and $\nu_1 \geq 0$. This case means the financial limit is binding while the firm uses up all the invested capacity for product 1 K_D, but not all capacity K_D. We obtain:

$$\begin{aligned} {Q_D^1}^* &= K_D \\ {Q_D^2}^* &= \frac{R_D}{y_D} - K_D \end{aligned}$$

This solution is valid if it satisfies all other constraints, $\lambda \geq 0$ and $\nu_1 \geq 0$, observe that, as ${Q_D^2}^* = \frac{R_D}{y_D} - K_D \leq K_D$, for $\max(K_D, K_D) \leq \frac{R_D}{y_D} < K_D + K_D$. We need to check only for for the nonnegativity of λ and ν_1. It follows that: The condition for $\lambda \geq 0$ is

$$\begin{aligned} \lambda &= \xi_2({Q_D^2}^*)^{\frac{1}{b}} - y_D \\ &= \xi_2(\frac{R_D}{y_D} - K_D)^{\frac{1}{b}} - y_D \\ &\geq 0 \\ \Rightarrow \xi_2^{-b} &\geq \left(\frac{R_D}{y_D} - K_D\right)\left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \end{aligned} \tag{6.39}$$

The condition for $\nu_1 \geq 0$ is

$$\begin{aligned} \nu_1 &= \xi_1({Q_D^1}^*)^{\frac{1}{b}} - y_D - \lambda \\ &\geq 0 \end{aligned}$$

After some algebra, this is equal to

$$\xi_1^{-b}\left(\frac{R_D}{y_D} - K_D\right) \geq \xi_2^{-b} K_D \tag{6.40}$$

Subcase 2.5: ${Q_D^1}^* + {Q_D^2}^* = \frac{R_D}{y_D}$, ${Q_D^2}^* = K_D$ and ${Q_D^1}^* \leq K_D$

This subcase is symmetric to the above subcase 3.4 and thus, the optimal production plan is:

$$\begin{aligned} {Q_D^2}^* &= K_D \\ {Q_D^1}^* &= \frac{R_D}{y_D} - K_D \end{aligned} \tag{6.41}$$

The validity condition is:

$$\xi_1^{-b} \geq \left(\frac{R_D}{y_D} - K_D\right)\left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \tag{6.42}$$

$$\xi_2^{-b}\left(\frac{R_D}{y_D} - K_D\right) \geq \xi_1^{-b} K_D \tag{6.43}$$

Subcase 2.6: $Q_D^1 + Q_D^2 = \frac{R_D}{y_D}$, ${Q_D^1}^* < K_D$ and ${Q_D^2}^* < K_D$

It follows that $\lambda \geq 0$ and $\nu_1 = \nu_2 = 0$. This case means the firm uses up all its available budget for production but the physical capacity constraints for both products are not binding. From (6.24) and (6.25) and $Q_D^1 + Q_D^2 = \frac{R_D}{y_D}$, we obtain:

$${Q_D^i}^* = \frac{R_D}{y_D}\left(\frac{\xi_i^{-b}}{\xi_1^{-b} + \xi_2^{-b}}\right) \tag{6.44}$$

This solution is valid if it satisfies all other constraints and $\lambda \geq 0$, observe that, as ${Q_D^i}^* = \frac{R_D}{y_D}\left(\frac{\xi_i^{-b}}{\xi_1^{-b}+\xi_2^{-b}}\right) > 0$ for $i = 1,2$, we will only need to check for (6.22), which implies:

$$\begin{aligned} {Q_D^i}^* &= \frac{R_D}{y_D}\left(\frac{\xi_1^{-b}}{\xi_1^{-b} + \xi_2^{-b}}\right) \\ &\leq K_D \\ \xi_1^{-b}\left(\frac{R_D}{y_D} - K_D\right) &\leq \xi_2^{-b} K_D & (6.45) \\ \xi_2^{-b}\left(\frac{R_D}{y_D} - K_D\right) &\leq \xi_1^{-b} K_D & (6.46) \end{aligned}$$

The condition for $\lambda \geq 0$ is $\lambda = \xi_i ({Q_D^i}^*)^{\frac{1}{b}} - y_D \geq 0$ for $i = 1,2$. After some algebra, the condition for (6.30) to be valid is:

$$\xi_1^{-b} + \xi_2^{-b} \geq \frac{R_D}{y_D}\left[\frac{y_D}{1 + \frac{1}{b}}\right]^{-b} \tag{6.47}$$

Combining all the subcases, we obtain the optimal production plan for $\max(K_D, K_D) \leq \frac{R_D}{y_D} < K_D +$

K_D (i.e. $\underline{B}_D \leq B^2 < \overline{B}_D$) as follows:

$$Q_D^*\left(B^2,\xi\right)' = \begin{cases} \left(\left[\frac{\left(1+\frac{1}{b}\right)\xi_1}{y_D}\right]^{-b}, \left[\frac{\left(1+\frac{1}{b}\right)\xi_2}{y_D}\right]^{-b}\right) & \text{if} \quad \xi \in \Omega^1_{D\underline{B}_D \leq B^2 < \overline{B}_D} \\ \left(K_D, \left[\frac{\left(1+\frac{1}{b}\right)\xi_2}{y_D}\right]^{-b}\right) & \text{if} \quad \xi \in \Omega^2_{D\underline{B}_D \leq B^2 < \overline{B}_D} \\ \left(\left[\frac{\left(1+\frac{1}{b}\right)\xi_1}{y_D}\right]^{-b}, K_D\right) & \text{if} \quad \xi \in \Omega^3_{D\underline{B}_D \leq B^2 < \overline{B}_D} \\ \left(K_D, \frac{R_D}{y_D} - K_D\right) & \text{if} \quad \xi \in \Omega^4_{D\underline{B}_D \leq B^2 < \overline{B}_D} \\ \left(\frac{R_D}{y_D} - K_D, K_D\right) & \text{if} \quad \xi \in \Omega^5_{D\underline{B}_D \leq B^2 < \overline{B}_D} \\ \left(\frac{R_D}{y_D}\frac{\xi_1{}^{-b}}{\xi_1{}^{-b}+\xi_2{}^{-b}}, \frac{R_D}{y_D}\frac{\xi_2{}^{-b}}{\xi_1{}^{-b}+\xi_2{}^{-b}}\right) & \text{if} \quad \xi \in \Omega^6_{D\underline{B}_D \leq B^2 < \overline{B}_D} \end{cases}$$

where

$$\Omega^1_{D\underline{B}_D \leq B^2 < \overline{B}_D} \quad \doteq \quad \xi : \begin{cases} \xi_1^{-b} + \xi_2^{-b} < \frac{R_D}{y_D}\left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \\ \xi_1^{-b} \leq K_D \left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \end{cases}$$

$$\Omega^2_{D\underline{B}_D \leq B^2 < \overline{B}_D} \quad \doteq \quad \xi : \begin{cases} \xi_2^{-b} \leq \left(\frac{R_D}{y_D} - K_D\right)\left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \\ \xi_1^{-b} \geq K_D \left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \end{cases}$$

$$\Omega^3_{D\underline{B}_D \leq B^2 < \overline{B}_D} \quad \doteq \quad \xi : \begin{cases} \xi_1^{-b} \leq \left(\frac{R_D}{y_D} - K_D\right)\left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \\ \xi_2^{-b} \geq K_D \left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \end{cases}$$

$$\Omega^4_{D\underline{B}_D \leq B^2 < \overline{B}_D} \quad \doteq \quad \xi : \begin{cases} \xi_2^{-b} \geq \left(\frac{R_D}{y_D} - K_D\right)\left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \\ \xi_1^{-b}\left(\frac{R_D}{y_D} - K_D\right) \geq \xi_2^{-b} K_D \end{cases}$$

$$\Omega^5_{D\underline{B}_D \leq B^2 < \overline{B}_D} \quad \doteq \quad \xi : \begin{cases} \xi_1^{-b} \geq \left(\frac{R_D}{y_D} - K_D\right)\left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \\ \xi_2^{-b}\left(\frac{R_D}{y_D} - K_D\right) \leq \xi_1^{-b} K_D \end{cases}$$

$$\Omega^6_{D\underline{B}_D \leq B^2 < \overline{B}_D} \doteq \xi : \begin{cases} \xi_1^{-b}\left(\frac{R_D}{y_D} - K_D\right) \leq \xi_2^{-b} K_D \\ \xi_2^{-b}\left(\frac{R_D}{y_D} - K_D\right) \leq \xi_1^{-b} K_D \\ \xi_1^{-b} + \xi_2^{-b} \geq \frac{R_D}{y_D}\left[\frac{y_F}{1+\frac{1}{b}}\right]^{-b} \end{cases}$$

Note that, for $\max(K_D, K_D) \leq \frac{R_D}{y_D} < K_D + K_D$:

$$\begin{aligned} \min\left(\left(\frac{R_D}{y_D} - K_D\right)^+, K_D\right) &= \frac{R_D}{y_D} - K_D \\ \min\left(\left(\frac{R_D}{y_D} - K_D\right)^+, K_D\right) &= \frac{R_D}{y_D} - K_D \end{aligned}$$

Thus, from (4.1), $\Omega^i_D = \Omega^i_{D\underline{B}_D \leq B^2 < \overline{B}_D}$ for $i = 1, 2, ..., 6$ and the solution in (4.1) is matched to the above solution.

Case 3: $B^2 \geq \overline{B}_D$

It follows that $\frac{R_D}{y_D} \geq 2K_D$. This case means the available budget is sufficient to cover full production for both products. Therefore, the financial constraint is never binding. Thus, it follows that $\lambda = 0$.

Subcase 3.1: ${Q^2_D}^* < K_D$ and ${Q^1_D}^* < K_D$

It follows that $v_1 = 0$ and $v_2 = 0$. This case means the firm does not use up all its available capacity investment K_D for production. From (6.24) and (6.25), we obtain:

$$\begin{aligned} \left(1 + \frac{1}{b}\right) \xi_i ({Q^i_D}^*)^{\frac{1}{b}} &= y_D \\ \Rightarrow {Q^i_D}^* &= \xi_i^{-b}\left[\frac{1+\frac{1}{b}}{y_D}\right]^{-b} \end{aligned} \tag{6.48}$$

This solution is valid if it satisfies all other constraints, observe that, as ${Q^i_D}^* = \xi_i^{-b}\left[\frac{1+\frac{1}{b}}{y_D}\right]^{-b} > 0$ for $i = 1, 2$, we need to check only for (6.21) and (6.22). Also, as $\frac{R_D}{y_D} \geq 2K_D$ in this case, it follows that if the solution satisfies (6.22), then (6.21) will also hold. The condition for (6.22) to hold is:

$$\begin{aligned} {Q^i_D}^* &= \xi_i^{-b}\left[\frac{1+\frac{1}{b}}{y_D}\right]^{-b} \\ &\leq K_D \\ \Rightarrow \xi_i^{-b} &\leq K_D\left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \end{aligned} \tag{6.49}$$

Subcase 3.2: ${Q_D^2}^* < K_D$ and ${Q_D^1}^* = K_D$

It follows that $v_2 = 0$ and $v_1 \geq 0$. This case means the firm uses up all the invested capacity for product 1 K_D, but not all its capacity for product 2. From (6.25), we obtain:

$$\begin{aligned}
\left(1+\frac{1}{b}\right)\xi_2({Q_D^2}^*)^{\frac{1}{b}} &= y_D \\
\Rightarrow {Q_D^2}^* &= \xi_2^{-b}\left[\frac{1+\frac{1}{b}}{y_D}\right]^{-b} \\
{Q_D^1}^* &= K_D
\end{aligned} \tag{6.50}$$

This solution is valid if it satisfies all other constraints and $v_1 \geq 0$, observe that, as ${Q_D^2}^* = \xi_2^{-b}\left[\frac{1+\frac{1}{b}}{y_D}\right]^{-b} > 0$, we need to check only for (6.21) and (6.22). Note that, as $\frac{R_D}{y_D} \geq 2K_D$ and $Q_D^1 = K_D$, any solution that satisfies (6.22) will also satisfies (6.21). It follows that:

$$\begin{aligned}
{Q_D^2}^* &= \xi_2^{-b}\left[\frac{1+\frac{1}{b}}{y_D}\right]^{-b} \\
&\leq K_D \\
\Rightarrow \xi_2^{-b} &\leq K_D\left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b}
\end{aligned} \tag{6.51}$$

The condition for $v_1 \geq 0$ is:

$$\begin{aligned}
v_1 &= \left(1+\frac{1}{b}\right)\xi_1({Q_D^1}^*)^{\frac{1}{b}} - y_D \\
&\geq 0 \\
\Rightarrow \xi_1^{-b} &\geq K_D\left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b}
\end{aligned} \tag{6.52}$$

Subcase 3.3: ${Q_D^1}^* < K_D$ and ${Q_D^2}^* = K_D$

This case is symmetric to the subcase 4.2 and thus, the optimal production plan is achieved as follows:

$$\begin{aligned}
{Q_D^1}^* &= \xi_1^{-b}\left[\frac{1+\frac{1}{b}}{y_D}\right]^{-b} \\
{Q_D^2}^* &= K_D
\end{aligned} \tag{6.53}$$

and the condition for it to be valid is:

$$\xi_1^{-b} \leq K_D\left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b}$$

$$\xi_2^{-b} \geq K_D \left[\frac{y_D}{1+\frac{1}{b}} \right]^{-b}$$

Subcase 3.4: ${Q_D^1}^* = K_D$ and ${Q_D^2}^* = K_D$

It follows that $v_1 \geq 0$ and $v_2 \geq 0$. This case means the firm uses up all its available capacity for production. The non-negativity of v ensures the validity of this solution. From (6.24) and (6.25):

$$\begin{aligned} v_i &= \left(1+\frac{1}{b}\right)\xi_i({Q_D^i}^*)^{\frac{1}{b}} - y_D \\ &= \left(1+\frac{1}{b}\right)\xi_i({K_D}^*)^{\frac{1}{b}} - y_D \\ &\geq 0 \\ \Rightarrow \xi_i^{-b} &\geq K_D \left[\frac{y_D}{1+\frac{1}{b}} \right]^{-b} \end{aligned} \tag{6.54}$$

Combining all the subcases, we obtain the optimal production plan for $\frac{R_D}{y_D} \geq 2K_D$ (i.e. $B^2 \geq \overline{B}_D$) as follows:

$$Q_D^*\left(B^2,\xi\right)' = \begin{cases} \left(\left[\frac{\left(1+\frac{1}{b}\right)\xi_1}{y_D} \right]^{-b}, \left[\frac{\left(1+\frac{1}{b}\right)\xi_2}{y_D} \right]^{-b} \right) & \text{if} \quad \xi \in \Omega^1_{DB^2 \geq \overline{B}_D} \\ \left(K_D, \left[\frac{\left(1+\frac{1}{b}\right)\xi_2}{y_D} \right]^{-b} \right) & \text{if} \quad \xi \in \Omega^2_{DB^2 \geq \overline{B}_D} \\ \left(\left[\frac{\left(1+\frac{1}{b}\right)\xi_1}{y_D} \right]^{-b}, K_D \right) & \text{if} \quad \xi \in \Omega^3_{DB^2 \geq \overline{B}_D} \\ (K_D, K_D) & \text{if} \quad \xi \in \Omega^4_{DB^2 \geq \overline{B}_D} \end{cases}$$

where

$$\Omega^1_{DB^2 \geq \overline{B}_D} \doteq \xi : \left\{ \ \xi_i^{-b} \leq K_D \left[\frac{y_D}{1+\frac{1}{b}} \right]^{-b} \right.$$

$$\Omega^2_{DB^2 \geq \overline{B}_D} \doteq \xi : \begin{cases} \xi_1^{-b} \geq K_D \left[\frac{y_D}{1+\frac{1}{b}} \right]^{-b} \\ \xi_2^{-b} \leq K_D \left[\frac{y_D}{1+\frac{1}{b}} \right]^{-b} \end{cases}$$

$$\Omega^3_{DB^2 \geq \overline{B}_D} \doteq \xi : \begin{cases} \xi_1^{-b} \leq K_D \left[\frac{y_D}{1+\frac{1}{b}} \right]^{-b} \\ \xi_2^{-b} \geq K_D \left[\frac{y_D}{1+\frac{1}{b}} \right]^{-b} \end{cases}$$

$$\Omega^4_{DB^2 \geq \overline{B}_D} \doteq \xi : \left\{ \quad \xi_i^{-b} \geq K_D \left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \right.$$

Note that, for $\frac{R_D}{y_D} \geq 2K_D$:

$$\min\left(\left(\frac{R_D}{y_D} - K_D\right)^+, K_D\right) = K_D$$

$$\min\left(\left(\frac{R_D}{y_D} - K_D\right)^+, K_D\right) = K_D$$

Thus, from (4.1), $\Omega_D{}^6$ region vanishes and also note that $\Omega^4_{DB^2 \geq \overline{B}_D} = \Omega^4_D + \Omega^5_D$. Therefore the above solution is the same as the solution given by (4.1) in proposition 5, which completes the proof. ■

Proof of Proposition 6 As shown in the proof of proposition 5, the Ω^i_D regions w.r.t. different stage 2 budget realizations are given by,

For $B^2 < \underline{B}_D$,

$$\Omega^1_{DB^2 < \underline{B}_D} \doteq \left\{ \xi : 0 \leq \xi_1^{-b} + \xi_2^{-b} < \left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \frac{R_D}{y_D} \right\}$$

$$\Omega^6_{DB^2 < \underline{B}_D} \doteq \left\{ \xi : \xi_1^{-b} + \xi_2^{-b} \geq \left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \frac{R_D}{y_D} \right\}.$$

For $\underline{B}_D \leq B^2 < \overline{B}_D$,

$$\Omega^1_{D\underline{B}_D \leq B^2 < \overline{B}_D} \doteq \xi : \begin{cases} \xi_1^{-b} + \xi_2^{-b} < \frac{R_D}{y_D}\left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \\ \xi_1^{-b} \leq K_D \left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \end{cases}$$

$$\Omega^2_{D\underline{B}_D \leq B^2 < \overline{B}_D} \doteq \xi : \begin{cases} \xi_2^{-b} \leq \left(\frac{R_D}{y_D} - K_D\right)\left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \\ \xi_1^{-b} \geq K_D \left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \end{cases}$$

$$\Omega^3_{D\underline{B}_D \leq B^2 < \overline{B}_D} \doteq \xi : \begin{cases} \xi_1^{-b} \leq \left(\frac{R_D}{y_D} - K_D\right)\left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \\ \xi_2^{-b} \geq K_D \left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \end{cases}$$

$$\Omega^4_{D\underline{B}_D\le B^2<\overline{B}_D} \doteq \xi : \begin{cases} \xi_2^{-b} \ge \left(\frac{R_D}{y_D} - K_D\right)\left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \\ \xi_1^{-b}\left(\frac{R_D}{y_D} - K_D\right) \ge \xi_2^{-b} K_D \end{cases}$$

$$\Omega^5_{D\underline{B}_D\le B^2<\overline{B}_D} \doteq \xi : \begin{cases} \xi_1^{-b} \ge \left(\frac{R_D}{y_D} - K_D\right)\left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \\ \xi_2^{-b}\left(\frac{R_D}{y_D} - K_D\right) \le \xi_1^{-b} K_D \end{cases}$$

$$\Omega^6_{D\underline{B}_D\le B^2<\overline{B}_D} \doteq \xi : \begin{cases} \xi_1^{-b}\left(\frac{R_D}{y_D} - K_D\right) \le \xi_2^{-b} K_D \\ \xi_2^{-b}\left(\frac{R_D}{y_D} - K_D\right) \le \xi_1^{-b} K_D \\ \xi_1^{-b} + \xi_2^{-b} \ge \frac{R_D}{y_D}\left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \end{cases}$$

And for $B^2 \ge \overline{B}_D$,

$$\Omega^1_{DB^2\ge\overline{B}_D} \doteq \xi : \left\{ \xi_i^{-b} \le K_D\left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \right.$$

$$\Omega^2_{DB^2\ge\overline{B}_D} \doteq \xi : \begin{cases} \xi_1^{-b} \ge K_D\left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \\ \xi_2^{-b} \le K_D\left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \end{cases}$$

$$\Omega^3_{DB^2\ge\overline{B}_D} \doteq \xi : \begin{cases} \xi_1^{-b} \le K_D\left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \\ \xi_2^{-b} \ge K_D\left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \end{cases}$$

$$\Omega^4_{DB^2\ge\overline{B}_D} \doteq \xi : \left\{ \xi_i^{-b} \ge K_D\left[\frac{y_D}{1+\frac{1}{b}}\right]^{-b} \right.$$

Recall that $\Pi_D \doteq \mathbb{E}\left[\Psi_D(K_D, B^2, \xi)\right]$ is the objective function in (4.2). Taking the first-order derivative w.r.t. K_D and after some algebra, we obtain

$$\frac{\partial \Pi_D}{\partial K_D} = \overline{B}^2 + B^1 - F_D - 2c_D K_D$$

$$
\begin{aligned}
&+\int_{B^l}^{\min(\max(B^l,\underline{B}_D),B^u)} H_D^1(K_D,B^2)\, dF(B^2) \\
&+\int_{\min(\max(B^l,\underline{B}_D),B^u)}^{\min(\max(B^l,\overline{B}_D),B^u)} H_D^2(K_D,B^2)\, dF(B^2) \\
&+\int_{\min(\max(B^l,\overline{B}_D),B^u)}^{B^u} H_D^3(K_D,B^2)\, dF(B^2).
\end{aligned} \tag{6.55}
$$

where

$$
H_D^1(K_D,B^2) = \iint_{\Omega^6_{DB^2<\underline{B}_D}} \left[\left(\frac{-2c_D}{y_D}\right)(1+1/b)\left(\frac{R_D}{y_D}\right)^{(1/b)}\left[\xi_1^{-b}+\xi_2^{-b}\right]^{\frac{-1}{b}}+2c_D\right] \mathrm{d}\Phi(\xi)
$$

and

$$
\begin{aligned}
H_D^2(K_D,B^2) &= \iint_{\Omega^2_{D\underline{B}_D\le B^2<\overline{B}_D}} \left[\xi_1 K_D^{1/b}\left(1+\frac{1}{b}\right)-y_D\right] \mathrm{d}\Phi(\xi) \\
&+\iint_{\Omega^3_{D\underline{B}_D\le B^2<\overline{B}_D}} \left[\xi_2 K_D^{1/b}\left(1+\frac{1}{b}\right)-y_D\right] \mathrm{d}\Phi(\xi) \\
&+\iint_{\Omega^4_{D\underline{B}_D\le B^2<\overline{B}_D}} \left[\left(\xi_1 K_D^{1/b}-\xi_2\left(\frac{2c_d+y_D}{y_D}\right)\left(\frac{R_D}{y_D}-K_D\right)^{1/b}\right)\left(1+\frac{1}{b}\right)+2c_D\right] \mathrm{d}\Phi(\xi) \\
&+\iint_{\Omega^5_{D\underline{B}_D\le B^2<\overline{B}_D}} \left[\left(\xi_2 K_D^{1/b}-\xi_1\left(\frac{2c_d+y_D}{y_D}\right)\left(\frac{R_D}{y_D}-K_D\right)^{1/b}\right)\left(1+\frac{1}{b}\right)+2c_D\right] \mathrm{d}\Phi(\xi) \\
&+\iint_{\Omega^6_{D\underline{B}_D\le B^2<\overline{B}_D}} \left[\left(\frac{-2c_D}{y_D}\right)(1+1/b)\left(\frac{R_D}{y_D}\right)^{(1/b)}\left[\xi_1^{-b}+\xi_2^{-b}\right]^{\frac{-1}{b}}+2c_D\right] \mathrm{d}\Phi(\xi) \\
H_D^3(K_D,B^2) &= \iint_{\Omega^2_{DB^2\ge\overline{B}_D}} \left[\xi_1 K_D^{1/b}\left(1+\frac{1}{b}\right)-y_D\right] \mathrm{d}\Phi(\xi) \\
&+\iint_{\Omega^3_{DB^2\ge\overline{B}_D}} \left[\xi_2 K_D^{1/b}\left(1+\frac{1}{b}\right)-y_D\right] \mathrm{d}\Phi(\xi) \\
&+\iint_{\Omega^4_{DB^2\ge\overline{B}_D}} \left[(\xi_1+\xi_2) K_D^{1/b}\left(1+\frac{1}{b}\right)-2y_D\right] \mathrm{d}\Phi(\xi)
\end{aligned}
$$

where $\Phi(.)$ is the cumulative distribution function of the random demand ξ. In parallel to the proof of proposition 2 for flexible technology, we can establish that Π_D is concave in K_D, and thus, the detailed proof is omitted here. It follows that the optimal capacity investment level with dedicated technology K_D^* is characterized by (4.3). ■

Proof of Corollary 4: We first note that (4.4) is directly obtained from the clearance assumption. We will only focus on the firm's production decision when $K_D \le \frac{R_D}{y_D} < 2K_D$. Under clearance assumption, the firm's stage 2 optimization problem is given by

$$\begin{aligned}\Psi_D = \max_{Q_D} \quad & R_D + \xi_1 (Q_D^1)^{1+\frac{1}{b}} + \xi_2 (Q_D^2)^{1+\frac{1}{b}} - y_D (Q_D^1 + Q_D^2) \\ \text{s.t.} \quad & y_D (Q_D^1 + Q_D^2) = R_D \\ & 0 \le Q_D^1 \le K_D \\ & 0 \le Q_D^2 \le K_D\end{aligned}$$

Substituting Q_D^2 with $\frac{R_D}{y_D} - Q_D^1$ and after some algebra, we obtain

$$\begin{aligned}\Psi_D = \max_{Q_D^1} \quad & \xi_1 (Q_D^1)^{1+\frac{1}{b}} + \xi_2 \left(\frac{R_D}{y_D} - Q_D^1 \right)^{1+\frac{1}{b}} \\ \text{s.t.} \quad & 0 \le Q_D^1 \le K_D\end{aligned}$$

The above single variable optimization problem is easily solved and its solution is characterized by (4.5), which completes the proof. ■

Proof of Proposition 7: Form Lemma 3, to demonstrate Proposition 7, it is sufficient to show the concavity of expected profit Π_D in stage 2 budget B^2. Taking the first-order derivative of Π_D in (4.6) w.r.t. B^2 and after some algebra, we obtain

$$\begin{aligned}\frac{\partial \Pi_D}{\partial B^2} &= \int_{B^l}^{\min(\max(B^l,\underline{B}_D),B^u)} \left[\frac{\partial G_D^{1c}}{\partial B^2}\right] \mathrm{d}F(B^2) \\ &\quad + \int_{\min(\max(B^l,\underline{B}_D),B^u)}^{\min(\max(B^l,\overline{B}_D),B^u)} \left[\frac{\partial G_D^{2c}}{\partial B^2}\right] \mathrm{d}F(B^2) \\ &\quad + \int_{\min(\max(B^l,\overline{B}_D),B^u)}^{B^u} \left[\frac{\partial G_D^{3c}}{\partial B^2}\right] \mathrm{d}F(B^2)\end{aligned} \tag{6.56}$$

where

$$\begin{aligned}\frac{\partial G_D^{1c}}{\partial B^2} &= \left(1 + \frac{1}{b}\right) \left(\frac{R_D}{y_D}\right)^{\frac{1}{b}} \left(\frac{1}{y_D}\right) \mathbb{E}\left[(\xi_1^{-b} + \xi_2^{-b})^{-\frac{1}{b}}\right] \\ \frac{\partial G_D^{2c}}{\partial B^2} &= \iint_{\Omega_D^{1c}} \left[\xi_2 \left(1 + \frac{1}{b}\right) \left(\frac{R_D}{y_D} - K_D\right)^{\frac{1}{b}} \left(\frac{1}{y_D}\right)\right] \mathrm{d}\Phi(\xi_1, \xi_2) \\ &\quad + \iint_{\Omega_D^{2c}} \left[\xi_1 \left(1 + \frac{1}{b}\right) \left(\frac{R_D}{y_D} - K_D\right)^{\frac{1}{b}} \left(\frac{1}{y_D}\right)\right] \mathrm{d}\Phi(\xi_1, \xi_2)\end{aligned}$$

$$+\iint_{\Omega_D^{3c}} \left[\left(1+\frac{1}{b}\right) \left(\frac{R_D}{y_D}\right)^{\frac{1}{b}} \left(\frac{1}{y_D}\right) (\xi_1^{-b}+\xi_2^{-b})^{-\frac{1}{b}} \right] \mathrm{d}\Phi(\xi_1,\xi_2)$$

$$\frac{\partial G_D^{3c}}{\partial B^2} = 1$$

It is easily obtained that G_D^{1c} and G_D^{2c} are concave in B^2. Moreover,

$$\left.\frac{\partial G_D^{1c}}{\partial B^2}\right|_{B^2=\underline{B}^2} = \left.\frac{\partial G_D^{2c}}{\partial B^2}\right|_{B^2=\underline{B}^2}$$

We note that under Assumption 2, we further obtain

$$\left.\frac{\partial G_D^{2c}}{\partial B^2}\right|_{B^2=\overline{B}^2} \geq \left.\frac{\partial G_D^{3c}}{\partial B^2}\right|_{B^2=\overline{B}^2} = 1$$

Therefore, we can conclude that Π_D is concave in B^2, which also completes the proof. ■

Proof of Proposition 8: With deterministic stage 2 budget, the firm's stage 1 profit Π_D can be directly derived from (4.6):

$$\Pi_D = \begin{cases} G_D^{3c} & \text{if } K_D \in \left[0, \min\left(\underline{K}_D, \frac{B^1-F_D}{2c_D}\right)\right] \\ G_D^{2c} & \text{if } K_D \in \left(\min\left(\underline{K}_D, \frac{B^1-F_D}{2c_D}\right), \min\left(\overline{K}_D, \frac{B^1-F_D}{2c_D}\right)\right] \\ G_D^{1c} & \text{if } K_D \in \left(\min\left(\overline{K}_D, \frac{B^1-F_D}{2c_D}\right), \frac{B^1-F_D}{2c_D}\right] \end{cases} \tag{6.57}$$

where

$$\begin{aligned} G_D^{1c} &= \left(\frac{R_D}{y_D}\right)^{1+\frac{1}{b}} \mathbb{E}\left[(\xi_1^{-b}+\xi_2^{-b})^{-\frac{1}{b}}\right] \\ G_D^{2c} &= \iint_{\Omega_D^{1c}} \left[\xi_1 K_D^{1+\frac{1}{b}} + \xi_2 \left(\frac{R_D}{y_D} - K_D\right)^{1+\frac{1}{b}}\right] \mathrm{d}\Phi(\xi_1,\xi_2) \\ &\quad + \iint_{\Omega_D^{2c}} \left[\xi_1 \left(\frac{R_D}{y_D} - K_D\right)^{1+\frac{1}{b}} + \xi_2 K_D^{1+\frac{1}{b}}\right] \mathrm{d}\Phi(\xi_1,\xi_2) \\ &\quad + \iint_{\Omega_D^{3c}} \left[\left(\frac{R_D}{y_D}\right)^{1+\frac{1}{b}} (\xi_1^{-b}+\xi_2^{-b})^{-\frac{1}{b}}\right] \mathrm{d}\Phi(\xi_1,\xi_2) \\ G_D^{3c} &= R_D + 2\overline{\xi} K_D^{1+\frac{1}{b}} - 2y_D K_D \end{aligned}$$

Here, $\underline{K}_D = \frac{B^2+B^1-F_D}{2c_D+2y_D}$ and $\overline{K}_D = \frac{B^2+B^1-F_D}{2c_D+y_D}$ are discussed as in the paper. $\frac{B^1-F_D}{c_D}$ represents the firm's capacity investment limit for each product. It is easy to establish that Π_D is continuous everywhere. Taking the first-order derivative w.r.t. K_D, we obtain

$$\frac{\partial \Pi_D}{\partial K_D} = \begin{cases} \frac{\partial G_D^{3c}}{\partial K_D} & \text{if } K_D \in \left[0, \min\left(\underline{K}_D, \frac{B^1-F_D}{2c_D}\right)\right] \\ \frac{\partial G_D^{2c}}{\partial K_D} & \text{if } K_D \in \left(\min\left(\underline{K}_D, \frac{B^1-F_D}{2c_D}\right), \min\left(\overline{K}_D, \frac{B^1-F_D}{2c_D}\right)\right] \\ \frac{\partial G_D^{1c}}{\partial K_D} & \text{if } K_D \in \left(\min\left(\overline{K}_D, \frac{B^1-F_D}{2c_D}\right), \frac{B^1-F_D}{2c_D}\right] \end{cases} \tag{6.58}$$

where

$$\begin{aligned}
\frac{\partial G_D^{1c}}{\partial K_D} &= \left(1+\frac{1}{b}\right)\left(\frac{R_D}{y_D}\right)^{\frac{1}{b}}\left(\frac{-2c_D}{y_D}\right)\mathbb{E}\left[(\xi_1^{-b}+\xi_2^{-b})^{-\frac{1}{b}}\right] \\
\frac{\partial G_D^{2c}}{\partial K_D} &= \iint_{\Omega_D^{1c}}\left[\xi_1\left(1+\frac{1}{b}\right)K_D^{\frac{1}{b}} - \xi_2\left(1+\frac{1}{b}\right)\left(\frac{R_D}{y_D}-K_D\right)^{\frac{1}{b}}\left(\frac{2c_D}{y_D}+1\right)\right] \mathrm{d}\Phi(\xi_1,\xi_2) \\
&\quad + \iint_{\Omega_D^{2c}}\left[-\xi_1\left(1+\frac{1}{b}\right)\left(\frac{R_D}{y_D}-K_D\right)^{\frac{1}{b}}\left(\frac{2c_D}{y_D}+1\right)+\xi_2\left(1+\frac{1}{b}\right)K_D^{\frac{1}{b}}\right] \mathrm{d}\Phi(\xi_1,\xi_2) \\
&\quad + \iint_{\Omega_D^{3c}}\left[\left(1+\frac{1}{b}\right)\left(\frac{R_D}{y_D}\right)^{\frac{1}{b}}\left(\frac{-2c_D}{y_D}\right)(\xi_1^{-b}+\xi_2^{-b})^{-\frac{1}{b}}\right] \mathrm{d}\Phi(\xi_1,\xi_2) \\
\frac{\partial G_D^{3c}}{\partial K_D} &= -2c_D + 2\overline{\xi}\left(1+\frac{1}{b}\right)K_D^{\frac{1}{b}} - 2y_D
\end{aligned}$$

It is easy to see that $\frac{\partial G_D^{1c}}{\partial K_D} \leq 0$. Under Assumption 3, we can also establish $\frac{\partial G_D^{2c}}{\partial K_D} \leq 0$. Thus, it is never for the firm to purchase physical capacity that is more than $\min\left(\underline{K}_D, \frac{B^1-F_D}{2c_D}\right)$ limit. We note that G_D^{1c} is concave in K_D and the first-best capacity investment level $K_D^0 = \left[\frac{\overline{\xi}\left(1+\frac{1}{b}\right)}{c_D+y_D}\right]^{-b}$. We then analyze two cases to clearly characterize the optimal capacity investment level K_D^*.

Case 1: $\frac{B^1-F_D}{2c_D} \leq \underline{K}_D \Longleftrightarrow B^2 \geq y_D\left(\frac{B^1-F_D}{c_D}\right)$

We note that Π_D is concave in K_D and the firm can not invest in physical capacity above the capacity limit $\frac{B^1-F_D}{2c_D}$, therefore we obtain

$$K_D^* = \begin{cases} K_D^0 & \text{if } K_D^0 \leq \frac{B^1-F_D}{2c_D} \\ \frac{B^1-F_D}{2c_D} & \text{if } K_D^0 > \frac{B^1-F_D}{2c_D} \end{cases} \tag{6.59}$$

Case 2: $\frac{B^1-F_D}{2c_D} > \underline{K}_D \Longleftrightarrow B^2 < y_D\left(\frac{B^1-F_D}{c_D}\right)$

In this case, it is never optimal for the firm to invest in physical capacity more than $\underline{K}_D$ as we have discussed above. The firm's optimal capacity investment level is given by

$$K_D^* = \begin{cases} K_D^0 & \text{if } K_D^0 \le \underline{K}_D \\ \underline{K}_D & \text{if } K_D^0 > \underline{K}_D \end{cases} \tag{6.60}$$

We note that if $B^1 \ge 2(c_D + y_D)K_D^0 + F_D$, (6.60) yields,

$$K_F^* = K_D^0 \tag{6.61}$$

After combining the two cases and some algebra, we obtain the results. ■

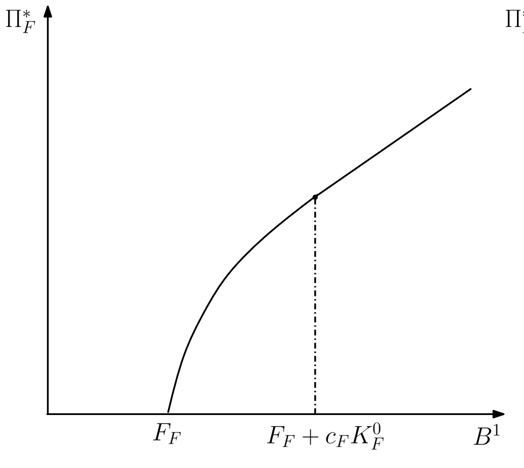
Proof of Proposition 9: Without production cost, Π_F in Corollary 3 and Π_D in Corollary 6 yield respectively,

$$\Pi_F^* = \begin{cases} B^2 + B^1 - F_F + \frac{c_F}{-(b+1)} K_F^0 & \text{if } B^1 > c_F K_F^0 + F_F \\ B^2 + M_F \left(\frac{B^1 - F_F}{c_F}\right)^{1+\frac{1}{b}} & \text{if } B^1 \le c_F K_F^0 + F_F \end{cases} \tag{6.62}$$

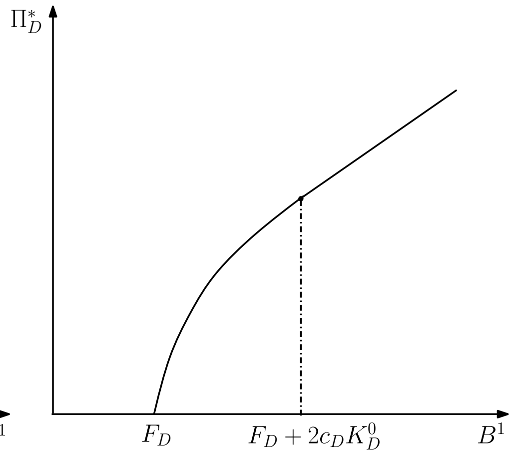
and

$$\Pi_D^* = \begin{cases} B^2 + B^1 - F_D + \frac{c_D}{-(b+1)} (2K_D^0) & \text{if } B^1 > 2c_D K_D^0 + F_D \\ B^2 + 2^{-\frac{1}{b}} \overline{\xi} \left(\frac{B^1 - F_D}{c_D}\right)^{1+\frac{1}{b}} & \text{if } B^1 \le 2c_D K_D^0 + F_D \end{cases} \tag{6.63}$$

where $K_F^0 = \left[\frac{(1+\frac{1}{b})M_F}{c_F}\right]^{-b}$ and $K_D^0 = \left[\frac{\overline{\xi}(1+\frac{1}{b})}{c_D}\right]^{-b}$. Figure 6.1 summarizes Π_F^* and Π_D^* with $y_T = 0$.



(a) Π_F^* with $y_T = 0$



(b) Π_D^* with $y_T = 0$

Figure 6.1: Π_T^* with $y_T = 0$.

We first note that Π_F and Π_D are continuous and differentiable everywhere in B^1. Recall that we have $F_F \geq F_D$. In order to explicitly characterize the variable cost threshold $\overline{c}_F$, we then analyze two case w.r.t. the ordering between F_F and $F_D \leq 2c_D K_D^0 + F_D$. For given B^1, we find the unique variable cost threshold $\overline{c}_F$ such that the firm is indifferent between the two technologies.

Case 1: $F_F \geq 2c_D K_D^0 + F_D$

In this case, the first unique variable cost threshold $\overline{c}_F^3$ solves

$$B^2 + B^1 - F_F + \frac{c_F}{-(b+1)} K_F^0 = B^2 + B^1 - F_D + \frac{c_D}{-(b+1)} (2K_D^0)$$

After some algebra, we obtain

$$\overline{c}_F^3 = \left[\frac{\left[\left(1+\frac{1}{b}\right) M_F\right]^{-b}}{(F_D - F_F)(b+1) + 2c_D K_D^0} \right]^{\frac{-1}{b+1}}$$

It is easy to establish that $\overline{c}_F^3$ is valid only for $B^1 \geq (F_F + c_F K_F^0)\big|_{\overline{c}_F^3} = F_D + 2c_D K_D^0 + b(F_D - F_F)$. And for $F_F \leq B^1 < F_D + 2c_D K_D^0 + b(F_D - F_F)$, the variable cost threshold $\overline{c}_F^2$ is given by

$$\overline{c}_F^2 = \left[\frac{M_F (B^1 - F_F)^{1+\frac{1}{b}}}{B^1 - F_D + \frac{2c_D K_D^0}{-(b+1)}} \right]^{\frac{b}{b+1}}$$

which uniquely solves the following equation:

$$B^2 + M_F \left(\frac{B^1 - F_F}{c_F} \right)^{1+\frac{1}{b}} = B^2 + B^1 - F_D + \frac{c_D}{-(b+1)} (2K_D^0)$$

It is easily verified that $\overline{c}_F^2$ increases in B^1 for $F_F \leq B^1 < F_D + 2c_D K_D^0 + b(F_D - F_F)$.

Case 2: $F_D \leq F_F < 2c_D K_D^0 + F_D$

In a similar fashion, we can establish that if $B^1 \geq F_D + 2c_D K_D^0 + b(F_D - F_F)$, the unique variable cost threshold is characterized by $\overline{c}_F^3$ and if $F_D + 2c_D K_D^0 \leq B^1 < F_D + 2c_D K_D^0 + b(F_D - F_F)$, it is $\overline{c}_F^2$. Finally, if $F_F \leq B^1 < F_D + 2c_D K_D^0$, the threshold $\overline{c}_F^1$ solves

$$B^2 + M_F \left(\frac{B^1 - F_F}{c_F} \right)^{1+\frac{1}{b}} = B^2 + 2^{-\frac{1}{b}} \overline{\xi} \left(\frac{B^1 - F_D}{c_D} \right)^{1+\frac{1}{b}}$$

and is given by

$$\overline{c}_F^1 = \left(\frac{M_F}{2^{-\frac{1}{b}} \overline{\xi}} \right)^{\frac{b}{b+1}} \left[\frac{B^1 - F_F}{B^1 - F_D} \right] c_D$$

With $F_F \geq F_D$, it is easy to demonstrate that $\overline{c}_F^1$ increases in stage 1 budget B^1. Combining the two cases and after some algebra, we obtain the result in (5.1). ■

Proof of Proposition 11: We first note that with symmetric production cost $y_F = y_D = y$ and no fixed cost $F_F = F_D = 0$, Π_D^* is given by (4.13) as depicted in Figure 4.5c, when $B^1 \leq 2c_D K_D^0$. It is easy to verify that when $c_F = c_D$, we have $\Pi_F^* \geq \Pi_D^*$ and $B^1 \leq 2c_D K_D^0 \leq c_F K_F^0$. Thus, Π_F^* is characterized by (3.12) as shown in Figure 3.2c, when $c_F = c_D$. As c_F increases from c_D, Π_F^* shifts correspondingly from Figure 6.2a to Figure 6.2b, and to Figure 6.2c finally.

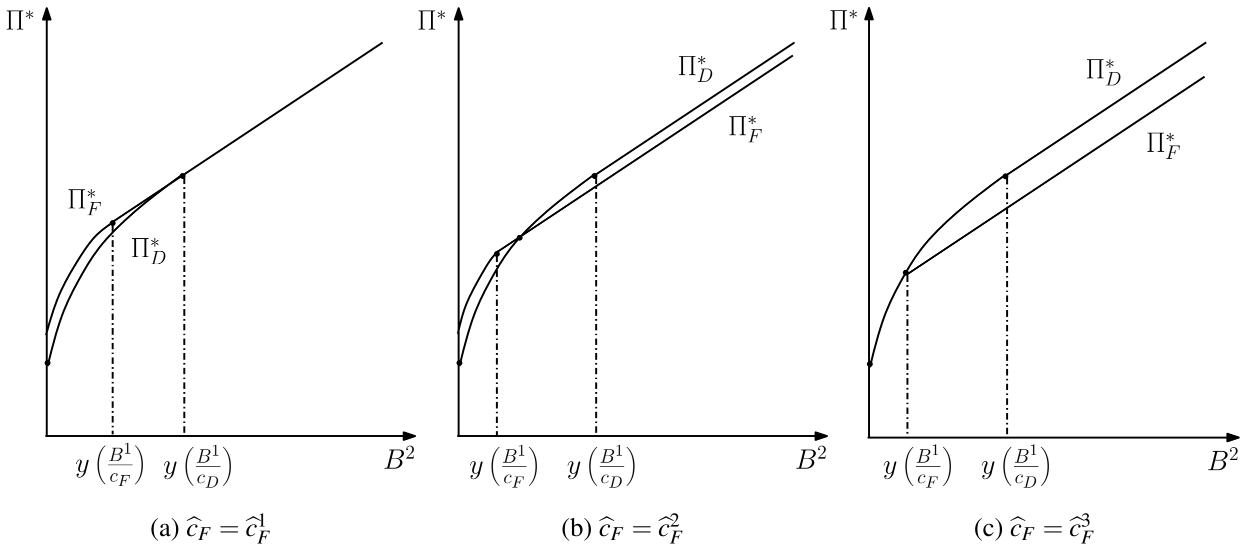


Figure 6.2: Variable cost threshold $\widehat{c}_F$

Thus, there exist three parallel variable cost thresholds $\widehat{c}_F^1 < \widehat{c}_F^2 < \widehat{c}_F^3$, which are characterized by (5.5), (5.6) and (5.7) respectively. It is easy to verify that $c_F K_F^0\big|_{c_F=\widehat{c}_F^3} > 2c_D K_D^0 \geq B^1$, that is, Π_F^* is given by (3.12) as shown in Figure 3.2c for all $c_F \in [c_D, \widehat{c}_F^3]$, which is consistent with Figure 6.2. The comparative statistic analysis results developed in the Proposition 11 can be easily derived by applying the implicit function theorem with the optimality conditions and thus, are omitted. ■