PROFIT MAXIMIZATION FOR SEMICONDUCTOR MANUFACTURERS: SCRAP OR NOT SCRAP?

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Profit Maximization for Semiconductor Manufacturers: Scrap or Not Scrap?

by

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Abstract

In the semiconductor industry, the fall-outs of the production, or yield loss, are no longer doomed to be scrapped. Thanks to the technology advancement, they can be, and are now, sold to a value conscious market (the low-end market). While the manufacturers are thrilled by this creative idea - turning scrap cost into sales profit, we wonder whether such practice is always beneficial? What are the conditions under which the manufacturers should consider switching back to the old practice - scrapping at a cost? Building upon a standard marketing model for two differentiated markets, we are able to characterize optimal decisions, including operational strategies-whether to scrap the low quality product and whether to downgrade the high quality product, and the corresponding production capacity, supply quantity to each market, and price for each market. We find that when both the yield and the scrap cost are small, the manufacturer should switch back to the old practice. Otherwise, the manufacturer could lose up to 72.7% profit increase, shown by our comprehensive numerical study. Moreover, we observe that the manufacturer may over sacrifice the low-end market with negative profit when balancing the profit earned from each market. Counter intuitively, the manufacturer may be worse off when l-market consumers are willing to pay more.

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Chapter 1

Introduction

In the semiconductor industry, in the past, the production yield of any new product is usually unstable; the fall-outs, or the yield loss, need to be scrapped. The traditional practice is selling only the regular yield to its demanding market, which we refer to as the *high-end market*. Thus production planning with yield uncertainty is an important topic for both practice and academia. There has been extensive research on this topic (for example, Lee and Yano 1988, Kazaz 2004). Today, as the technology advances rapidly, the production process has become much more mature and stable, with little yield variation. As a matter of fact, production for any product will ramp only when the yield is stable enough. Thus, yield uncertainty is no longer a primary factor.

Furthermore, the fall-outs show reasonably good quality and significant demand for them has emerged. The current practice is selling the fall-outs to a value conscious market, which we refer to as the *low-end market*, in addition to selling the regular yield to the high-end market.

The current practice can be described by a coproduction system, in which multiple products are produced simultaneously in each batch or continuous production run (Bitran and Gibert 1994). The final products can be completely different in nature (i.e., horizontally differentiated), such as gasoline, kerosene, diesel, etc. in an oil refining process. Alternatively, the final products may be the same in nature, but varied in one or more key performance factors (i.e., vertically differentiated), such as the regular yield (the high-quality output) and the fall-outs (the low-quality output) in the semiconductor industry. As noted in Tomlin and Wang (2008), high-quality outputs could be used to fulfill the demand for low-quality outputs, but not visa versa. This, however, requires either down-conversion or down-grading of the high-quality outputs. Down-conversion means converting a high-quality product to a lower-quality product at a conversion cost, while down-grading is the practice of direct substitution. For example, high-quality memory chips can be down-graded at no additional cost by simply being affixed with a lower brand name or being labeled as low-quality chips.

The semiconductor industry is currently enjoying a creative trick: selling, instead of scrapping, the fall-outs to turn scrap losses into profits. Take DRAM (Dynamic Random Access Memory) memory chips as an example. There are different quality levels of memory chips: high-quality ones are with full data sheet parts, verified good and tested, while the low-quality ones are the fall-out parts, confirmed as not being full data sheet parts. On one hand, the low-quality DRAMs may be sold under a different brand name at a lower price to recover the manufacturing cost partially or even to make profits, such as Spectek (for Micron), Elixir (for Nanya), and Aeneon (for Infineon). Most module manufacturers sell memory chips of both quality levels to cater the demands for both the high-end and low-end markets. For the high-end market, it is supplied with high-quality branded chips; for the low-end market, it is supplied with unbranded chips, which include fall-outs and downgraded high-quality chips. Similar arrangements can also be applied to other semiconductor products, such as Flash memory, SRAM, Fusion memory etc. On the other hand, the quality requirements for DRAM memory chips are dependent on the final consumer applications and the low-quality DRAMs can be diverted to produce certain electronic products with less stringent requirements. For example, Samsung is offering a wide spectrum of consumer electronics devices - digital TVs, DVD recorders, and digital cameras - as well as hard disk drives, printers, networking equipment, automotive devices and other digital applications, among which the DRAM quality requirements are very much application specific. For instance, the printers/networking equipment can take a full range of DRAM offerings, while high-quality cameras will require high-density and high-performance DRAMs.

Although the semiconductor manufacturers are thrilled by the magic of turning scrap costs into sales profits, we wonder whether it is always the best strategy? When should they switch back to the traditional practice: scrapping at a cost? What does the choice of strategy depend on? Intuitively since the yield is fixed, the manufacturer is obviously better off if the fall-outs are now generating profits, rather than generating scrapping costs. However, because the price for each market is driven by its own supply, supplying the right amount of the fall-outs may drive up the supply of the high-quality products and thus lowering the price for high-end market. Thus the total profit earned from both markets might be worse than that with scrapping. Thus to answer the above questions, we build a singe-period model, by generalizing a standard marketing model for two differentiated markets. Consumers in high-end market only demand high-quality products, while consumers in low-end market are price-sensitive, but indifferent to product quality, and thus consume low-quality products as well as downgraded high-quality products, if available. The production flow and decisions (indicted by question marks) are shown in the Figure 1.1.

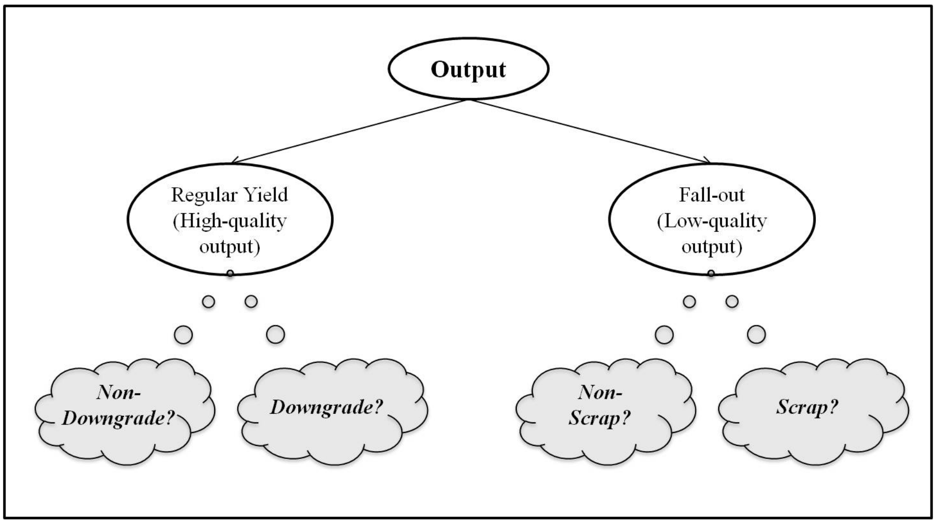

Figure 1.1: *Manufacturer Production Flow and Decisions*

We characterize the optimal decisions, including operational strategies - whether to scrap (major strategy) the low-quality product and whether to downgrade (minor strategy) the high-quality product, and the supporting production capacity, supply to each market, and price for each market for each strategy. We find that manufacturer with low scrap cost and low yield should switch back to the scrapping practice. Indeed, we characterize explicitly conditions on external factors (e.g., the difference between the two markets) and internal factors (e.g., the yield and scrap cost) for a manufacturer under which one particular operational strategy should be adopted. As discussed above, there is extensive literature on production control with yield uncertainty (for the past practice) and on coproduction systems (for the current practice). To the best of our knowledge, however, we are the first to compare the two practices and identify conditions for certain strategy to be optimal. Thus, we not only contribute to the relevant literature, but also provide the semiconductor manufacturers with valuable guidance on how to choose optimal operational strategies.

The rest of the paper is organized as follows. Chapter 2 reviews the existing literature related to our project. We will introduce the model formulation in Chapter 3. It is followed by the analysis of the past practice - scrapping strategy is adopted and the analysis of the current practice - non-scrapping strategy is adopted in Chapter 4 and Chapter 5 respectively. Complete comparison between scrapping strategy and non-scrapping is stated in Chapter 6, following with the numerical study is in Chapter 7. Conclusion and future research opportunities will be discussed in Chapter 8.

Chapter 2

Literature Review

2.1 Operations Management Literature Review

There are considerable studies on single-product random yield problems. Yano and Lee (1995) provide an extensive and good review of lot sizing problem with random yield in production or procurement, and discuss important issues related to the modeling of costs, yield uncertainty and performance.

The joint quantity-and-pricing problem has been studied extensively, however a large number of the existing studies are focusing on perfectly reliable supply and a single product. Van Meighem and Dada (1999) investigate operational recourse actions and show that the benefit from production postponement is minimal if price postponement is also implemented. Chen and Simchi-Levi (2004), Monahan et al. (2004), and Xu and Hopp (2004) compare the dynamic pricing with static pricing in a multiperiod setting. Li and Zheng (2006) consider the joint quantity-and-price setting problem of a single product with supply uncertainty and establish a periodic review model with inventory replenishment and pricing setting at the beginning of each period. Kazaz (2004), Kazaz (2008) and Kazaz and Webster (2010) consider a single-product production planning and pricing problem with yield uncertainty and investigate the impact of yield-dependent cost.

Comparably, there is less focus for joint quantity-and-price problem with multiple products. Bish and Wang (2004) study the benefits of flexible resource as compared to dedicated resource, while Chod and Rudi (2005) focus on a single flexible resource and investigate the effect of demand variability on the optimal resource level. Price-dependent aggregate demand models are used in both papers, however product differentiation is horizontal instead of vertical as in our paper.

Coproduction is a special type of multiproduct production system with a single input and production process whereas multiple products are produced simultaneously. Furthermore, among all the multiple-product systems, coproduction system with vertical product differentiation is characterized with downward flexibility, that is, the high-level product supply can be used to satisfy the low-level product demand. Most existing coproduction system literature consider the production quantity and substitution decisions under the following assumptions: price and customer preferences are exogenous, and each customer class has a different preferred product.

Bitran and Dasu (1992), Bitran and Leong (1992) and Bitran and Gilbert (1994) all study the multi-period, multi-product, deterministic-demand coproduction problem with random yield. Bitran and Dasu (1992) assume that the production yield is discrete and the given production output quantities are given. It determines the optimal allocation policies for single and multi-period problems. Bitran and Leong (1992) consider the same scenario, but assume the demand for each of the products must be satisfied with a certain probability and the quantity to be substituted is determined before the demand is observed. Both studies use deterministic models to approximate the stochastic problem. The same problem is also studied in Bitran and Gilbert (1994), with the objective to minimize the expected production, inventory holding and storage costs over a finite horizon, and where two downgrading policies are evaluated.

Gerchak and Grosfeld-Nir (1992) incorporate coproduction problem into a single period model with demands that must be satisfied to minimize the total setup and variable production costs. Gerchak et al (1996) evaluate several different models of a single-period, deterministic-demand, two-product substitution problem with random yield, and identify structuring properties of the optimal policy. Gerchak and Grosfeld-Nir (1999) formulate a multiple-lot-sizing production-to-order (MLPO) problem and solve for the optimal lot size by minimizing the total cost. Hsu and Bassok (1999) study a single-period, N-product, full downward substitution problem and formulate it as a two-stage stochastic program. Different solution methods for finding the optimal production decision under demand uncertainty and random yield are developed.

Netessine et al. (2002) investigate the resource-investment problem in a downwardly flexible system, however, separate investment is required for each resource and no yield uncertainty is involved.

Tomlin and Wang (2008) assess a single-period, two-product coproduction problem with supply uncertainty, stochastic demand and downward flexibility, for the optimal production quantity, product pricing, down-conversion quantity, and allocation decisions, which is one of the two studies most closely related to ours. Utility-maximizing customer models are used. This paper mainly studies product down-conversion, with down-conversion cost involved. In contrast, we characterize situations under which monopoly manufacturer should prefer downgrading to scrapping or vice versa. For their two-class coproduction model, they do not characterize the heterogeneity of consumer preferences for the two classes, that is, the two classes may prefer different or the same products, thus priority-based allocation policy is proposed. We differentiate the two consumer classes by the highest willingness-to-pay and investigate the impact of some market parameters, which include the size and highest willingness-to-pay for the low-end market relative to the high-end market.

2.2 Marketing and Economics Literature Review

Many marketing and economics literature have studied the marketing competition strategy with product and demand vertical differentiation. There are a wide range of empirical studies on heterogeneity of consumer preferences. Soberman and Parker

(2004) summarize the empirical studies about the existence of heterogeneity of consumer preferences for national brands and private labels, and mention that some consumers are willing to pay more for name-brand products while the others believe that private-label products are the same as name brands. Pauwels and Srinivasan (2004) empirically show that the new entrants of private-label products may be beneficial for the name-brand goods if consumers consider the quality of the name-brand as higher than that of the private-label. Thus it is crucial for firms that produce high-end products to create the perception of its brand as premium to high-end consumers. Randall et al. (1998) show that the presence of high-quality products in its product line can enhance brand equity and investigate the correlation of brand equity between the brand and its other high-quality products in its product line.

Mussa and Rosen (1978) consider the monopoly pricing problem for quality-differentiated products. It analyzes the utility functions and proposes optimal price-quality schedule to allocate customers along the quality spectrum by a process of self-selection.

Competition in heterogeneous markets features either cannibalization within the firm's own product line or competition across firms, while the monopoly problem will mainly focus on cannibalization. Moorthy (1984) studies the problem of implementing market segmentation through consumer self-selection and shows that the fundamental effect of consumer self-selection is cannibalization, and thus a monopolist must determine the optimal product and price for the whole product line simultaneously.

Moorthy and Png (1992) study two-period, two differentiated customer segments, stationary and known demand for a durable product problem. It compares the simultaneous and sequential introduction of low-end product by analyzing the trade-off between reducing cannibalization and the postponement of profits. Seller's marginal costs are assumed to be quadratic in quality. Desai (2001) investigates whether the cannibalization problem affects the price and quality decisions in a model with consumer differentiated in quality valuations and taste preferences for both the monopoly and duopoly. In the model where the market is made up of two segments, with one segment valuing quality more than the other, it is concluded that the monopolist finds it optimal to provide each segment with its preferred quality. When both segments are incompletely covered, under some conditions, the monopolist's price and quality choices are not determined by the cannibalization problem. Takeyama (2002) integrates durable goods time inconsistency problem with the self-selection issues of the static price discrimination problem in a two-period framework. It suggests that cannibalization of high-quality markets by low-quality goods may in fact be beneficial for durable-goods producers. Ghose, Telang and Krishnan (2005) investigate the competitive implications of newly emerging secondary markets on supply-chain profits and new good prices. The motivation is to capture additional surplus from consumers who were unable to buy in the new good market. It finds that new good prices might be lower with secondary markets, under both monopolistic and competitive scenarios.

Several theoretical literature has discussed the impact of competition from new entrants. Coughlan and Soberman (2005) consider a manufacturer distribution problem and show that manufacturer's own outlet stores may benefit independent primary retailers. Chen and Riordan (2007) build a monopolistic competition model with horizontal differentiation and reveal that under certain condition, new entry may increases the profits of the existing firms.

Ishibashi and Matsushima (2009) develop a quantity competition model and a price competition model for two high-end firms (producing high-quality products only), with the existence of vertical differentiation of products and demands, which is the other paper most closely related to our paper. It is shown that, under certain conditions, firms in the high-end market can earn more with the presence of the low-end firms than without. Without the low-end firms, the high-end firms will not be able to maintain high prices because each firm has the incentive to compete for the price-sensitive customers in the low-end market. The emergence of the low-end firms induces the high-end firms not to overproduce and sell only to high-end consumers as the low-end market is unprofitable for the high-end firms. Thus, the existence of low-end firms may help high-end firms. In our paper, we will study the profit maximization model with the optimal supply allocation and pricing strategies for the monopoly manufacturer, who produces high-quality and low-quality products and supplies to high-end and low-end markets.

Chapter 3

Model Formulation

We will focus on a monopoly manufacturer model with a single product for which the production output has two different quality levels, High-quality (H-product) and Low-quality (L-product). We believe that our monopoly model fits the practice reasonably well. Samsung is seen to be leading and dominating the memory market. For example, data for the third quarter of the past 3 years from IHS iSuppli Research, the world leader in technology, media, and telecommunications market intelligence and advisory services, show that Samsung's market share in DRAM market has soared from 29.5% to 40.7%. It is now almost the same as the total of all the other large manufacturers, including Hynix, Elpida and Micron.

Following the coproduction literature, we assume H-product and L-product incur the same manufacturing cost. These products can be used to serve two differentiated consumer groups, high-end market (h-market) and low-end market (l-market). Note that throughout the paper, we will use upper case letters, H and L, for the two outputs of different quality; we will use lower case letters, h and l, for the two different types of consumer markets. The manufacturer will need to make decisions on different combinations of major strategy, scrapping L-product or not, and minor strategy, downgrading H-product or not. We assume that scrapping will incur additional costs, while downgrading does not (since it is through a direct substitution, as explained earlier).

3.1 Different Strategies to Explore

As discussed above, the manufacturer may choose to combine any scrapping strategy, scrap (S) or not scrap ($\bar{S}$) L-product, with any downgrading strategy, downgrade (D) or not downgrade ($\bar{D}$) H-product. Thus we will compare the four combined strategies listed below in Table 3.1 to identify conditions under which each strategy is optimal.

S **Strategy**	**L-product is scrapped**
$S\bar{D}$	H-product is sold to h-market only
SD	H-product is sold to both h-market and l-market (after downgrading)
$\bar{S}$ **Policy**	**L-product is not scraped, but sold to l-market**
$\bar{S}\bar{D}$	H-product is sold to h-market only
$\bar{S}D$	H-product is sold to both h-market and l-market (after downgrading)

Table 3.1: *Scrapping and Downgrading Strategies*

3.2 Notation

Demands for both markets are assumed linear to the prices charged by the manufacturer. The demand model for each market will be carefully introduced in the next section. To help the readers, we now explain the notation used in the model. Please refer to Figure 3.1, describing our problem setup in details, when going through the notation list below.

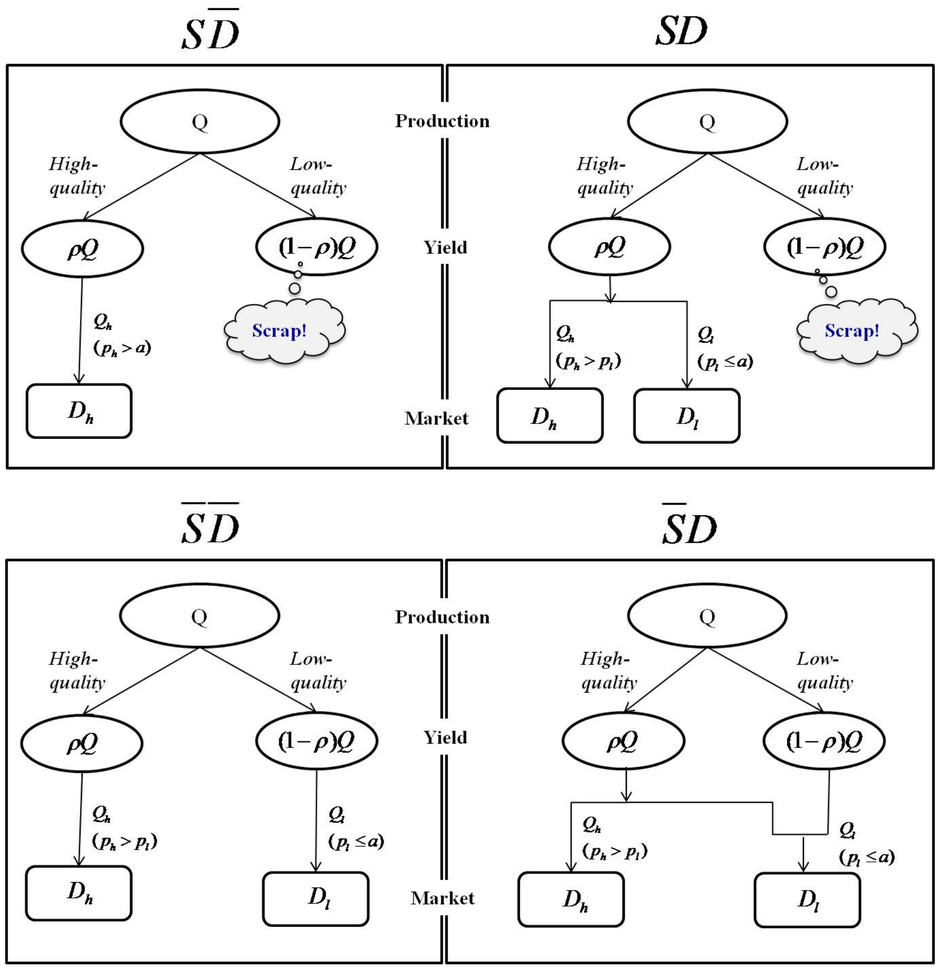

Figure 3.1: *Scrap and Non-Scrap Strategies*

Parameters:

- $\rho \in (0, 1)$: the production yield, i.e., the portion of the production output that is H-product. Thus $1 - \rho$ represents the portion of the production output that is L-product.
- $a \in (0, 1)$: the highest willingness-to-pay for l-market consumers.
- b: the size of l-market. Note that the size of h-market is 1.
- D_i: the price-dependent linear customer demand in i-market, $i = h, l$.
- c: unit scrapping cost of L-product.

Decisions:

- Q: the total production capacity invested for the product.
- $Q_h \in [0, \rho Q]$: the supply quantity to h-market.
- $Q_l \in [0, Q]$: the total supply quantity to l-market, which consists of the downgraded H-product and L-product, if not scraped. For S strategy, $Q_l = \rho Q - Q_h$; for $\bar{S}$ strategy, $Q_l = Q - Q_h$.
- $p_i (= r_i - c)$: the gross margin/price for sales in i-market, $i = h, l$, where r_i is the price charged to i-market customers and c is the unit production cost. Since the cost, c for each unit, is the same for the supply to both markets, setting p_i is equivalent to setting r_i. Thus we will focus on p_i and will also refer to it as the price throughout the paper.

3.3 Price-Dependent Linear Demand Function

We follow the linear demand model used in Ishibashi and Matsushima (2009), denoted by IM, where the demand is linear to the gross margin for each market. In contrast, IM studies a same supply to each market and thus assumes $p_h = p_l$. For our model with differentiated supply, however, we must assume the following to guarantee the existence of l-market.

Assumption 1 $p_h > p_l$.

Note that if $p_h \leq p_l$, consumers in the value conscious market, i.e., l-market, would buy H-product only and thus l-market would disappear.

Following IM, we assume for h-market that its consumers demand H-product only and their willingness-to-pay is uniformly distributed on $[0, 1]$. The h-market size is assumed 1. Thus, the price-dependent demand function for h-market, D_h, can be derived as:

$$D_h(p_h) = \begin{cases} 1 - p_h, & \text{if } p_h \in [0, 1]; \\ 0, & \text{if } p_h \in (1, \infty). \end{cases}$$

When confusion does not arise, we will omit the dependent variable in the price functions, such as, using D_h for $D_h(p_h)$.

Similarly, for l-market, we assume its consumers are price sensitive, but indifferent to H-product and L-product, with willingness-to-pay uniformly distributed on $[0, a]$, $a \in (0, 1)$. Recall that the size of l-market is denoted by b. Thus, the demand function for l-market, D_l, is given as:

$$D_l(p_l) = \begin{cases} b(1 - p_l/a), & \text{if } p_l \in (-\infty, a]; \\ \\ 0, & \text{if } p_l \in (a, \infty). \end{cases}$$

To keep the model general and practical, we allow the gross margin p_l to be negative, which reflects the observed practice that manufacturers may choose to sell low-quality products at a price lower than their production cost to partially recover the manufacturing cost. Therefore, b indeed represents the maximum demand in l-market if the gross margin p_l is non-negative; however when we allow for negative gross margin p_l, the demand can exceed b.

Chapter 4

S Strategy: L-Product Is Scrapped

In this section, we study the case that the manufacturer has decided to use scrap (S) strategy. In other words, we analyze the manufacturer's optimal decisions in the past practice. Note that the manufacturer still need to choose how to optimally allocate the supply to h-market and l-market. In other words, the manufacturer needs to decide whether to downgrade (D) H-product, and thus use SD strategy and serve both markets, or not downgrade H-product ($\bar{D}$), and thus adopt $S\bar{D}$ strategy and serve h-market only. The manufacturer also need to determine the corresponding optimal decisions, including the production capacity, the downgrading quantity, and the price charged to each market. Please refer to the top two figures in Figure 3.1 for the flow chart.

To determine the price for each market, we match its supply to the demand as follows:

$$Q_h = D_h = 1 - p_h \quad \text{and} \quad Q_l = \rho Q - Q_h = D_l = b(1 - \frac{p_l}{a}), \tag{4.1}$$

which implies the following price function:

$$p_h = 1 - Q_h \quad \text{and} \quad p_l = \frac{a}{b}(b + Q_h - \rho Q). \tag{4.2}$$

Note that the market prices are now functions of Q, the production capacity, and Q_h, the supply amount to h-market. The supply amount to l-market, Q_l is also a function of Q and Q_h. Therefore, the manufacturer's decisions reduces to Q and Q_h only. We thus state the manufacturer's profit maximization problem as:

$$\begin{aligned} \max \quad & \Pi_S(Q_h, Q) = p_h Q_h + p_l Q_l - c(1-\rho)Q \\ s.t. \quad & 0 \leq Q_h \leq \rho Q, \\ & Q_l = \rho Q - Q_h, \\ & p_l < p_h \leq 1. \end{aligned} \tag{4.3}$$

Note that this is a non-linear program and the subscript in the profit function $\Pi_S(Q_h, Q)$ represents the chosen strategy.

Here it is most appropriate to explain the real meaning of the scrap cost c. Let $\alpha \in [0, 1]$ denote the portion of manufacturing cost that can be recovered in scrapping (via using the scrapped for evaluation or re-work), where $\alpha = 0$ and $\alpha = 1$ correspond to zero recovery and full recovery, respectively. Then the manufacturer's profit function can be rewritten as $\Pi_S(Q_h, Q) = (r_h - c)Q_h + (r_l - c)Q_l - c(1-\rho)Q + \alpha c(1-\rho)Q =$

$p_hQ_h + p_lQ_l - c(1-\rho)Q$, where $c = (1-\alpha)c$. This implies that c indeed represents the unit non-recoverable manufacturing cost in scrapping. More importantly, the scrapping cost can be viewed as an indirect cost for H-product, the only supply to both markets.

Definition 1 *The "indirect cost due to scrapping" for each unit of supply is* $\frac{c(1-\rho)}{\rho}$.

Note that at the supply amount of $\rho Q = Q_h + Q_l$ for H product, the total scrap cost is $c(1-\rho)Q$. Therefore, $\frac{c(1-\rho)Q}{\rho Q} = \frac{c(1-\rho)}{\rho}$ can be perceived as the scrapping cost incurred for each unit of supply.

We solve the manufacturer's problem as follows. We start by solving a relaxed program by removing the first constraint. We then incorporate this constraint using the joint concavity of the objective function. We thus identify when to adopt SD or $S\bar{D}$ strategy and the corresponding optimal production quantity and allocation to the respective marekts.

The following assumption must be made such that the scrapping strategies are worth considering.

Assumption 2 $1 - \frac{c(1-\rho)}{\rho} > 0$, *i.e.*, $c < \frac{\rho}{1-\rho}$.

Note that 1 is the highest willingness to pay for h-market consumers. This assumption is used to guarantee a positive profit earned from the sales in h-market. If this assumption is not satisfied, the manufacturer, who decides to scrap, will not profit from any production and thus should not produce at all.

As shown below, the choice between the single-market strategy ($S\bar{D}$) and the dual-market strategy (SD) depends on internal parameters, c and ρ, and external/market parameter a. Indeed, it requires the manufacturer to assess the profitability of the l-market.

Proposition 1 *If $a < \frac{c(1-\rho)}{\rho}$, the manufacturer should choose the non-downgrading strategy $S\bar{D}$; if $a \geq \frac{c(1-\rho)}{\rho}$, the manufacturer should choose the downgrading strategy SD. Details of each policy are presented in Table 4.1.*

Strategy	**Optimal If**	p_h^*	p_l^*	Q_h^*	Q_l^*
$S\bar{D}$	$a < \frac{c(1-\rho)}{\rho}$	$\frac{1+c(1-\rho)/\rho}{2}$	$-$	$\frac{1-c(1-\rho)/\rho}{2}$	0
SD	$a \geq \frac{c(1-\rho)}{\rho}$	$\frac{1+c(1-\rho)/\rho}{2}$	$\frac{a+c(1-\rho)/\rho}{2}$	$\frac{1-c(1-\rho)/\rho}{2}$	$\frac{b-cb(1-\rho)/a\rho}{2}$

(Continued..)

Strategy	**Optimal If**	Q^*	Π^*
$S\bar{D}$	$a < \frac{c(1-\rho)}{\rho}$	$\frac{1-c(1-\rho)/\rho}{2\rho}$	$\frac{[1-c(1-\rho)/\rho]^2}{4}$
SD	$a \geq \frac{c(1-\rho)}{\rho}$	$\frac{1+b-(a+b)c(1-\rho)/a\rho}{2\rho}$	$\frac{(1-c(1-\rho)/\rho)^2+\frac{b}{a}(a-c(1-\rho)/\rho)^2}{4}$

Table 4.1: *Optimal Policy for Scrapping Strategies*

Note that the choice between SD and $S\bar{D}$ depends on the relative size of a and $\frac{c(1-\rho)}{\rho}$, as shown in Figure 4.1, where a is the highest willingness-to-pay in l-market and $\frac{c(1-\rho)}{\rho}$ is the indirect cost due to scrapping for each unit of supply. If this indirect cost exceeds what l-market consumers can pay, i.e., $\frac{c(1-\rho)}{\rho} > a$, entering l-market brings no profit and thus the manufacturer should serve h-market only. Otherwise, it is profitable for the manufacturer to enter l-market and thus both markets should be served. Note that the choice between SD and $S\bar{D}$ is independent of the size of h-market, b. As explained above, it is solely determined by the profitability when entering l-market. b, however, does play a role in the optimal production capacity when SD is chosen, i.e., when both markets are served.

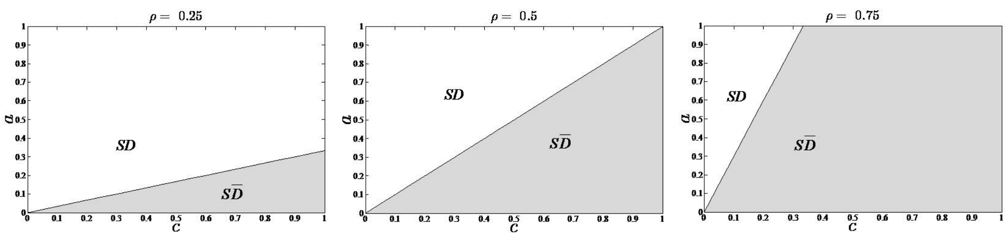

Figure 4.1: *Scrap Strategy Optimal Downgrading Policy: a vs. c*

Corollary 1 *Comparing $S\bar{D}$ to SD, we find Q_h^* stays unchanged, but Q^* increases and so does the profit.*

When Scrapping strategies are implemented, the two types of consumers are served separately with completely controlled supply amount, thus the optimal amount of supply to h-market is constant, regardless whether the low-end consumers are served, i.e., regardless whether $S\bar{D}$ or SD strategy is chosen. However, when SD strategy is chosen, the manufacturer will benefit by expanding his production capacity with the additional downgraded H-product catering for the low-end consumers.

Corollary 2 *As ρ increases, $S\bar{D}$ is more preferred; p_h^* and p_l^* drop; but Q_h^* and Q_l^**

rise and so does the profit, regardless of the choice of downgrading.

Intuitively, as the yield increases, for a same amount of supply, less would be needed to be scraped and thus the indirect cost, $\frac{c(1-\rho)}{\rho}$, will drop. This will induce the manufacturer to produce more, which drives down the prices for both markets, p_h^* and p_l^*, and more likely to serve l-market. That is, the single-market policy $S\bar{D}$ is more preferred.

Chapter 5

$\bar{S}$ Strategy: L-Product is Not Scrapped but Sold

In this section, we study the case that the manufacturer has decided to use non-scrapping ($\bar{S}$) strategy. In other words, we analyze the manufacturer's optimal decisions in the current practice. Similarly as in the previous section, we determine for the manufacturer whether to adopt the downgrading option (i.e., choosing between $\bar{S}D$ and $\bar{S}\bar{D}$ strategies), and the corresponding decisions of production capacity, the allocation of the supply to each market, and the price charged to each market. Please refer to the bottom two figures in Figure 3.1 for the flow chart.

To determine the price for each market, we match its supply to the demand as follows:

$$Q_h = D_h = 1 - p_h \quad \text{and} \quad Q_l = (\rho Q - Q_h) + (1 - \rho)Q = D_l = b(1 - \frac{p_l}{a}), \tag{5.1}$$

which implies the following price function: To assure the existence of both h-market and l-market, we assume $p_h > p_l$ and $p_l \leq a$. For dual-market strategies, basic transformation leads to the following price function:

$$p_h = 1 - Q_h \quad \text{and} \quad p_l = \frac{a}{b}(b + Q_h - Q). \tag{5.2}$$

Plugging in the profit functions, we state the manufacturer's profit maximization problem as:

$$\begin{aligned} \max \qquad & \Pi_{\bar{S}}(Q_h, Q) = p_h Q_h + p_l Q_l \\ s.t. \qquad & 0 \leq Q_h \leq \rho Q, \\ & Q_l = Q - Q_h, \\ & p_l < p_h \leq 1. \end{aligned} \tag{5.3}$$

We follow the solution procedure for the scraping case stated in the previous section and obtain the following optimal policy. To express the policy, we define $p^*_{h,\bar{S}\bar{D}} = \frac{(2a+ab+b)\rho^2-(ab+4a)\rho+2a}{2a(1-\rho)^2+2b\rho^2}$, $p^*_{l,\bar{S}\bar{D}} = \frac{a[(a+2b+1)\rho^2-(2a+1)\rho+a]}{2a(1-\rho)^2+2b\rho^2}$, $Q^*_{h,\bar{S}\bar{D}} = \frac{b\rho[\rho+a(1-\rho)]}{2a(1-\rho)^2+2b\rho^2}$, $Q^*_{l,\bar{S}\bar{D}} = \frac{b(1-\rho)[\rho+a(1-\rho)]}{2a(1-\rho)^2+2b\rho^2}$, $Q^*_{\bar{S}\bar{D}} = \frac{b[\rho+a(1-\rho)]}{2a(1-\rho)^2+2b\rho^2}$, $\Pi^*_{\bar{S}\bar{D}} = \frac{b[\rho+a(1-\rho)]^2}{4a(1-\rho)^2+4b\rho^2}$.

Proposition 2 *If $0 \leq \rho \leq \frac{1}{1+b}$, the manufacturer should choose the non-downgrading strategy $\bar{S}\bar{D}$; if $\frac{1}{1+b} < \rho \leq 1$, the manufacturer should choose the downgrading strategy $\bar{S}D$. Details of each policy are presented in Table 5.1.*

We first note that the choice of downgrading or not depends on the comparison between the yield ρ and $\frac{1}{1+b}$, which is illustrated in Figure 5.1. We also note that when

Strategy	Optimal if	p_h^*	p_l^*	Q_h^*	Q_l^*	Q^*	Π^*
$\bar{S}\bar{D}$	$0 \leq \rho \leq \frac{1}{1+b}$	$p_{h,\bar{S}\bar{D}}^*$	$p_{l,\bar{S}\bar{D}}^*$	$Q_{h,\bar{S}\bar{D}}^*$	$Q_{l,\bar{S}\bar{D}}^*$	$Q_{\bar{S}\bar{D}}^*$	$\Pi_{\bar{S}\bar{D}}^*$
$\bar{S}D$	$\frac{1}{1+b} < \rho \leq 1$	$\frac{1}{2}$	$\frac{a}{2}$	$\frac{1}{2}$	$\frac{b}{2}$	$\frac{1+b}{2}$	$\frac{1+ab}{4}$

Table 5.1: *Optimal Policy for Non-Scrap Strategies*

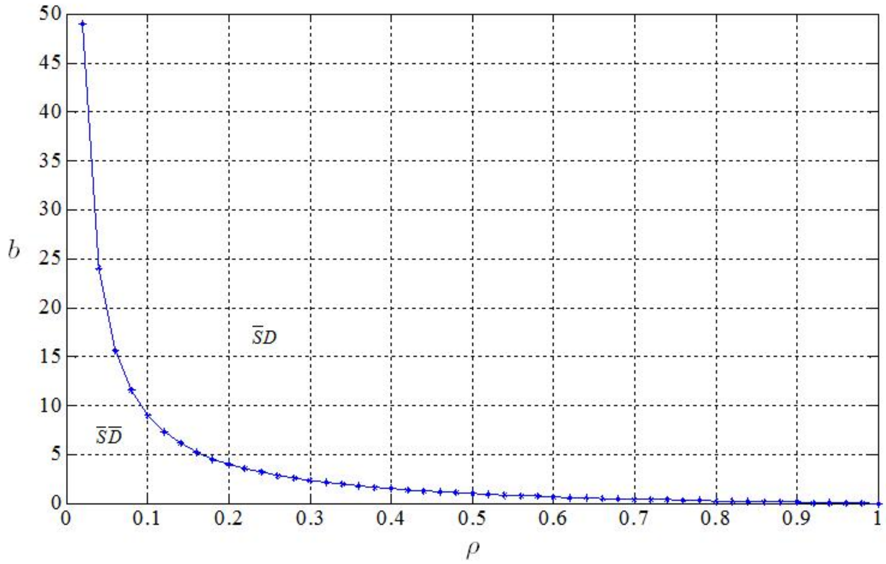

Figure 5.1: *Non-Scrap Strategy Optimal Downgrading Policy: ρ vs. b*

the yield is large enough, i.e., when $\rho > \frac{1}{1+b}$, the manufacturer can use the downgrading strategy, $\bar{S}D$ strategy, to optimize the supply to both markets, extracting the maximum profit from each market. Specifically, the manufacturer should produce to serve half of the potential market, i.e., $Q_h^* = \frac{1}{2}$ and $Q_l^* = \frac{b}{2}$, and charge half of the maximum price the consumers are willing to pay for both markets, i.e., $p_h^* = \frac{1}{2}$ and $p_l^* = \frac{a}{2}$. We refer to this solution as the *global optimal*.

To help explain the intuition behind the optimal policy, we need to introduce the following definition.

Definition 2 *The "optimal ratio of supply" to h-market and l-market is the ratio of the respective market size,* $1 : b$*.*

The optimal ratio of supply is naturally achieved when $\rho = \frac{1}{1+b}$ as $\rho Q : (1-\rho)Q = 1 : b$; it can be manipulated via downgrading when $\rho > \frac{1}{1+b}$. As shown above, the optimal ratio of supply leads to the global optimal for the manufacturer. When $\rho < \frac{1}{1+b}$, however, the optimal ratio of supply is unreachable; downgrading will further distort the ratio of supply. In this case, the manufacturer should not downgrade and supply $Q^*_{h,\bar{S}\bar{D}}$ and $Q^*_{l,\bar{S}\bar{D}}$ to h-market and l-market, respectively.

Corollary 3 *Comparing* $\bar{S}\bar{D}$ *to* $\bar{S}D$*, we find that* $Q^*_{h,\bar{S}\bar{D}} < \frac{1}{2}$*;* $Q^*_{l,\bar{S}\bar{D}} > \frac{b}{2}$*;* $Q^*_{\bar{S}\bar{D}} \geq \frac{1+b}{2}$ *if* $\rho \in [\frac{a}{a+b}, \frac{1}{1+b}]$ *and* $Q^*_{\bar{S}\bar{D}} < \frac{1+b}{2}$ *if* $\rho < \frac{a}{a+b}$*.*

When the yield ρ is less than $\frac{1}{1+b}$, the optimal ratio of supply is not attainable and neither is the global optimal. Specifically, the optimal supply to h-market, $Q^*_{h,\bar{S}\bar{D}}$, is less than the global optimal, $\frac{1}{2}$ (referred to as *under-supply*); the optimal supply to l-market, $Q^*_{l,\bar{S}\bar{D}}$, is, however, more than the global optimal, $\frac{b}{2}$ (referred to as *over-supply*). Consequently, h-market consumers will suffer the resulted higher price, higher than the global optimal, $\frac{1}{2}$; l-market consumers will benefit from the resulted lower price, lower than the global optimal, $\frac{a}{2}$.

Combining the supply to both markets, the total production, $Q^*_{\bar{S}\bar{D}}$, however, may or may not exceed the global optimal, $\frac{1+b}{2}$. It depends on the magnitude of the production yield ρ. The manufacturer with low yield (lower than $\frac{a}{a+b}$) should produce less than the global optimal, while the manufacturer with high yield (higher than $\frac{a}{a+b}$) should produce more than the global optimal.

We find two counter-intuitive phenomena for the manufacturer utilizing $\bar{S}\bar{D}$ strategy. First, when balancing the profit earned from each market, the manufacturer's decision can be a little extreme - over sacrificing the profitability in l-market (such that negative gross margin is incurred) for better profitability in h-market. Second, a better l-market with its consumers' willing to pay more can make the manufacturer worse off (i.e., may lower his profit).

Corollary 4 *When* $\frac{2a+1-\sqrt{1-8ab}}{2(a+2b+1)} < \rho < \frac{2a+1+\sqrt{1-8ab}}{2(a+2b+1)} (< \frac{1}{1+b})$ *and* $ab < \frac{1}{8}$*, we find: (1)* $p^*_{l,\bar{S}\bar{D}} < 0$*, with minimum occurred at* $\rho = \frac{2a+1}{2(a+2b+1)}$*, and (2)* $\Pi^*_{\bar{S}\bar{D}}$ *decreases in* a*, albeit it always increases in* b*.*

Such an extreme sacrifice, however, only occurs when $ab < \frac{1}{8}$, where a and b are the highest willingness-to-pay and the market size, respectively, for l-market. ab can be interpreted as the profit upper bound for l-market. Similarly, the profit upper bound for h-market is 1. As such, ab can also be viewed as the relative market profitability of l-market, compared to h-market. Thus the l-market is worth sacrificing only when its relative market profitability is no more than $\frac{1}{8}$. Clearly, the sacrifice motive is justified as the profit obtained from h-market can compensate the loss in l-market. A typical behavior of the optimal supplies and profits with respect to the yield, ρ, is shown in Figure 5.2.

a*b	a	b
0.10	0.25	0.40

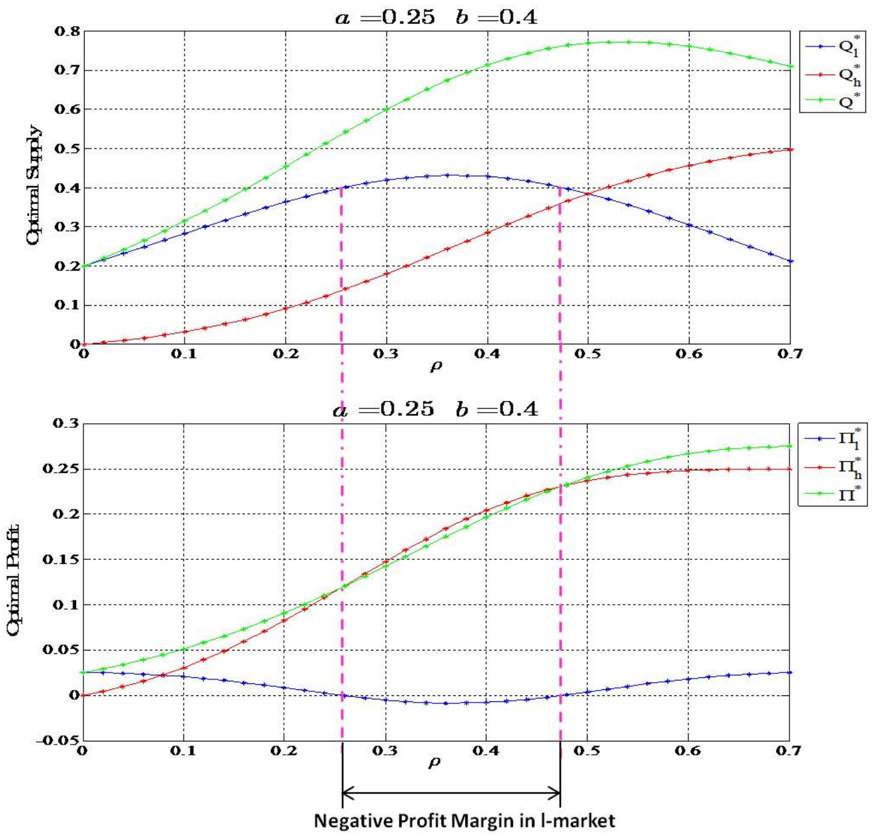

Figure 5.2: *Negative Gross Margin in l-market*

Under the same condition, a better l-market with a higher a leads to a lower profit for the manufacturer surprisingly. One examples is shown in Figure 5.3.

With higher willingness-to-pay in l-market, the L-product price will be set higher, thus lower demand will be incurred and lower production capacity will be required for the same production yield. But in total, higher profits from l-market can be generated. Since the manufacturer will produce less, lower supply of H-product will

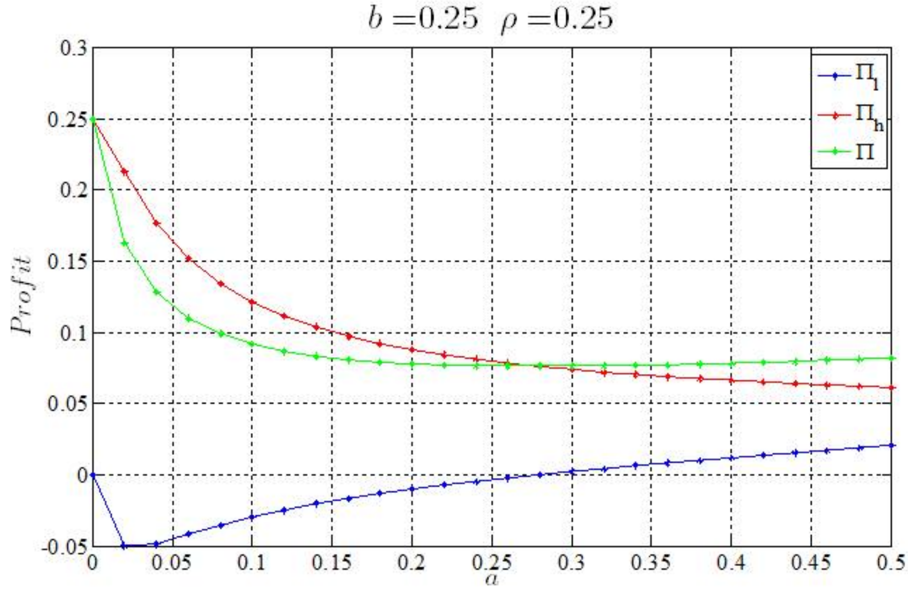

Figure 5.3: *Profit Decrease in a*

enable the manufacturer to charge a higher price for H-product. Consequently, the profit generated from H-product will be slightly lower for the low production yield region, but it can be compensated by the higher L-product profits. Therefore, in total, the increasing in a will lead to higher overall profits with reasonably large a.

$$a\uparrow \;\Rightarrow \left\{ \begin{array}{lcl} p_l^*(\downarrow)\uparrow & Q_l^*\downarrow & \Pi_l^*(\downarrow)\uparrow \\ & Q^*\downarrow & \\ p_h^*\uparrow & Q_h^*\downarrow & \Pi_h^*\downarrow \end{array} \right\} \Rightarrow \Pi^*(\downarrow)\uparrow,$$

where $p_l^*(\downarrow)\uparrow$,$\Pi_l^*(\downarrow)\uparrow$, and $\Pi^*(\downarrow)\uparrow$ mean p_l^*, Π_l^* and Π^* first decrease then increase in a respectively.

The impact of b on the manufacturer's profit is, however, always positive. With larger l-market market size, thus larger market capability for L-product, for the same production yield, the manufacturer can produce more, which results in larger supply for H-product but lower H-product price. But on the whole, increasing of l-market size will lead to higher profit from H-product. On the other hand, with larger b, the manufacturer will produce more, but not too much to drive down the L-product price. Thus, with larger l-market market size will lead to higher price, and supply for L-product. Since non-downgrading strategy is only preferred when $0 \le \rho \le \frac{1}{b+1}$, whereas global optimal can be achieved when $\frac{1}{b+1} < \rho \le 1$ with downgrading strategy. When b is increasing, the production yield requirement to achieve global optimal will be lower.

$$
b \uparrow \ \Rightarrow \left\{ \begin{array}{ccc} p_l^* \uparrow & Q_l^* \uparrow & \Pi_l^*(\downarrow)\uparrow \\ & Q^* \uparrow & \\ p_h^* \downarrow & Q_h^* \uparrow & \Pi_h^* \uparrow \end{array} \right\} \Rightarrow \Pi^* \uparrow
$$

Chapter 6

Comparison Between S Strategy and $\bar{S}$ Strategy

Using the results from the previous two sections, we can now allow the manufacturer to choose freely between all the strategies, SD, $S\bar{D}$, $\bar{S}D$ and $\bar{S}\bar{D}$. As shown in the previous section, $\bar{S}D$ strategy achieves the global optimal when $\rho > \frac{1}{1+b}$. Thus, when the yield falls in this range, $\bar{S}D$ is the best strategy, regardless of the other model parameters. However, when $\rho \leq \frac{1}{1+b}$, the manufacturer might be better off by switching back to the past practice - scrapping.

To help describe the optimal conditions, we define $c_1 = \frac{a\rho}{1-\rho}$, $c_2 = \frac{\rho}{1-\rho}(1-2\sqrt{\Pi^*_{\bar{S}\bar{D}}})$, $c_3 = \frac{a\rho}{(a+b)(1-\rho)}\left[(1+b) - \sqrt{b\frac{2(1-a)(1+b)\rho+ab+2a-1}{a(1-\rho)^2+b\rho^2}}\right]$, $\rho_1 = \frac{1+b-a-\sqrt{b^2+2b-ab}}{1-a}$, $\rho_2 = \frac{1}{2(1+b)} - \frac{a}{2(1-a)}$, and $\rho_3 = \frac{1}{1+b}$.

Proposition 3 *Comparing the profit for various strategies, we obtain the comparison* *results summarized in Table 6.1 below.*

Strategy	**Optimal If**	Π^*
$S\bar{D}$	$c_1 \leq c < c_2,\ 0 \leq \rho < \rho_1$	$\frac{[1-\frac{c(1-\rho)}{\rho}]^2}{4}$
SD	$c < c_1 \wedge c_3,\ \rho_2 \leq \rho \leq \rho_3$ or $c < c_1,\ 0 \leq \rho < \rho_2$	$\frac{(1-\frac{c(1-\rho)}{\rho})^2+\frac{b}{a}(a-\frac{c(1-\rho)}{\rho}^2}{4}$
$\bar{S}\bar{D}$	otherwise	$\frac{b[\rho+a(1-\rho)]^2}{4a(1-\rho)^2+4b\rho^2}$
$\bar{S}D$	$\rho > \frac{1}{1+b}$	$\frac{1+ab}{4}$

Table 6.1: *Optimal Policy with Free Choice of Strategies*

Recall that out of the four strategies listed in Table 6.1, only the first, $S\bar{D}$, corresponds to that the manufacturer sells to a single market, i.e., the h-market. A close examination of the optimal conditions for this strategy shows that if the other market, i.e., the l-market, is profitable enough, $S\bar{D}$ strategy is never optimal and thus the manufacturer should always sell to both markets.

Corollary 5 *If $b \geq \frac{(1-a)^2}{a}$, the manufacturer should always sell to both markets.*

This result implies the profitability of l-market can be characterized by its market size, b, and its customers' highest willingness-to-pay, a. It also provides a benchmark that is simple and easy to check. Specifically, if b or/and a is large enough such that condition $b \geq \frac{(1-a)^2}{a}$ is satisfied, the manufacturer should always serve both markets.

For example, if the l-market consumers are willing to pay half of what the h-market consumers are willing to pay, i.e., if $a = \frac{1}{2}$, the size of h-market needs to be at least half of the size of h-market such that l-market is worth entering for the manufacturer.

Chapter 7

Numerical Study

To better understand the strategy selection conditions and the impacts of the model parameters, we supplement our analytical results with a through numerical study.

7.1 Strategy Preference Based on c and ρ

To help visualize when each of the four strategies, $S\bar{D}, SD, \bar{S}\bar{D}$ and $\bar{S}D$, is optimal and thus should be adopted, we provide Figure 7.1, consisting of four figures all drawn in the scrap cost, c, versus the production yield, ρ. Note that in each figure, the zones in which each strategy is optimal are shaded in different colors.

For reference convenience, we next define the curves that separate the various optimal zones. The curve associated with $c = \frac{a\rho}{1-\rho}$ is dividing the zone associated with scraping strategies into two, one for $S\bar{D}$ and the other for SD. Thus, we define this curve as $S\bar{D}/SD$ *critical curve*. For any combination of c and ρ on this curve,

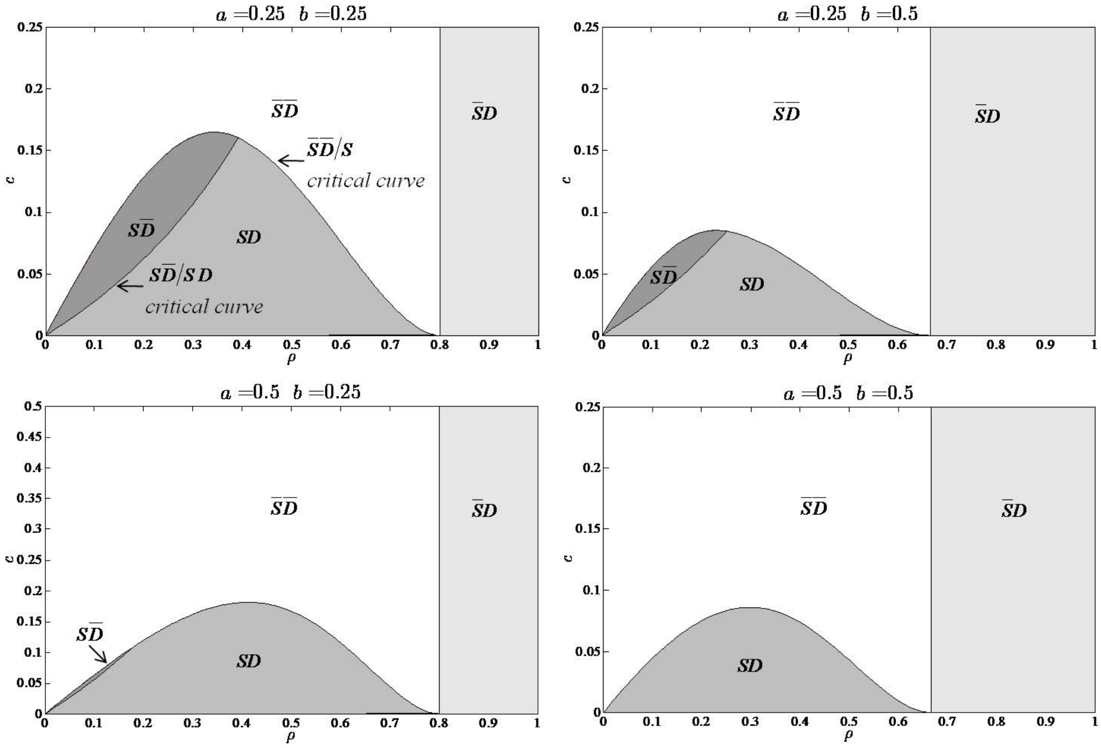

Figure 7.1: *Policy Preference Based on ρ and c*

the manufacturer is indifferent to $S\bar{D}$ and SD. Above the curve, the scrapping cost is high and thus the manufacturer should cover h-market only; below the curve, scrap cost is lower and the manufacturer should cover both markets. Similarly, we define the bell-shaped curve as $\bar{S}\bar{D}/S$ *critical curve*, where S includes $S\bar{D}$ and SD. For any combination of c and ρ on this curve, the manufacturer is indifferent to $\bar{S}\bar{D}$ and $S\bar{D}$ or SD.

7.1.1 Strategy Shifting in a

Comparing the top two figures to their respective figures below in Figure 7.1, we find that as a increases, the region for $S\bar{D}$ shrinks and the $S\bar{D}/SD$ critical curve shifts up. This means that $S\bar{D}$ strategy is less preferred. Recall that $S\bar{D}$ strategy is the only single-market strategy among the four. As the l-market consumers are willing to pay a higher price (i.e., as a, the highest willingness to pay for l-market, increases towards 1, the highest willingness to pay for h-market), the difference between the two markets will be smaller. Therefore, l-market will be more attractive; the manufacturer would have more incentive to cover both markets.

7.1.2 Strategy Shifting in b

Comparing the two figures to the left to their respective figures to the right in Figure 7.1, we find that as the size of l-market, b, increases, the region for S shrinks and so does the $\bar{S}\bar{D}$/S critical curve. This means that S strategy is less preferred. The *$\bar{S}\bar{D}/S$ critical curve* corresponds to the balance between the scrap cost in S strategy and the gross margin loss from over-supplying l-market in $\bar{S}\bar{D}$ strategy. Recall that over-supply is defined in comparison to the global optimal. We can further quantify the degree of over-supply to l-market by $\frac{Q_l - b/2}{b/2}$. Intuitively, we note that under $\bar{S}\bar{D}$ strategy, as b increases, the degree of over-supply drops, constrained by the profit maximization for h-market, and so does the gross margin loss due to over-supply. Given the same scrap cost, this raises the manufacturer's incentive to adopt $\bar{S}\bar{D}$

strategy.

In addition, we find that as the size of l-market, b, increases, $\bar{S}D$ strategy is more preferred. As Figure 7.1 shows, the vertical black line at $\rho = \frac{1}{1+b}$ separates the region for $\bar{S}D$ from the other three regions. We define $\frac{1}{1+b}$ as the *critical yield level* at which the ratio of the supply is the optimal ratio, $1 : b$. Clearly, as the size of l-market, b, increases, the critical yield level will decrease. This corresponds to the expansion of the region for $\bar{S}D$ strategy in Figure 7.1, i.e., $\bar{S}D$ strategy is more preferred.

7.1.3 Strategy Shifting in ρ

For any given scrap cost, c, as the yield, ρ, increases from 0 to 1, the optimal strategy will evolve as: $\bar{S}\bar{D} \rightarrow S\bar{D} \rightarrow SD \rightarrow \bar{S}\bar{D} \rightarrow \bar{S}D$, when c is low; $\bar{S}\bar{D} \rightarrow \bar{S}D$, when c is high. As shown in Figure 7.2, we define the corresponding ranges of ρ as *Segment I, II, III, IV, V*. Recall that as discussed earlier and in Section 5, when ρ is below the critical yield level, $\frac{1}{1+b}$ (i.e., in Segments I-IV), the manufacturer, who chooses $\bar{S}\bar{D}$, will over-supply l-market, but under-supply h-market.

In *Segment I*, ρ is very small and thus the indirect cost due to scrapping, $\frac{c(1-\rho)}{\rho}$, is very high and the dominant output is L-product. The manufacturer should not scrap L-product but collect most of his profit from l-market sales. Note that for this strategy H-product costs the same as L-product, but is sold at a much higher price. Therefore, the manufacturer should not consider downgrading any H-product. In summary, when ρ is very small, $\bar{S}\bar{D}$ strategy should be adopted.

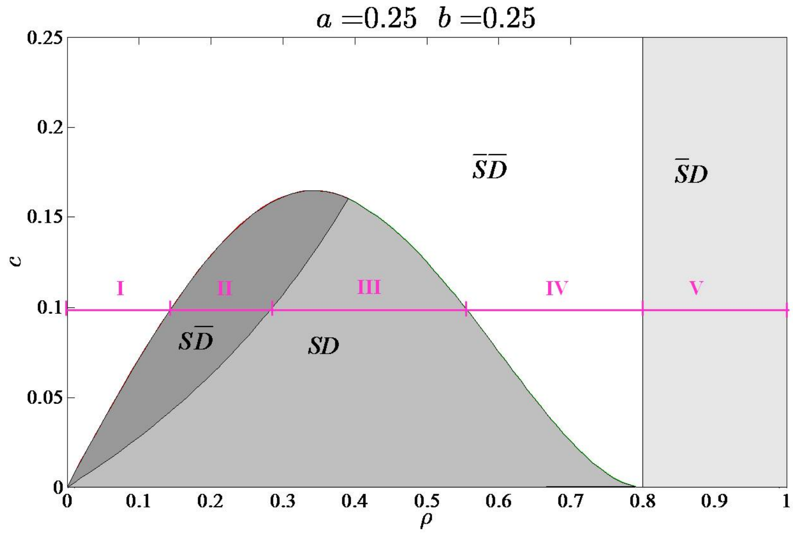

Figure 7.2: *Policy Preference Evolved with* ρ

As ρ increases, the indirect cost $\frac{c(1-\rho)}{\rho}$ drops and the output of H-product increases. In Segments II and III, the indirect cost due to scrapping drops below the gross margin loss due to over-supply, making S strategy preferred (to $\bar{S}$ strategy). Moving from Segment II to III, the indirect cost drops and goes below the gross margin earned in l-market. Thus the manufacturer will earn profits from both market by downgrading some H-product.

As ρ further increases to Segment IV, although the indirect cost due to scrapping further drops, it cannot catch up with the reduction of the gross margin loss due to over-supply. Therefore, the manufacturer will choose $\bar{S}\bar{D}$ strategy. Finally, as ρ crosses the critical yield level, $\frac{1}{1+b}$, (i.e., in Segment V), the manufacturer should use downgrading to achieve the optimal supply ratio to both market, i.e., should adopt

$\bar{S}D$ strategy.

7.2 Sensitivity Study for $\bar{S}\bar{D}$ Strategy

In this section, we focus on $\bar{S}\bar{D}$ strategy and study how the manufacturer's profit and optimal decisions are affected by the l-market parameters, a and b. The reasons of focusing on this strategy are as following. First, the sensitivity study for the other three strategies are straightforward and important results are already stated in analytically. Second, $\bar{S}\bar{D}$ is the only one among the four under which the optimal ratio of supply is not attainable. Under SD and $S\bar{D}$, the manufacturer has a single source of supply (i.e., H-product) and thus can completely control the ratio of supply to the two markets. The selection between them solely depends on the profitability of entering l-market. Under $\bar{S}D$, the manufacturer can leverage on the downgrading option to control the ratio of supply to the two markets, albeit two sources of supply (i.e., H-product and L-product). Third, as $\bar{S}D$ is associated with the global optimal, only manufacturers who are currently utilizing $\bar{S}\bar{D}$ strategy could be better off switching back to the past practice and use either SD or $S\bar{D}$.

Our numerical results show that the two parameters for l-market, the highest willingness-to-pay a and the market size b, will affect the manufacturer's optimal profit in a different degree. A same percentage of increase or decrease in b will result in a higher percentage change of the optimal profit than that in a will, as shown in Figure 7.3. This implies that b has a stronger impact on the profit; the manufacturer would prefer a doubled l-market size to a doubled highest willingness-to-pay.

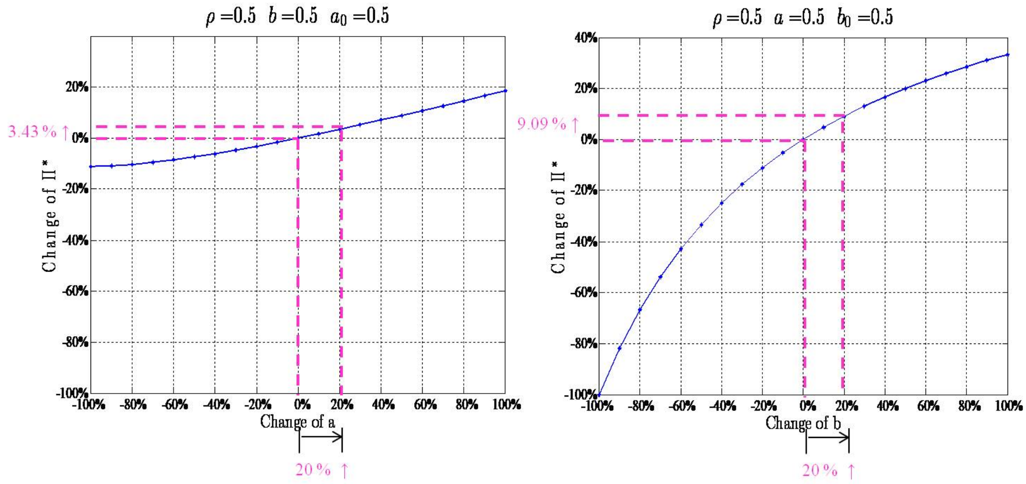

Figure 7.3: *Impact of Increase/Decrease in a and b on Optimal Profit*

Chapter 8

Conclusions

In our paper, we extend the joint quantity-and-price setting problem in the semiconductor manufacturing industry and integrate it with supply and demand differentiation. Building upon a standard marketing model for two differentiated markets, we come out with supply strategy (scrapping and downgrading) and market strategy (single market vs. dual market), supported with the optimization of product pricing, production capacity, and product profit. We find that when both the production yield and the scrapping cost are small, the manufacturer should switch back to the old practice. Otherwise, the manufacturer could lose up to 72.7% profit increase, shown by our numerical study (softcopy excel calculation file is included in the CD). We observe that when the yield fraction is large, with the downward flexibility, the global optimum can be achieved with the best economical setting and maximal profitability. Moreover, we observe that the manufacturer may sacrifice the low-end market with negative profit when balancing the profit earned from each market. Counter intuitively, the manufacturer may be worse off when l-market consumers are willing to pay more.

Following the analysis for monopoly manufacturer in the market, the market competition strategy for duopoly manufacturers can be explored. Furthermore, in our paper, we assume that the consumers in high-end market only demand high-quality product and will never buy low-quality product regardless the price difference for both. Future research can be extended to the product cannibalization.

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Appendix

Proof of Table 4.1:

Step 1. We first ignore the constraints on Q_h for single-market and dual-market cases for analysis convenience. We will incorporate them later after we fragment this scrap model into two sub-models.

Using price expressions derived, we can rewrite the manufacturer's profit function as:

$$\begin{aligned}
\max \qquad & \Pi_S(Q_h, Q) = p_h Q_h + p_l Q_l - c_s(1-\rho)Q \\
& \qquad\qquad = (1-Q_h)Q_h + \frac{a}{b}(b+Q_h-\rho Q)(\rho Q - Q_h) - c_s(1-\rho)Q \\
s.t. \qquad & 0 \le Q_h \le \rho Q, \\
& p_h > p_l.
\end{aligned}$$

Differentiating the profit function with respect to (w.r.t.) Q_h, we obtain

$$\begin{aligned}
\frac{\partial \Pi_S(Q_h, Q)}{\partial Q_h} &= 1 - 2Q_h + \frac{a}{b}(b+Q_h-\rho Q)(-1) + \frac{a}{b}(\rho Q - Q_h) \\
&= \frac{1}{b}[b - ab + 2a\rho Q - 2(a+b)Q_h],
\end{aligned}$$

which is decreasing in Q_h. Thus we know that for any given Q, $\Pi_S(Q_h, Q)$ is concave in Q_h and has a unique optimum. Let $\tilde{Q}_h$ denote this value, that is, $\frac{\partial \Pi_S(\tilde{Q}_h, Q)}{\partial \tilde{Q}_h} = 0$. $\tilde{Q}_h$ represents the optimal supply to h-market without considering all the constraints. Solving the first order condition, we have

$$\tilde{Q}_h = \frac{b - ab + 2a\rho Q}{2(a+b)} > 0. \tag{8.1}$$

Using the concavity property of $\Pi_S(Q_h, Q)$, we can compare $\tilde{Q}_h$ to ρQ, the maximum supply available for the h-market, and obtain the following two scenarios to be analyzed individually:

1. $S\bar{D}$ strategy is optimal: $Q_h^* = \arg_{0 \le Q_h \le \rho Q} \Pi_S(Q_h, Q) = \rho Q$, when $Q \le \frac{1-a}{2\rho}$, that is, where $\tilde{Q}_h \ge \rho Q$. In this case, the manufacturer with low capacity is better off by focusing on the h-market only.

2. SD strategy is optimal: $Q_h^* = \arg_{0 \le Q_h \le \rho Q} \Pi_S(Q_h, Q) = \tilde{Q}_h$, when $Q > \frac{1-a}{2\rho}$, that is, where $\tilde{Q}_h < \rho Q$. In this case, the manufacturer with high capacity should enter both markets.

Step 2. We then solve for the optimal strategy when $S\bar{D}$ strategy, serve high-end market only, is optimal.

As discussed earlier, we now incorporate the constraints on $Q_h(= \rho Q)$ to meet the price requirement for the h-market, i.e., we should guarantee $Q_h = \rho Q < \frac{a}{a+b}\rho Q + \frac{b}{a+b}(1-a)$, such that $p_h > p_l$, from which we obtain $Q < \frac{1-a}{\rho}$. We start by rewriting the manufacturer's profit function as a single-variable function as follows:

$$\begin{aligned}
\max \qquad & \Pi_{S\bar{D}}(Q) = p_h(\rho Q) = \rho Q - \rho^2 Q^2 - c_s(1-\rho)Q \\
s.t. \qquad & Q \le \frac{1-a}{2\rho}, (\ i.e.,\ \tilde{Q}_h \ge \rho Q), \\
& 0 \le Q < \frac{1-a}{\rho}, (\ i.e.,\ p_h > p_l).
\end{aligned} \tag{8.2}$$

Solving the non-linear Equation 8.2, we first note that $\Pi_{S\bar{D}}(Q)$ is a concave quadratic function of Q with a single optimum at $Q = \frac{\rho - c_s(1-\rho)}{2\rho^2} > 0$. Using the convexity property of $\Pi_{S\bar{D}}(Q)$, we can conclude that the optimal capacity, denoted by $Q^*_{S\bar{D}}$, satisfies $Q^*_{S\bar{D}} = \frac{\rho - c_s(1-\rho)}{2\rho^2} \wedge \frac{1-a}{2\rho} \wedge \frac{1-a}{\rho} = \frac{\rho - c_s(1-\rho)}{2\rho^2} \wedge \frac{1-a}{2\rho}$, where $a \wedge b = \min\{a, b\}$ and $a \vee b = \max\{a, b\}$.

Further, we can check the difference for the two potential optimum,

$$\frac{\rho - c_s(1-\rho)}{2\rho^2} - \frac{1-a}{2\rho} \quad = \quad \frac{1}{2\rho^2}[a\rho - c_s(1-\rho)].$$

We then conclude that the optimal production capacity for case $S\bar{D}$ is:

$$Q^*_{S\bar{D}} = \begin{cases} \frac{1-a}{2\rho} & \text{if } a > \frac{c_s(1-\rho)}{\rho},\ (S_1 \preceq S_2), \\ \frac{1-c_s(1-\rho)/\rho}{2\rho} & \text{if } a \leq \frac{c_s(1-\rho)}{\rho},\ (S_1 \succ S_2), \end{cases}$$

where $a \preceq b$ means a is not better than b, and $a \succ b$ means a is superior to b.

If $a > \frac{c_s(1-\rho)}{\rho}$, $S\bar{D}$ strategy is not better than SD strategy. We will further explore this case in the next step.

Therefore, $S\bar{D}$ strategy is preferred when $a \leq \frac{c_s(1-\rho)}{\rho}$, and the optimal production capacity is $\frac{1-c_s(1-\rho)/\rho}{2\rho}$.

The optimal strategy for $S\bar{D}$ strategy is shown in table below.

Strategy	**Optimal If**	p_h^*	p_l^*	Q_h^*	Q_l^*	Q^*	Π^*
$S\bar{D}$	$a \leq \frac{c_s(1-\rho)}{\rho}$	$\frac{1+c_s(1-\rho)/\rho}{2}$	$-$	$\frac{1-c_s(1-\rho)/\rho}{2}$	0	$\frac{1-c_s(1-\rho)/\rho}{2\rho}$	$\frac{[1-c_s(1-\rho)/\rho]^2}{4}$

Step 3. We then solve for the optimal strategy when SD strategy, serve both markets, is optimal.

As shown earlier, in this case the optimal quantity of H-product allocated to the h-market is $Q_h(=\tilde{Q}_h = \frac{b-ab+2a\rho Q}{2(a+b)})$. We can analyze this two-market scrap model similarly to we do for the single market scrap model. We incorporate the constraints on Q_h to meet the requirement for p_h and p_l, where $\tilde{Q}_h$ is defined by (8.1). That is, we should guarantee $\tilde{Q}_h \leq \frac{a}{a+b}\rho Q + \frac{b}{a+b}(1-a)$, i.e., $p_h > p_l$, and $\tilde{Q}_h < \rho Q$, i.e., $p_l \leq a$, where the first inequality is automatically satisfied. We then rewrite the manufacturer's profit function as follows:

$$\begin{aligned}
\max \qquad & \Pi_{SD}(Q) = p_h\tilde{Q}_h + p_l(\rho Q - \tilde{Q}_h) - c_s(1-\rho)Q \\
& \qquad = \frac{1}{4(a+b)}[-4a\rho^2Q^2 + 4a(b+1)\rho Q + b(1-a)^2] - c_s(1-\rho)Q \\
s.t. \qquad & Q > \frac{1-a}{2\rho}, (\ i.e.,\ \bar{Q}_h < \rho Q), \\
& Q \geq \frac{1-a}{2\rho}, (\ i.e.,\ p_l \leq a).
\end{aligned} \tag{8.3}$$

Solving the non-linear Equation 8.3, we first note that $\Pi_{SD}(Q)$ is a concave quadratic function of Q with a single optimum at $Q = \frac{(1+b)-(a+b)c_s(1-\rho)/a\rho}{2\rho} > 0$. Using the convexity property of $\Pi_{SD}(Q)$, we can compare this optimum with the constraints and obtain the following result for the optimal capacity, denoted by Q^*_{SD}, $Q^*_{SD} = \frac{1-a}{2\rho} \vee \frac{(1+b)-(a+b)c_s(1-\rho)/a\rho}{2\rho}$.

With further comparison, using the fact that

$$\frac{1-a}{2\rho} - \frac{(1+b)-(a+b)c_s(1-\rho)/a\rho}{2\rho} = \frac{(a+b)[c_s(1-\rho)/\rho - a]}{2a\rho},$$

we can obtain the optimal production capacity for case SD is:

$$Q^*_{SD} = \begin{cases} \frac{(1+b)-(a+b)c_s(1-\rho)/a\rho}{2\rho} & \text{if } a > \frac{c_s(1-\rho)}{\rho},\ (S_2 \succ S_1), \\ \frac{1-a}{2\rho} & \text{if } a \leq \frac{c_s(1-\rho)}{\rho},\ (S_2 \preceq S_1). \end{cases}$$

Therefore, SD strategy is preferred when $a > \frac{c_s(1-\rho)}{\rho}$, and the optimal production capacity is $\frac{(1+b)-(a+b)c_s(1-\rho)/a\rho}{2\rho}$.

The optimal strategy for SD strategy is shown as in table below.

Case	**Optimal If**	p_h^*	p_l^*	Q_h^*	Q_l^*
SD	$a > \frac{c_s(1-\rho)}{\rho}$	$\frac{1+c_s(1-\rho)/\rho}{2}$	$\frac{a+c_s(1-\rho)/\rho}{2}$	$\frac{1-c_s(1-\rho)/\rho}{2}$	$\frac{b-bc_s(1-\rho)/a\rho}{2}$

(Continued..)

Case	**Optimal If**	Q^*	Π^*
SD	$a > \frac{c_s(1-\rho)}{\rho}$	$\frac{1+b-(a+b)c_s(1-\rho)/a\rho}{2\rho}$	$\frac{(1-c_s(1-\rho)/\rho)^2+\frac{b}{a}(a-c_s(1-\rho)/\rho)^2}{4}$

Combining results in the previous two tables, we can obtain the optimal policy for scrap strategy as in Table 4.1. ∎

Proof of Table 5.1:

Step 1. We first ignore the constraints on Q_h for single-market and dual-market cases for analysis convenience. We will incorporate them later after we fragment this non-scrap model into two sub-models.

Using the expressions derived, we then rewrite the manufacturer's profit function as:

$$\begin{aligned}\max \qquad \Pi_{\bar{S}}(Q_h, Q) &= p_h Q_h + p_l Q_l - c_s(1-\rho)Q \\ &= (1-Q_h)Q_h + \frac{a}{b}(b+Q_h-Q)(Q-Q_h)\end{aligned}$$

$$s.t. \qquad 0 \leq Q_h \leq \rho Q,$$

$$p_h > p_l,$$

Differentiating the profit function w.r.t Q_h, we get

$$\begin{aligned}
\frac{\partial \Pi_{NS}(Q_h, Q)}{\partial Q_h} &= 1 - 2Q_h + \frac{a}{b}(b + Q_h - Q)(-1) + \frac{a}{b}(Q - Q_h) \\
&= \frac{1}{b}[b - ab + 2aQ - 2(a+b)Q_h],
\end{aligned}$$

which is a decreasing function of Q_h. Thus for any given Q, $\Pi_{NS}(Q_h, Q)$ is concave in Q_h and has a unique optimum. Let $\tilde{Q}_h$ denote this value, i.e., $\frac{\partial \Pi_{\bar{S}}(\tilde{Q}_h, Q)}{\partial \tilde{Q}_h} = 0$. Note that the subscript stands for non-scrapping strategy. $\tilde{Q}_h$ represents the optimal supply to h-market without considering all the constraints. Thus, based on the concavity of the objective function, the optimal Q_h without considering the price constraints is

$$\tilde{Q}_h = \frac{b - ab + 2aQ}{2(a+b)} > 0. \tag{8.4}$$

Thus,

$$Q_h^* = \frac{b - ab + 2aQ}{2(a+b)} \wedge \rho Q.$$

Further, based on the concavity property of $\Pi_{\bar{S}}(Q_h, D)$, we can compare $\tilde{Q}_h$ to ρQ, the maximum available supply to h-market, and obtain the following two scenarios to be analyzed individually:

1. Model $\bar{S}\bar{D}$ is optimal: $Q_h^* = \arg_{0 \leq Q_h \leq \rho Q} \Pi_{\bar{S}}(Q_h, Q) = \rho Q$, when

$$\begin{cases} Q \leq \frac{b(1-a)}{2[(a+b)\rho - a]} & \text{if } \rho > \frac{a}{a+b}, \\ Q \geq 0 & \text{if } \rho \leq \frac{a}{a+b}. \end{cases}$$

The price for all the H-product will be too high for the l-market customers, thus they will only purchase L-product. In this case, the manufacturer with low capacity is better off by focusing on the h-market only.

2. Model $\bar{S}D$ is optimal: $Q_h^* = \arg_{0 \le Q_h \le \rho Q} \Pi_{\bar{S}}(Q_h, Q) = \tilde{Q}_h$, when $Q > \frac{b(1-a)}{2[(a+b)\rho - a]}$ and $\rho > \frac{a}{a+b}$. The price for $\rho Q - Q_h$ portion of H-product is lower than the highest willingness to pay of l-market customers, thus customers in l-market will purchase both H-products and L-product. In this case, the manufacturer with high capacity should enter both markets.

Step 2. We then solve for the optimal policy when $\bar{S}\bar{D}$ strategy, H-product sold to h-market market only, is optimal.

Incorporating the constraints on $Q_h(= \rho Q)$ to meet the price requirement for h-market, i.e., we should guarantee $Q_h = \rho Q < \frac{a}{a+b}Q + \frac{b}{a+b}(1-a)$, such that $p_h > p_l$, and $Q_h \le Q$, such that $p_l \le a$. We start by rewriting the manufacturer's profit function as a single-variable function as follows:

$$
\begin{aligned}
\max \quad & \Pi_{\bar{S}\bar{D}}(Q) = p_h(\rho Q) + p_l(1-\rho)Q = (1-\rho Q)\rho Q + \frac{a}{b}(b + \rho Q - Q)(Q - \rho Q), \\
s.t. \quad & \rho \le \frac{a}{a+b}, (\ i.e.,\ \bar{Q}_{h_s} \ge \rho Q), \\
or \quad & \rho > \frac{a}{a+b}, \quad \& \quad Q \le \frac{b(1-a)}{2[(a+b)\rho - a]}, (\ i.e.,\ \bar{Q}_{h_s} \ge \rho Q).
\end{aligned}
\tag{8.5}
$$

Taking first order condition for non-linear Equation 8.5 w.r.t Q, we have,

$$
\frac{d\Pi_{\bar{S}\bar{D}}(Q)}{dQ} = \rho - 2\rho^2 Q + a(1-\rho) - \frac{2a(1-\rho)^2}{b}Q,
$$

which implies that $\Pi_{\bar{S}\bar{D}}(Q)$ has a single optimum at $Q = \frac{b[\rho+a(1-\rho)]}{2a(1-\rho)^2+2b\rho^2} > 0$. Using the convexity property of $\Pi_{\bar{S}\bar{D}}(Q)$, we can analyze the constraints and conclude that the optimal capacity, denoted by $Q^*_{\bar{S}\bar{D}}$, satisfies

$$Q^*_{\bar{S}\bar{D}} = \begin{cases} \frac{b[\rho+a(1-\rho)]}{2a(1-\rho)^2+2b\rho^2} \wedge \frac{b(1-a)}{2[(a+b)\rho-a]} & \text{if } \rho > \frac{a}{a+b}, \\ \frac{b[\rho+a(1-\rho)]}{2a(1-\rho)^2+2b\rho^2} & \text{if } \rho \le \frac{a}{a+b}. \end{cases}$$

Then we can obtain the optimal production capacity for case $\bar{S}\bar{D}$ is:

$$Q^*_{\bar{S}\bar{D}} = \begin{cases} \frac{b[\rho+a(1-\rho)]}{2a(1-\rho)^2+2b\rho^2} & \text{if } 0 < \rho \le \frac{1}{b+1}, \ (NS_1 \succ NS_2), \\ \frac{b(1-a)}{2[(a+b)\rho-a]} & \text{if } \frac{1}{b+1} < \rho \le 1, \ (NS_1 \preceq NS_2). \end{cases}$$

Therefore, $\bar{S}\bar{D}$ strategy is preferred when $0 < \rho \le \frac{1}{b+1}$, and the optimal production capacity is $\frac{b[\rho+a(1-\rho)]}{2a(1-\rho)^2+2b\rho^2}$.

The optimal decisions for $\bar{S}\bar{D}$ strategy are summarized in table below.

Strategy	**Optimal If**	p_h^*	p_l^*	Q_h^*
$\bar{S}\bar{D}$	$0 < \rho \le \frac{1}{b+1}$	$\frac{(2a+ab+b)\rho^2-(ab+4a)\rho+2a}{2a(1-\rho)^2+2b\rho^2}$	$\frac{a[(a+2b+1)\rho^2-(2a+1)\rho+a]}{2a(1-\rho)^2+2b\rho^2}$	$\frac{b\rho[\rho+a(1-\rho)]}{2a(1-\rho)^2+2b\rho^2}$

(Continued..)

Strategy	**Optimal If**	Q_l^*	Q^*	Π^*
$\bar{S}\bar{D}$	$0 < \rho \le \frac{1}{b+1}$	$\frac{b(1-\rho)[\rho+a(1-\rho)]}{2a(1-\rho)^2+2b\rho^2}$	$\frac{b[\rho+a(1-\rho)]}{2a(1-\rho)^2+2b\rho^2}$	$\frac{b[\rho+a(1-\rho)]^2}{4a(1-\rho)^2+4b\rho^2}$

Step 3. We then solve for the optimal policy when $\bar{S}D$ strategy, H-product sold to both markets, is optimal.

As shown earlier, in this case the optimal quantity of H-product allocated to h-market is $Q_h(= \tilde{Q}_h = \frac{b-ab+2aQ}{2(a+b)})$. We can analyze this two-market non-scrap model similarly as we do for the previous models. We incorporate the constraints on Q_h to meet the requirement for p_h and p_l. That is, we should guarantee $\tilde{Q_h} < \frac{a}{a+b}Q + \frac{b}{a+b}(1-a)$, such that $p_h > p_l$, and $\tilde{Q_h} \leq Q$, such that $p_l \leq a$, where the first inequality is automatically satisfied. We then rewrite the manufacturer's profit function as follows:

$$\begin{aligned}
\max \qquad & \Pi_{\bar{S}D}(Q) = p_h\tilde{Q_h} + p_l(Q - \tilde{Q_h}) \\
& \qquad = \frac{1}{4(a+b)}[-4aQ^2 + 4a(b+1)Q + b(1-a)^2] \\
s.t. \qquad & Q > \frac{b(1-a)}{2[(a+b)\rho - a]} \quad with \;\; \rho > \frac{a}{a+b}, \;\; i.e., \;\; \tilde{Q_h} \leq \rho Q \\
& Q \geq \frac{1-a}{2}, \;\; i.e., \;\; p_l \leq a.
\end{aligned} \tag{8.6}$$

Solving the non-linear Equation 8.6, we first note that $\Pi_{\bar{S}D}(Q)$ is a concave quadratic function of Q with a single optimum at $Q = \frac{1+b}{2} > 0$. Using the convexity property of $\Pi_{\bar{S}D}(Q)$, we can compare this optimum with the constraints and obtain the following result for the optimal capacity, denoted by $Q^*_{\bar{S}D}$:

$$Q^*_{\bar{S}D} = \begin{cases} \frac{b(1-a)}{2[(a+b)\rho - a]} & \text{if } 0 < \rho \leq \frac{1}{b+1}, \text{ and where } NS_2 \preceq NS_1, \\ \frac{1+b}{2} & \text{if } \frac{1}{b+1} < \rho \leq 1, \text{ and where } NS_2 \succ NS_1. \end{cases}$$

Therefore, $\bar{S}D$ strategy is preferred when $\frac{1}{b+1} < \rho \leq 1$, and the optimal production capacity is $\frac{1+b}{2}$.

The optimal decisions for $\bar{S}D$ strategy is summarized in table below.

Strategy	**Conditions**	p_h^*	p_l^*	Q_h^*	Q_l^*	Q^*	Π^*
$\bar{S}D$	$\frac{1}{1+b} < \rho \leq 1$	$\frac{1}{2}$	$\frac{a}{2}$	$\frac{1}{2}$	$\frac{b}{2}$	$\frac{1+b}{2}$	$\frac{1+ab}{4}$

Combining results in the previous two tables, we can obtain the optimal policy for scrap strategy as in Table 5.1. ∎

Proof of Corollary 3: As summarized in Table 5.1, for case $\bar{S}\bar{D}$, $Q_l^* = \frac{b(1-\rho)[\rho+a(1-\rho)]}{2a(1-\rho)^2+2b\rho^2}$. The global optimal of product supply to l-market is $\frac{b}{2}$. $Q_l^* - \frac{b}{2} = \frac{b\rho[1-(1+b)\rho]}{2a(1-\rho)^2+2b\rho^2}$. Since $0 \leq \rho \leq \frac{1}{1+b}$, $Q_l^* - \frac{b}{2} \leq 0$, that is, $Q_l^* \geq \frac{b}{2}$. Therefore, under $\bar{S}\bar{D}$ strategy, the manufacturer will 'over-supply' l-market.

Meanwhile, $Q_h^* = \frac{b\rho[\rho+a(1-\rho)]}{2a(1-\rho)^2+2b\rho^2}$. The global optimal of product supply to h-market is $\frac{1}{2}$. $Q_h^* - \frac{1}{2} = \frac{a(1-\rho)[(a+b)\rho-a]}{2a(1-\rho)^2+2b\rho^2}$. Since $0 \leq \rho \leq \frac{1}{1+b}$ and $\frac{a}{a+b} < \frac{1}{1+b}$, $Q_h^* - \frac{1}{2} < 0$, that is, $Q_h^* < \frac{1}{2}$. Therefore, under $\bar{S}\bar{D}$ strategy, the manufacturer will 'under-supply' h-market. ∎

Proof of Corollary 4: From Table 5.1, we can see that $p_{l,\bar{S}\bar{D}}^* = \frac{a[(a+2b+1)\rho^2-(2a+1)\rho+a]}{2a(1-\rho)^2+2b\rho^2}$. Solving equation $\frac{a[(a+2b+1)\rho^2-(2a+1)\rho+a]}{2a(1-\rho)^2+2b\rho^2} < 0$, we can obtain $\frac{2a+1-\sqrt{1-8ab}}{2(a+2b+1)} < \rho < \frac{2a+1+\sqrt{1-8ab}}{2(a+2b+1)}$, which only exists when $ab < \frac{1}{8}$. Similarly, from $\Pi_{\bar{S}\bar{D}}^* = \frac{b[\rho+a(1-\rho)]^2}{4a(1-\rho)^2+4b\rho^2}$, we can obtain that $\frac{\partial \Pi_{\bar{S}\bar{D}}^*}{\partial a} = \frac{b(1-\rho)[\rho+a(1-\rho)][(2b+a+1)\rho^2-(2a+1)\rho+a]}{[2a(1-\rho)^2+2b\rho^2]^2}$. Let $\frac{\partial \Pi_{\bar{S}\bar{D}}^*}{\partial a} < a$, we can get the same constraint as above. Therefore, we obtain Proposition 4. ∎

Proof of Corollary 5: As summarized in Table 6.1, case $S\bar{D}$ is preferred when $0 \leq \rho \leq \frac{1-ab}{1-ab+2b}$ and $\rho < \frac{1+b-a-\sqrt{b^2+2b-ab}}{1-a}$. If $\rho < 0$, case $S\bar{D}$ will never be preferred. From $\frac{1+b-a-\sqrt{b^2+2b-ab}}{1-a} \leq 0$, we obtain that $b \geq \frac{(1-a)^2}{a}$. Therefore, Proposition 5 is proven. ∎

Proof of Table 6.1:

Step 1. We first compare $S\bar{D}$ strategy and $\bar{S}\bar{D}$ strategy.

$\bar{S}\bar{D}$ strategy is preferred when ρ is small. When ρ is small, $\frac{1-\rho}{\rho}$ will be large, thus $a < \frac{c_s(1-\rho)}{\rho}$ is easier to be satisfied. Comparing the optimal profits for $S\bar{D}$ strategy and $\bar{S}\bar{D}$ strategy, when there is no down-grading for H-product,

$$\begin{aligned} & \Pi^*_{S\bar{D}} - \Pi^*_{\bar{S}\bar{D}} \\ = \quad & \frac{[1 - c_s(1-\rho)/\rho]^2}{4} - \frac{b[\rho + a(1-\rho)]^2}{4a(1-\rho)^2 + 4b\rho^2} \end{aligned}$$

and together with the assumption that $c_s < \frac{\rho}{1-\rho}$, we can obtain that,

if $0 \le \rho < \frac{1-ab}{1-ab+2b}$, i.e. $2\sqrt{\Pi^*_{\bar{S}\bar{D}}} < 1$, then

$$\begin{cases} \Pi^*_{S\bar{D}} > \Pi^*_{\bar{S}\bar{D}} & \text{when } c_s < \frac{\rho}{1-\rho}(1 - 2\sqrt{\Pi^*_{\bar{S}\bar{D}}}), \\ \Pi^*_{S\bar{D}} \le \Pi^*_{\bar{S}\bar{D}} & \text{when } \frac{\rho}{1-\rho}(1 - 2\sqrt{\Pi^*_{\bar{S}\bar{D}}}) \le c_s < \frac{\rho}{1-\rho}; \end{cases}$$

if $\frac{1-ab}{1-ab+2b} \le \rho \le 1$, i.e. $2\sqrt{\Pi^*_{\bar{S}\bar{D}}} \ge 1$, then $\Pi^*_{S\bar{D}} \le \Pi^*_{\bar{S}\bar{D}}$.

Combined with the preferred condition for $S\bar{D}$ strategy and $\bar{S}\bar{D}$ strategy, the above correlations can be re-written as below:

$$\begin{cases} \Pi^*_{S\bar{D}} > \Pi^*_{\bar{S}\bar{D}} & \text{when } c_s < \frac{\rho}{1-\rho}(1 - 2\sqrt{\Pi^*_{\bar{S}\bar{D}}}),\ a \le \frac{c_s(1-\rho)}{\rho},\ \ \&\ \ 0 \le \rho < \frac{1-ab}{1-ab+2b}, \\ \Pi^*_{S\bar{D}} \le \Pi^*_{\bar{S}\bar{D}} & \text{when } \frac{\rho}{1-\rho}(1 - 2\sqrt{\Pi^*_{\bar{S}\bar{D}}}) \le c_s < \frac{\rho}{1-\rho},\ \ \&\ \ 0 \le \rho < \frac{1-ab}{1-ab+2b},\ \ \text{or}\ \ \frac{1-ab}{1-ab+2b} \le \rho \le \frac{1}{b+1}. \end{cases}$$

In the above expressions, we note that we need to check the concurrence of constraint $c_s < \frac{\rho}{1-\rho}(1 - 2\sqrt{\Pi^*_{\bar{S}\bar{D}}})$ and constraint $a \le \frac{c_s(1-\rho)}{\rho}$, i.e., $c_s \ge \frac{a\rho}{1-\rho}$. After further comparison, we can obtain that, only when $0 \le \rho < \frac{1+b-a-\sqrt{b^2+2b-ab}}{1-a}$,

$\frac{a\rho}{1-\rho} < \frac{\rho}{1-\rho}(1 - 2\sqrt{\Pi^*_{\bar{S}\bar{D}}})$, i.e., $2\sqrt{\Pi^*_{\bar{S}\bar{D}}} < 1 - a$. Furthermore, $\Pi^*_{\bar{S}\bar{D}}$ is a increasing function of ρ, thus we can obtain $\frac{1+b-a-\sqrt{b^2+2b-ab}}{1-a} < \frac{1-ab}{1-ab+2b}$.

Thus,

$$\begin{cases} \Pi^*_{S\bar{D}} > \Pi^*_{\bar{S}\bar{D}} & \text{when } \frac{a\rho}{1-\rho} \leq c_s < \frac{\rho}{1-\rho}(1 - 2\sqrt{\Pi^*_{\bar{S}\bar{D}}}), \text{ and } 0 \leq \rho < \frac{1+b-a-\sqrt{b^2+2b-ab}}{1-a} \\ \Pi^*_{S\bar{D}} \leq \Pi^*_{\bar{S}\bar{D}} & \text{when } \frac{\rho}{1-\rho}(1 - 2\sqrt{\Pi^*_{\bar{S}\bar{D}}}) \leq c_s < \frac{\rho}{1-\rho}, \ \& \ 0 \leq \rho < \frac{1-ab}{1-ab+2b}, \text{ or } \frac{1-ab}{1-ab+2b} \leq \rho \leq \frac{1}{b+1}. \end{cases}$$

When the production yield and the scrap cost is low, the manufacturer should scrap the L-product; when the scrap cost is high, the manufacturer should sell the L-product.

Step 2. We then compare SD strategy and $\bar{S}\bar{D}$ strategy.

$\bar{S}\bar{D}$ strategy is preferred when ρ is small. When ρ is small, $\frac{1-\rho}{\rho}$ will be large, thus $a \geq \frac{c_s(1-\rho)}{\rho}$ is more difficult to be satisfied. Comparing the optimal profits for SD strategy and $\bar{S}\bar{D}$ strategy,

$$\begin{aligned} & \Pi^*_{SD} - \Pi^*_{\bar{S}\bar{D}} \\ = & \ \frac{(1 - c_s(1-\rho)/\rho)^2 + \frac{b}{a}(a - c_s(1-\rho)/\rho)^2}{4} - \frac{b[\rho + a(1-\rho)]^2}{4a(1-\rho)^2 + 4b\rho^2} \\ = & \ \frac{1}{4}\left[\frac{(a+b)(1-\rho)^2}{a\rho^2}c_s^2 - \frac{2(1+b)(1-\rho)}{\rho}c_s + \frac{a(1-(1+b)\rho)^2}{a(1-\rho)^2 + b\rho^2}\right] \end{aligned}$$

Define R_1 and R_2 ($R_1 \leq R_2$) as two roots for equation, and $\Delta = b\frac{2(1-a)(1+b)\rho+ab+2a-1}{a(1-\rho)^2+b\rho^2}$,

$$\frac{(a+b)(1-\rho)^2}{a\rho^2}c_s^2 - \frac{2(1+b)(1-\rho)}{\rho}c_s + \frac{a(1-(1+b)\rho)^2}{a(1-\rho)^2 + b\rho^2} = 0,$$

if $\Delta \geq 0$, i.e., $\rho \geq \frac{1}{2(1+b)} - \frac{a}{2(1-a)}$, then we can obtain,

$$R_1 = \frac{a\rho}{(a+b)(1-\rho)}\left[(1+b) - \sqrt{b\frac{2(1-a)(1+b)\rho + ab + 2a - 1}{a(1-\rho)^2 + b\rho^2}}\right] \tag{8.7}$$

$$R_2 = \frac{a\rho}{(a+b)(1-\rho)}\left[(1+b) + \sqrt{b\frac{2(1-a)(1+b)\rho + ab + 2a - 1}{a(1-\rho)^2 + b\rho^2}}\right]; \tag{8.8}$$

otherwise, if $\Delta < 0$, then R_1 and R_2 do not exist.

Note that

$$(1+b)^2 - b\frac{2(1-a)(1+b)\rho + ab + 2a - 1}{a(1-\rho)^2 + b\rho^2} = \frac{(a+b)[(1+b)\rho - 1]^2}{a(1-\rho)^2 + b\rho^2} \geq 0,$$

which implies $R_1 \geq 0$.

Summarizing the above results of comparing the optimal profit for SD strategy and $\bar{S}\bar{D}$ strategy, we can obtain that,

if $[\frac{1}{2(1+b)} - \frac{a}{2(1-a)}] \leq \rho \leq 1$,

$$\begin{cases} \Pi^*_{SD} > \Pi^*_{\bar{S}\bar{D}} & \text{when } c_s < R_1 \text{ or } c_s > R_2; \\ \Pi^*_{SD} \leq \Pi^*_{\bar{S}\bar{D}} & \text{when } R_1 \leq c_s \leq R_2; \end{cases}$$

if $0 \leq \rho < [\frac{1}{2(1+b)} - \frac{a}{2(1-a)}]$, then $\Pi^*_{SD} > \Pi^*_{\bar{S}\bar{D}}$.

Combined with the preferred condition for SD strategy and $\bar{S}\bar{D}$ strategy, the above correlations can be re-written as below:

$$\begin{cases} \Pi^*_{SD} > \Pi^*_{\bar{S}\bar{D}} & \text{when } c_s < R_1,\ a > \frac{c_s(1-\rho)}{\rho},\ \text{ and } \ \frac{1}{2(1+b)} - \frac{a}{2(1-a)} \leq \rho \leq \frac{1}{1+b}, \\ & \text{or when } a > \frac{c_s(1-\rho)}{\rho},\ \text{ and } \ 0 \leq \rho < \frac{1}{2(1+b)} - \frac{a}{2(1-a)}; \\ \Pi^*_{SD} \leq \Pi^*_{\bar{S}\bar{D}} & \text{when } R_1 \leq c_s < \frac{a\rho}{(1-\rho)},\ \text{ and } \ \frac{1}{2(1+b)} - \frac{a}{2(1-a)} \leq \rho \leq \frac{1}{1+b}. \ \blacksquare \end{cases}$$