The Inventory Billboard Effect on Information Sharing in Competing Supply Chains with Production Diseconomies

XIN WEI

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by

Xin Wei

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<u>Thesis Committee:</u>

Zhengping Wu (Supervisor/Chair)

Assistant Professor of Operations Management

Singapore Management University

Yun Fong Lim

Assistant Professor of Operations Management

Singapore Management University

Rong Li

Assistant Professor of Operations Management

Singapore Management University

Singapore Management University

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Abstract

The Billboard effect in operations management indicates that the increasing shelf-space allocated to a product has a positive effect on the product demand. This paper studies the billboard effect on the vertical information sharing strategy of competing supply chains in an environment with production diseconomies. We consider a model of two competing supply chains. Each supply chain consists of one retailer and one manufacturer, and the retailers engage in Cournot (quantity) competition. We analyze how equilibrium information sharing strategy, wholesale price and retail quantity are affected by the billboard effect. Our results show that with the existence of production diseconomies, information sharing benefits the supply chain and the billboard effect increases the value of information sharing.

Key words: billboard effect; information sharing; production diseconomy; supply chain management; competition

Contents

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Chapter 1

Introduction

The billboard effect originated from the hotel industry. Hotels listed on third-party distributors' website, commonly known as online travel agents (OTA), gain a reservation benefit in addition to direct sales. The benefit, often called the billboard effect, involves a boost in reservation through the hotel's own distribution channels (including its website), due to the hotel's being listed on the OTA website. In operations management, the billboard effect indicates that the increasing shelf-space allocated to a product has a positive effect on the product demand. The promotional role of inventory can be explained from the perspective of marketing. As stated by Balakrishnan et al. (2008), tall stacks of an item may enhance product visibility, kindle latent demand, signal a popular product, or provide consumers with an assurance of future availability. From the perspective of operations management, the increasing inventory provides a higher service rate and thus attracts more customer demand in the competition with outside options. Previous research provides empirical evidence of the promotional effect of inventory on demand. Wolfe (1968) studies the retail sales histories of style merchandise, such as women's dresses, coats, and sports clothes, and shows that within the selling season unit sales of each style are proportional to the amount of inventory displayed. The promotional effect also exists in other industries. Koschat (2008) documents the major findings of a market research in magazine industry conducted by a major US magazine publisher. It presents empirical evidence that inventory motivates demand, and it quantifies the magnitude of inventory effect. This market study develops an accurate and comprehensive understanding of the promotional effect of inventory on demand.

We consider a model of two competing supply chains and involve the billboard effect to study the effect of billboard effect coefficient on the vertical information sharing strategy of competing supply chains in an environment with production diseconomies. We consider a model of two competing supply chains, each consisting of one manufacturer and one retailer. The manufacturer i provides the exact retail quantity to her retailer i and the retailers engage in a Cournot (quantity) competition. We consider the case with production diseconomies, so the manufacturers produce with an increasing marginal cost.

The billboard effect is very similar to the concept of shelf-space-dependent demand involved in previous research. Most of the work considering the billboard effect studied its effect on supply chain coordination or the optimal inventory policies under such shelf-space-dependent demand. Our research on the billboard effect on the information sharing strategy is quite new.

The billboard effect coefficient, denoted by β, positively affects the customer demand, so $\beta > 0$ and we assume the market size is in the form of $a = \alpha + \beta q$, where α stands for the deterministic customer demand and βq stands for the demand stimulated by the billboard effect. Most of the previous research involving the shelf-space-dependent demand assumes that the entire inventory of a product is displayed on shelf so that the shelf-space exactly equals to the inventory of the product. We first follow the above assumption and consider the case where the billboard effect coefficient is the same for both of the retailers. In this case, the increased demand is a function of the aggregate retail quantities of two retailers. Later we release the assumption to the general case in which each retailer i has his individual billboard effect coefficient, denoted by

β_i and study the impact of a supply chain's billboard effect on its own and the parallel supply chain's information sharing strategy.

Intuitively, the information sharing between the retailer and the manufacturer enlarges the manufacturer's bargaining power and therefore reduces the retailer and the supply chain's profits. While with the consideration of production diseconomies, the information sharing makes it possible for the manufacturer to adjust wholesale price and hence to decrease the variance of retail quantity. This adjustment lowers the production cost and thus improves the profit of the supply chain. In the presence of the inventory billboard effect, both of the above effects may be affected and the tradeoff is a subject worthy of study. We will investigate the optimal information sharing strategy for the two competing supply chains and explore the impact of the inventory billboard effect on it.

We analyze how the equilibrium information sharing strategy, wholesale price, and retail quantity are affected by the billboard effect. Our results show that in the presence of production diseconomies, information sharing benefits the supply chain and the billboard effect increases the value of information sharing and thus pronounces supply chain's preference to be communicative.

The rest of the thesis is organized as follows. We summarize the related literature in Chapter 2. Chapter 3 describes the model framework. Chapters 4 and 5 study the inventory billboard effect on the information sharing equilibrium in the cases without and with production diseconomies, respectively. Chapter 6 extends the basic model in two directions, the scarcity effect and the imperfect information with investment. Chapter 7 concludes the thesis.

Chapter 2

Literature Review

This thesis is closely related to three categories of the existing literature. The first category is the literature on shelf-space-dependent demand. It is well known that retailers can affect sales quantity of a product by increasing the shelf-space allocated to the product. Marketing researchers and practitioners have extensively exploited the motivational effect of shelf-space on demand and sales. An explanation is that the increase in a product's shelf-space may induce more consumers to buy it because the consumers believe this product is popular. Other reasons include that tall stacks of an item may enhance product visibility, kindle latent demand , signal a popular product, or provide consumers with an assurance of future availability. Wolfe (1968) studies the retail sales histories of style merchandise, such as women's dresses, coats, and sports clothes, and shows that within the selling season unit sales of each style are proportional to the amount of inventory displayed. Koschat (2008) documents the major findings of a market research in magazine industry conducted by a major US magazine publisher. It presents the empirical evidence that inventory motivates sales, and quantifies the magnitude of the inventory effect. It shows that an inventory decrease for one brand can result a decrease of demand for itself, and an increase of demand for a competing brand. This market research develops an accurate and comprehensive understanding of the promotional effect of inventory on demand. In operations management, Cachon and Olivares (2010) conduct an empirical investigation of U.S. automobile industry on the inventory billboard effect and use detailed data from automobile dealerships to measure the extent that inventory drives sales quantity. Because the increased shelf-space requires higher inventories for retailers, which incur higher inventory costs, choosing an optimal shelf-space is a problem of inventory management.

The inventory billboard effect, or shelf-space-dependent demand, has been involved in many research literature. Gerchak and Wang (1994) develop an approach to modeling periodic-review production/inventory problems where the demand in any period depends on the starting inventory level in a very general deterministic form, multiplied by a random variable. They show that there exists a unique critical inventory level for all periods, with which an order-up-to type policy is optimal, and the way to determine the level is similar to a single-period model. Wang and Gerchak (2001) consider the supply chain coordination problem in a situation where retailers face demand rates of a product that positively depend on the shelf-space devoted to the product by themselves and their competitors. They show that in a supply chain with a manufacturer and two competitive retailers, when the demand is a function of the aggregate inventory, the manufacturer can coordinate the supply chain by offering an inventory costs subsidy to retailers. While when the demand is a function of the individual inventory, it depends on whether the retailers are centrally controlled.

In operations management, the billboard effect or the shelf-space-dependent demand can also be understood from the perspective of service rate. Robinson (1991) states that demand may increase as a function of service level. Customers have a preference for avoiding stockouts, and thus they are more likely to switch stores after experiencing a stockout than after finding goods in stock. The greater inventory a store holds, the more likely the customer is able to obtain the product by visiting this store and the less likely she would switch to another option. So increasing shelf-space increases the availability of a product and leads to a higher demand for the product. Dana and Petruzzi (2001) consider a firm's price and inventory policy when it faces uncertain demand that depends on both the price and the inventory level (shelf-space). They measure the service rate competition by assuming that consumers choose between visiting the firm and an exogenous outside option. They show that when the firm internalizes the effect of its inventory on its demand, it maintains a higher inventory level,provides a higher fill rate, attracts more customer demands and earns higher profits. Petruzzi et al. (2009) study the newsvendor problem when consumers are heterogeneous either in their valuation of the newsvendor's product, in their valuations of an outside option, or in both valuations. The outside option may be interpreted as a search cost. They develop a framework includes both the newsvendor model with price-dependent demand and the model with endogenous demand, and make improvement on Dana and Petruzzi's work.

Ernst and Powell (1995) also study a model in which the service level affects the distribution of demand and investigate the optimal inventory policies under service-sensitive demand. They model the response of long-run demand to the service level of the retailer, and determine optimal order-up-to inventory policies in the presence of service-sensitive demand. They extent the analysis dealing with deterministic demand to a more difficult case of stochastic demand, and propose a model of service sensitive demand in which the mean and the standard deviation of the long-run demand change independently as the retailer changes its service level. Similar to Wang and Gerchak (2001), Ernst and Cohen (1992) also study the coordination problem in a manufacturer/retailer inventory system, but from the perspective of service level. The demand is stochastic, and increases as a function of the service level offered to the market by the dealer. Extending the model Ernst and Cohen (1993) introduced for dealers, they adopt a profit maximization perspective and consider the impact of dealer's performance on the manufacturer's profit.

Many other papers also work on the relationship between availability and profitability. For example, Baker and Urban (1988), Datta and Pal (1990), Balakrishnan et al. (2004) develop static inventory-control models in which the demand is assumed as an increasing function of the firm's inventory level; Schwartz (1966), Schwartz (1970), Fergani (1976) and Hall and Porteus (2000) study dynamic models in which the future demand depends on customers' past experience with stockouts.

The second category of related literature is the research on information sharing in supply chains in a competitive environment. Information sharing has gained the interests of both academic and practitioners. In practice, information sharing happens between different levels of a supply chain, including information sharing between companies and investors, manufacturers and retailers, retailers and consumers. Information sharing may also happen among the parties in the same level of a vertical supply chain, such as multiple retailers receiving goods from a common manufacturer.

Li (1985) studies the incentives for Cournot oligopolists to share information about a common parameter (the uncertainty about the demand function) or about firm-specific parameters (the uncertainty about individual cost functions). They show that no information sharing is the unique equilibrium when the uncertainty is about the common demand, while complete information sharing is the the unique equilibrium when the private costs are uncertain. But the nonpooling equilibrium converges to the situation where the pooling strategies are adopted as the total amount of information increases, and the efficiency is achieved in the competitive equilibrium as the number of firms gets large.

Li (2002) considers the incentives for information sharing in a supply chain with horizontal competition (competition among multiple retailers). They model the problem with a two-level supply chain in which there are one manufacturer and many retailers. Each retailer possesses some private information about the downstream market demand or their individual cost, and they are engaged in a Cournot competition.They analyze both the ”direct effect” and the ”leakage effect” of vertical information sharing on retailers' information sharing strategies. They also identify the conditions under which the information may be traded when voluntary information sharing is impossible.

Many of these work shows that the manufacturer is better off by acquiring information from more retailers while each retailer is worse off by disclosing his information to the manufacturer due to both direct effect and leakage effect. So retailers have no incentive to share information with manufacturer voluntarily and thus no information sharing is the unique equilibrium. One thing to mention is that, as more retailers already disclose their information, the incremental gain to the manufacturer from the next retailer's information, and the loss for the next retailer to disclose his information become smaller.

Ha et al. (2011) extend the research by incorporating production diseconomies in the model and study how the vertical information sharing equilibrium in competing supply chains is affected by the diseconomy of scale. They consider a model of two supply chains, each consisting of one manufacturer and one retailer, with the retailers engaging in Cournot competition or Bertrand competition. They show that in the presence of production diseconomies, information sharing benefits a supply chain when the production diseconomy is large, competition is less intense, and the information is less accurate under Cournot competition. The results under Bertrand competition may be quite different from that of Cournot competition. Another contribution of their work is that they quantify and analyze three effects of information sharing between the retailer and the manufacturer in a supply chain, which are the direct, the competitive and the spillover effects.

The third related research stream considers the impact of production diseconomies. In practice, some production technologies exhibit diseconomies of scale, which means the capacity is increasingly more expensive and larger production quantity results in higher average production cost. Some empirical studies in practical industries show that production may have a diseconomy of scale in some industries. Griffin (1972) studies the U.S. petroleum refining industry and suggests an alternative process analysis approach that, instead of a statistical cost function, the marginal costs may rise and the average cost function may be U-shaped. Mollick (2004) investigates data from the Japanese vehicle industry and suggests that the auto-making industry operates in the range of increasing marginal costs in most of its products. In the previous work, it is quite common to model the phenomenon of production diseconomy using a quadratic cost function, such as Eichenbaum (1989) and Anand and Mendelson (1997).

Chapter 3

The Model Setup

We consider two competing supply chains which provide the same product to the market. Each supply chain consists of a manufacturer and a retailer, and the retailers are engaged in Cournot competition (quantity competition). The inverse demand function for retailer i is

$$p = a + \theta - q_i - q_j, \tag{3.1}$$

where a is the market potential and θ is the uncertainty of customer demand with zero mean and variance σ^2. Since we consider the model in which the customer demand is dependent on the shelf-space allocated to the product by the retailer, the market size is a function of q_i and q_j. Denote the dependent factor for retailer i by β_i. So the market potential becomes

$$a = \alpha + \beta_i q_i + \beta_j q_j. \tag{3.2}$$

If $0 \leq \beta_i < 1$, the inventory displays billboard effect, as $\beta_i \geq 0$ implies that the market size increases in retail quantity, while $\beta_i < 1$ ensures that $\partial p / \partial q_i = -(1 - \beta_i) < 0$, i.e., the market price decreases in the retail quantity. If $\beta_i < 0$, the inventory displays scarcity effect where the market size decreases in inventory. We focus on the former case in

Chapters 4 and 5. In this case, the inverse demand function for retailer i is

$$p = \alpha + \theta - (1-\beta_i)q_i - (1-\beta_j)q_j. \tag{3.3}$$

Each retailer may have his individual billboard effect coefficient. The individual billboard effect coefficient may be interpreted in three ways. First, it is possible that a retailer only displays a fraction of his entire inventory on the shelf in practice, so the individual billboard effect coefficient can be regarded as a measure of the ratio of the displayed quantity to the whole inventory and thus may be different for each retailer. Second, retailers facing different markets have different customer volume, and therefore the extent of billboard effect may differ. Third, observing the same shelf-space allocated to a product (displayed retail quantity), customers have different likelihood to be stimulated and to buy the product.

We assume the production cost per unit product is the same for two manufacturers. In the presence of production diseconomies, the production cost of manufacturer i is quadratic in the production quantity q_i, $C = cq_i^2/2$, where the parameter c is a measure of the production diseconomy. We first assume that each retailer has access to perfect information about the demand uncertainty. Later we will release this assumption to the case where each retailer observes a demand signal Y_i, with accuracy $1/s_i$, where $s_i = E\left[Var[Y_i|\theta]\right]/\sigma^2$.

We consider a multi-stage game and the sequence of events and decisions is as follows:

1. Before the retailers obtain the information on demand uncertainty, each manufacturer i decides whether to offer a payment to buy the information from retailer i, and retailer i decides whether to accept the payment and share the information with his manufacturer. If the retailer i has an agreement with manufacturer i on information sharing and shares the information with the manufacturer, we say

that supply chain i is communicative (denoted by C). Otherwise, supply chain i is non-communicative (denoted by N).

2. Each retailer obtains the perfect information on demand uncertainty and discloses it to his manufacturer if the supply chain is communicative.

3. Each manufacturer makes her decision on wholesale price w_i and retailer i determines his retail quantity q_i based on the wholesale price w_i in the Cournot competition.

4. Manufacturer i produces exactly q_i products and supplies them to retailer i. Market price is realized and parties in supply chains earn their profits.

We make the assumptions that whether supply chain i is communicative or non-communicative is observable to its parallel supply chain, while the wholesale price w_i of supply chain i is unobservable to supply chain j.

We solve the problem in a backward procedure.

1. Given any information sharing arrangement (X_i, X_j) in Stage 1, we solve the equilibrium wholesale prices and equilibrium retail quantities.

2. Based on the equilibrium wholesale prices and retail quantities, we calculate the ex ante profits for each party and the supply chains under different information sharing arrangements.

3. We solve the information sharing equilibrium in the first stage.

Chapter 4

The Competition without Production Diseconomies

We first consider the case without production diseconomies to investigate the information sharing equilibrium and the impact of the billboard effect coefficient. The results of this section will serve as the benchmark and be compared with the case with production diseconomies to analyze the effect of the scale of diseconomy. For simplicity, we first assume two competing supply chains have common billboard effect coefficient, $\beta_1 = \beta_2 = \beta$.

Because the information on demand uncertainty is perfect, the realized demand uncertainty is exactly the same with the demand information retailers obtained. Given any wholesale price w_i set by the manufacture i, retailer i maximizes his profit

$$\max_{q_i} \Pi_{R_i} = (p - w_i)q_i = \left[\alpha + \theta - w_i - (1-\beta)q_j\right] q_i - (1-\beta)q_i^2$$

with the retail quantity

$$q_i(w_i, q_j) = \frac{\alpha + \theta - w_i - (1-\beta)q_j}{2(1-\beta)}. \tag{4.1}$$

Anticipating retailer's retail quantity, manufacturer sets the optimal wholesale price w_i to maximize her profit. Since there is no scale of diseconomy, the marginal production cost is a constant. Without loss of generality, we standardize the production cost to zero. If supply chain i is *communicative*, manufacturer i maximizes her profit $\max_{w_i} \Pi_{M_i} = w_i q_i$ with wholesale price $w_i(q_j) = (\alpha + \theta - (1-\beta)q_j)/2$. If supply chain i is *non-communicative*, manufacturer i anticipates the retailer's retail quantity and maximizes her expected profit $\max_{w_i} \Pi_{M_i} = w_i E[q_i]$ with the wholesale price $w_i = (\alpha - (1-\beta)E[q_j])/2$. Plugging the wholesale price into equation (4.1), we get the optimal retail quantity of supply chain i in response to that of supply chain j.

Lemma 1. *(a) If supply chain i is communicative, the retail quantity of supply chain i in response to that of supply chain j is*

$$q_i(q_j) = \frac{\alpha + \theta - (1-\beta)q_j}{4(1-\beta)}. \tag{4.2}$$

(b)If supply chain i is non-communicative, the retail quantity of supply chain i in response to that of supply chain j is

$$q_i(q_j) = \frac{\alpha + 2\theta + (1-\beta)E[q_j] - 2(1-\beta)q_j}{4(1-\beta)}. \tag{4.3}$$

Because of the assumption that the wholesale price w_i is unobservable to the parallel supply chain, there is only the retailers's Cournot competition. The equilibrium retail quantities are obtained by solving $q_1 = q_1(q_2)$ and $q_2 = q_2(q_1)$ simultaneously.

Lemma 2. *The equilibrium retail quantity and wholesale price are shown in Table 4.1.*

Table 4.1: Equilibrium retail quantity and wholesale price

	CC	NN	CN	NC
q_i	$\frac{\alpha}{5(1-\beta)} + \frac{\theta}{5(1-\beta)}$	$\frac{\alpha}{5(1-\beta)} + \frac{\theta}{3(1-\beta)}$	$\frac{\alpha}{5(1-\beta)} + \frac{\theta}{7(1-\beta)}$	$\frac{\alpha}{5(1-\beta)} + \frac{3\theta}{7(1-\beta)}$
w_i	$\frac{2}{5}\alpha + \frac{2}{5}\theta$	$\frac{2}{5}\alpha$	$\frac{2}{5}\alpha + \frac{2}{7}\theta$	$\frac{2}{5}\alpha$

***Remark* 1.** $q_i^{(N,X_j)}$ is more responsive to the demand information and thus is more variable than $q_i^{(C,X_j)}$. Given any information sharing strategy of supply chain j, the equilibrium retail quantity q_i is more responsive to the demand information and thus more variable when supply chain i is non-communicative. Information sharing in supply chain i makes the equilibrium retail quantity q_i less variable because the shared information allows the manufacturer to adjust the wholesale price, which makes the retailer adjust the quantity in the opposite direction.

***Remark* 2.** $q_j^{(C,X_j)}$ is more responsive to demand information than $q_j^{(N,X_j)}$. The information sharing in supply chain i makes the retail quantity of supply chain j more responsive to the information and thus more variable. This is because the retail quantities of two supply chains are positively substitutable and the less variable q_i makes q_j more variable.

***Remark* 3.** The equilibrium retail quantity q_i can be regarded as the sum of the deterministic term $\alpha/(5(1-\beta))$ and the uncertain term about θ. The billboard effect coefficient β serves as a scale factor to both the deterministic term and the uncertain term. It magnifies the responsiveness of retailer to the information and thus increases the variance of retail quantity (the θ term).

***Remark* 4.** Manufacturer charges a more responsive price in communicative supply chain compared to non-communicative supply chain. It means that information sharing makes the manufacturer have more bargaining power and make better pricing decision.

Lemma 3. *The payoff matrix of ex ante profits for retailers, manufacturers and supply chains are shown in Table 4.2, Table 4.3 and Table 4.4.*

Table 4.2: Payoff Matrix for Retailers

		R 2 C	*R 2* N
R 1	C	$\left(\frac{\alpha^2}{25(1-\beta)}+\frac{\sigma^2}{25(1-\beta)}, \frac{\alpha^2}{25(1-\beta)}+\frac{\sigma^2}{25(1-\beta)}\right)$	$\left(\frac{\alpha^2}{25(1-\beta)}+\frac{\sigma^2}{49(1-\beta)}, \frac{\alpha^2}{25(1-\beta)}+\frac{9\sigma^2}{49(1-\beta)}\right)$
	N	$\left(\frac{\alpha^2}{25(1-\beta)}+\frac{9\sigma^2}{49(1-\beta)}, \frac{\alpha^2}{25(1-\beta)}+\frac{\sigma^2}{49(1-\beta)}\right)$	$\left(\frac{\alpha^2}{25(1-\beta)}+\frac{\sigma^2}{9(1-\beta)}, \frac{\alpha^2}{25(1-\beta)}+\frac{\sigma^2}{9(1-\beta)}\right)$

Table 4.3: Payoff Matrix for Manufacturers

		M 2 C	*M 2* N
M 1	C	$\left(\frac{2\alpha^2}{25(1-\beta)}+\frac{2\sigma^2}{25(1-\beta)}, \frac{2\alpha^2}{25(1-\beta)}+\frac{2\sigma^2}{25(1-\beta)}\right)$	$\left(\frac{2\alpha^2}{25(1-\beta)}+\frac{2\sigma^2}{49(1-\beta)}, \frac{2\alpha^2}{25(1-\beta)}\right)$
	N	$\left(\frac{2\alpha^2}{25(1-\beta)}, \frac{2\alpha^2}{25(1-\beta)}+\frac{2\sigma^2}{49(1-\beta)}\right)$	$\left(\frac{2\alpha^2}{25(1-\beta)}, \frac{2\alpha^2}{25(1-\beta)}\right)$

Table 4.4: Payoff Matrix for Supply Chains

		SC 2 C	*SC 2* N
SC 1	C	$\left(\frac{3\alpha^2}{25(1-\beta)}+\frac{3\sigma^2}{25(1-\beta)}, \frac{3\alpha^2}{25(1-\beta)}+\frac{3\sigma^2}{25(1-\beta)}\right)$	$\left(\frac{3\alpha^2}{25(1-\beta)}+\frac{3\sigma^2}{49(1-\beta)}, \frac{3\alpha^2}{25(1-\beta)}+\frac{9\sigma^2}{49(1-\beta)}\right)$
	N	$\left(\frac{3\alpha^2}{25(1-\beta)}+\frac{9\sigma^2}{49(1-\beta)}, \frac{3\alpha^2}{25(1-\beta)}+\frac{3\sigma^2}{49(1-\beta)}\right)$	$\left(\frac{3\alpha^2}{25(1-\beta)}+\frac{\sigma^2}{9(1-\beta)}, \frac{3\alpha^2}{25(1-\beta)}+\frac{\sigma^2}{9(1-\beta)}\right)$

Theorem 1. *Without the information payment, retailers prefer information sharing strategy* NN *while manufacturers prefer* CC*. The information sharing equilibrium for supply chains is* NN*. Information sharing makes the manufactures better off but makes the retailers and the supply chains worse off.*

From the ex ante profit functions we find that the ex ante profits for each party and the supply chain increase in the variance of retail quantity. When supply chain is non-communicative, the retail quantity is more responsive to the demand information and thus more variable, therefore retailers obtain higher profits. It is because the higher responsiveness indicates that retailers make better use of the demand information and make better decisions to earn more profits in non-communicative supply chains. When retailers share information with their manufacturers, informed manufacturers make use of the information and have more bargaining power to adjust the wholesale price. Therefore retailers' retail quantities are less responsive to the information and get close to the situation of no information, which means they are losing the advantage of demand information. So both of the retailers prefer not to share the information with their manufacturers.

Information sharing allows the manufacturers to have more price bargaining power. From manufacturer's ex ante profit function, we see that the manufacturer in communicative supply chain benefits from the more responsive wholesale price and earns more profits. As Li and Zhang (2002) state, the manufacturer seeks more economic rent with better information through pricing and hurts the benefits of retailer and supply chain. From the perspective of Economics, complete information sharing reduces both the expected total social benefits and the expected consumer surplus.

***Remark* 5.** The supply chain's competition is a case of *prisoner's dilemma*. Both of the supply chains choose no information sharing strategy, even though they will gain greater profits if they both choose information sharing strategy. The process can be explained as follows. Both of the supply chains achieve optimal profits when they are both communicative (C). Given supply chain j is communicative, supply chain i can be better off if it turns to be non-communicative (N) and thus supply chain i switch to no information sharing strategy. Given supply chain i is non-communicative, supply chain j betters off with strategy N and thus also switch to be non-communicative (N). When both of them choose no information sharing, they achieve the equilibrium so NN is the information sharing equilibrium but a case of *prisoner's dilemma*.

Chapter 5

The Competition with Production Diseconomies

Now we consider the model with production diseconomies, and investigate the information sharing equilibrium and the impact of billboard effect coefficient in the presence of production diseconomies. We assume the production cost per unit product is the same for the two manufacturers. The production cost of manufacturer i, denoted by C, is quadratic in the production quantity q_i. Define $C = cq_i^2/2$, where the parameter c is a measure of the production diseconomy. The two competing supply chains may have individual billboard effect coefficient, denoted by β_1 and β_2 respectively. So the market clearing price is $\alpha + \beta_1 q_1 + \beta_2 q_2$. The inverse demand function turns to be $p = \alpha + \theta - (1-\beta_1)q_1 - (1-\beta_2)q_2$.

5.1 Equilibrium Retail Quantity

With the optimal retail quantity $q_i = [\alpha + \theta - w_i - (1-\beta_j)q_j]\big/[2(1-\beta_i)]$, retailer i's profit function is $\Pi_{R_i} = (1-\beta_i)q_i^2$, which is the same as the previous section. If supply chain i is *communicative*, Manufacturer i maximizes her profit $\max\limits_{w_i} \Pi_{M_i} = w_i q_i - cq_i^2/2$;

if supply chain i is *non-communicative*, Manufacturer i has anticipation about the retail quantity of retailer i and maximizes her expected profit $\max_{w_i} \Pi_{M_i} = w_i E[q_i] - cE\left[q_i^2\right] \Big/ 2$. Following the similar analysis procedure, we obtain the equilibrium retail quantity and corresponding wholesale price.

Lemma 4. *The equilibrium retail quantity under any information sharing arrangement is*

$$q_i^{(X_i,X_j)} = A_i + B_i^{(X_i,X_j)}\theta, \tag{5.1}$$

where $A_i = E[q_i] = \frac{[3(1-\beta_j)+c]\alpha}{[4(1-\beta_i)+c][3(1-\beta_j)+c]+[3(1-\beta_i)+c](1-\beta_j)}$ *is the deterministic solution, and*

$$\begin{aligned}
B_i^{CC} &= \frac{3(1-\beta_j)+c}{[4(1-\beta_i)+c][3(1-\beta_j)+c]+[3(1-\beta_i)+c](1-\beta_j)}, \\
B_i^{NN} &= \frac{1}{3(1-\beta_i)}, \\
B_i^{CN} &= \frac{1}{7(1-\beta_i)+2c}, \\
B_i^{NC} &= \frac{3(1-\beta_j)+c}{[7(1-\beta_j)+2c](1-\beta_i)}.
\end{aligned}$$

The equilibrium retail quantities under any information sharing arrangement consist of the deterministic quantity and the uncertain term resulted from the demand uncertainty. Information sharing arrangement affects the equilibrium retail quantity through $B_i^{X_i,X_j}$, the retailer's responsiveness to the demand uncertainty and also stands for the variability of retail quantity. It can be shown that A_i and $B_i^{(X_i,X_j)}$ increase in β_i or β_j or both. So the increase of the billboard effect coefficient of either supply chain will increase the retail quantities of both supply chains. It makes sense since an increasing billboard effect coefficient stimulates larger market demand and both the competing supply chains increase their retail quantities to respond to it. The other observations about equilibrium retail quantity are the same as Chapter 4.

The equilibrium wholesale price for communicative supply chain is $w_i = [2(1-\beta_i)+c]q_i$, the equilibrium wholesale price for non-communicative supply chain is $w_i = [2(1-\beta_i)+c]E[q_i]$.

Corollary 1. *When the two supply chains have the same billboard effect coefficient, the equilibrium retail quantity is shown below.*

Table 5.1: Equilibrium retail quantity when $\beta_1 = \beta_2$

CC	NN	CN	NC
$\frac{\alpha}{5(1-\beta)+c} + \frac{\theta}{5(1-\beta)+c}$	$\frac{\alpha}{5(1-\beta)+c} + \frac{\theta}{3(1-\beta)}$	$\frac{\alpha}{5(1-\beta)+c} + \frac{\theta}{7(1-\beta)+2c}$	$\frac{\alpha}{5(1-\beta)+c} + \frac{[3(1-\beta)+c]\theta}{[7(1-\beta)+2c](1-\beta)}$

Comparing with the equilibrium retail quantities in the case without production diseconomies, for communicative supply chain, equilibrium q_i is less responsive because informed manufacturer makes greater price adjustment due to the production diseconomies; for non-communicative supply chain, the responsiveness of retail quantity to information is the same with that in the case without production diseconomies. So non-communicative supply chain is less effective in reducing average production cost.

Because the billboard effect coefficient β serves as the scaler factor in the denominator of $B_i^{(X_i,X_j)}$, it increases the variance of the retail quantity. From the ex ante profit function, it can be verified that increased variance of retail quantity increases retailers' ex ante profits while may decrease manufacturers' ex ante profits. So from the perspective of supply chain, the billboard coefficient β has two effects that work in the opposite directions, and the trade off between these two effects is determined by the coefficient β and parameter c. The detailed analysis will be illustrated in the following section.

5.2 Ex ante Profits

The differences in ex ante profits among different information sharing arrangements lie in the uncertain term (profits resulted from the demand uncertainty). To clarify the analysis, we focus on the uncertain parts of profits only. The ex ante profits of retailers and manufacturers are shown below.

Table 5.2: Payoff Matrix for Retailers

Retailer 1 \ *Retailer 2*	C	N
C	$(1-\beta_1)\left(B_1^{CC}\right)^2\sigma^2,\ (1-\beta_2)\left(B_2^{CC}\right)^2\sigma^2$	$(1-\beta_1)(B_1^{CN})^2\sigma^2,\ (1-\beta_2)(B_2^{CN})^2\sigma^2$
N	$(1-\beta_1)\left(B_1^{NC}\right)^2\sigma^2,\ (1-\beta_2)\left(B_2^{NC}\right)^2\sigma^2$	$(1-\beta_1)(B_1^{NN})^2\sigma^2,\ (1-\beta_2)(B_2^{NN})^2\sigma^2$

Table 5.3: Payoff Matrix for Manufacturers

M 1 \ *M 2*	C	N
C	$\left[2(1-\beta_1)+\frac{c}{2}\right]\left(B_1^{CC}\right)^2\sigma^2, \left[2(1-\beta_2)+\frac{c}{2}\right]\left(B_2^{CC}\right)^2\sigma^2$	$\left[2(1-\beta_1)+\frac{c}{2}\right](B_1^{CN})^2\sigma^2, -\frac{c}{2}(B_2^{CN})^2\sigma^2$
N	$-\frac{c}{2}\left(B_1^{NC}\right)^2\sigma^2, \left[2(1-\beta_2)+\frac{c}{2}\right]\left(B_2^{NC}\right)^2\sigma^2$	$-\frac{c}{2}(B_1^{NN})^2\sigma^2, -\frac{c}{2}(B_2^{NN})^2\sigma^2$

Lemma 5. *In the presence of production diseconomies, without the consideration of the payment for information, retailers prefer* NN *while manufacturers prefer* CC. *The condition for information sharing is that the information sharing makes the supply chain better off.*

The retailers' and manufacturers' preference can be obtained from Table 5.2 and Table 5.3. Similar to the previous case, information sharing makes retailers' profits worse off and neither of the retailers has the incentive to share information with their manufacturers voluntarily. Both manufacturers prefer information sharing since they can benefit from the shared information through increased bargaining power and better pricing decisions. Additionally, in the presence of production diseconomies, the more variable retail quantity results in higher average production cost. Information sharing allows manufacturers to adjust the wholesale price according to the demand uncertainty information. This price adjustment induces the retailer's order quantity to adjust in the opposite direction and thus lowers the variance of retail quantity. Manufacturers benefit from the lowered average production cost so the information sharing equilibrium for manufacturers is *CC*.

Serving as the scaler factor, the billboard effect coefficient magnifies both the manufacturer's advantage of being informed and the disadvantage of not being informed and therefore pronounce the manufacturer's preference for information sharing. The reason is that, because of the driving effect of retail quantity on demand, the billboard effect coefficient increases both the expectation and the variability of retail quantity, which greatly increases the average production cost due to the production diseconomies. So the billboard effect coefficient β works in the same direction with increasing production cost c and increase the value of information sharing for manufacturers.

Voluntary information sharing between retailer and manufacturer is impossible, so it is natural to investigate the condition for information sharing with payment. For manufacturers, they pay for the information only when the incremental profit brought by shared information covers the information sharing cost. While for retailers, they share information with manufacturers only when the payment is no less than the profit decrease. It can be verified that the information sharing with payment is possible only when information sharing brings benefit to the whole supply chain, or information sharing is the equilibrium for supply chain. Now we analyze the equilibrium for supply chains. The payoff matrix for supply chains is shown in Table 5.4.

Table 5.4: Payoff Matrix for Supply Chains

		SC 2 C	N
SC 1	C	$\left[3(1-\beta_1)+\frac{c}{2}\right]\left(B_1^{CC}\right)^2, \left[3(1-\beta_2)+\frac{c}{2}\right]\left(B_2^{CC}\right)^2$	$\left[3(1-\beta_1)+\frac{c}{2}\right](B_1^{CN})^2, \left[1-\beta_2-\frac{c}{2}\right](B_2^{CN})^2$
	N	$\left[1-\beta_1-\frac{c}{2}\right]\left(B_1^{NC}\right)^2, \left[3(1-\beta_2)+\frac{c}{2}\right]\left(B_2^{NC}\right)^2$	$\left[1-\beta_1-\frac{c}{2}\right](B_1^{NN})^2, \left[1-\beta_2-\frac{c}{2}\right](B_2^{NN})^2$

5.3 Information Sharing Equilibrium

Based on the ex ante profits shown in Table 5.4, we analyze supply chains' information sharing equilibrium. To simplify the analysis process, we define

$$z_i = \frac{c}{1-\beta_i}, \ \ z_i > 0.$$

Obviously z_i increases in c and β_i. Comparing the profits in different information sharing arrangements, we find that supply chain l's decision is determined by two critical values of β_i (or z). Since each supply chain has his individual billboard effect coefficient, the critical values of supply chain i's coefficient β_i may also be affected by β_j, the coefficient of the parallel supply chain.

Proposition 1. *Supply chain i's information sharing decision is determined by the relationship among z_i and two critical values t_i^C and t_i^N, where*

$$t_i^C = f(z_j) = \frac{\sqrt{8z_j^2+64z_j+127}-2z_j-7}{z_j+4}, \ \ t_i^N = 1.28. \tag{5.2}$$

The superscript C and N stand for communicative *and* noncommunicative *supply chain for the parallel supply chain.*

***Remark* 6.** Given supply chain j is communicative, supply chain i's decision is affected by the billboard effect coefficient of the competing supply chain j. It can be verified that the critical value t_i^C decreases in z_j and thus decreases in β_j and c. It means that increasing β_j and c make the range for supply chain i to be non-communicative get smaller. When β_j, the billboard effect coefficient of supply chain j, increases, the market demand increases, and therefore the retail quantities and variance of retail quantity of both supply chains increase. So the value of information sharing for supply chain is pronounced and the range for supply chain i to choose no information sharing gets smaller. The effect of c works in the same direction.

***Remark* 7.** Given supply chain j is non-communicative, supply chain i's decision is not affected by supply chain j's billboard effect coefficient β_j, but only determined by his own coefficient β_i.

***Remark* 8.** It can be verified that $t_i^C < t_i^N = 1.28$, so the range for supply chain i to choose no information sharing is smaller when supply chain j shares information, compared to that in the case when supply chain j chooses no information sharing. It can be explained from two perspective. Firstly, the fact that supply chain j is communicative indicates billboard coefficient β_j is high, and thus as what we explained in Remark 6, the expectation and variance of retail quantity q_i and therefore the value of information sharing increase, so supply chain i prefers information sharing more. On the other hand, as explained in Remark 2, the information sharing in supply chain j makes the retail quantity of supply chain i q_i more variable and thus makes the production cost higher. So supply chain i is more likely to choose information sharing.

Corollary 2. *When two supply chains have the same billboard effect coefficient, the two critical values of z_i are $t^C = 1.02$ and $t^N = 1.28$.*

Given supply chain j's information sharing strategy is X_j (N or C), supply chain i's information sharing decision is determined by the critical value $t_i^{X_j}$. When $z_i < t_i^{X_j}$, supply chain i chooses to be non-communicative; when $z_i > t_i^{X_j}$, supply chain i chooses to be communicative. The reason is as follows: when supply chain i's billboard effect coefficient is relatively large and thus z_i is large, supply chain i's equilibrium retail quantity gets larger and more variable (as stated in Remark 3), therefore the average production cost of supply chain i increases due to the production diseconomies. In this condition the benefit of information sharing gets larger so supply chain i prefers to choose to be communicative, which allows manufacturer to adjust wholesale price to reduce the variation of retail quantity and thus reduce the average production cost. When the billboard effect is relatively low and the value of information sharing is small, the incremental profit of communicative manufacturer cannot cover the profit loss of retailer, so supply chain chooses no information sharing. Combing the choice of two supply chains, we obtain the information sharing equilibrium.

Theorem 2. *With production diseconomies and the individual billboard effect coefficient, the information sharing equilibrium for supply chains (with respect to z) is*

$$\begin{cases} NN & if \;\; z_2 < t_2^C \;\&\; z_1 < 1.28 \;\; or \;\; (t_2^C <) z_2 < 1.28 \;\&\; z_1 < t_1^C \\ NC & if \;\; z_2 > 1.28 \;\&\; z_1 < t_1^C \\ CN & if \;\; z_2 < t_2^C \;\&\; z_1 > 1.28 \\ CC & if \;\; z_2 > 1.28 \;\&\; t_1^C < z_1 < 1.28 \;\; or \;\; z_2 > t_2^C \;\&\; z_1 > 1.28 \\ CC \;\&\; NN & if \;\; t_2^C < z_2 < 1.28 \;\&\; t_1^C < z_1 < 1.28 \end{cases}$$

Corollary 3. *When two supply chains have the same billboard effect coefficient, the information sharing equilibrium for supply chains is*

$$\begin{cases} CC & if \;\; z = \frac{c}{1-\beta} > 1.28 \\ CC \;\&\; NN & if \;\; 1.02 < z = \frac{c}{1-\beta} \leq 1.28 \\ NN & if \;\; 0 \leq z = \frac{c}{1-\beta} \leq 1.02 \end{cases}$$

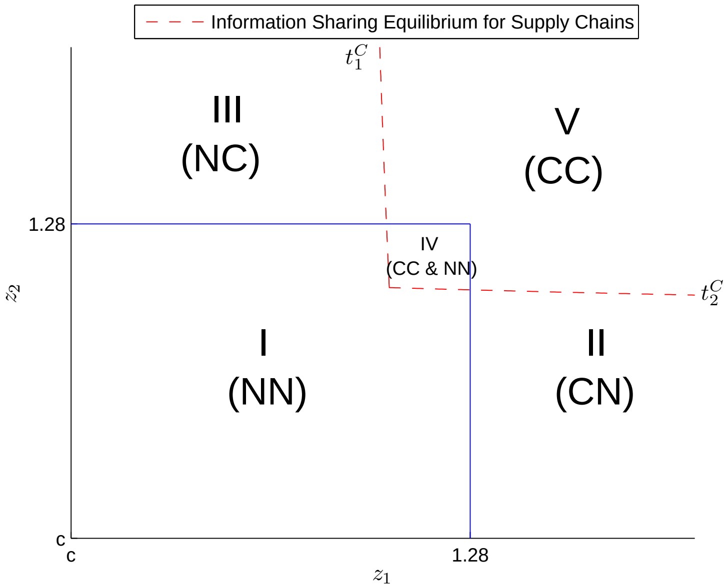
***Remark* 9.** For the situation with two equilibria *CC & NN*, equilibrium *CC* dominates equilibrium *NN*.

In the case without production diseconomies, the information sharing equilibrium for supply chains is *NN* and it is a problem of *prisoner's dilemma*. Now there exists the production disecoomies. Because the non-communicative supply chain has a more variable retail quantity, which increases the average production cost, the production diseconomies has a negative effect on the non-communicative supply chain. The increasing billboard effect coefficient β_i pronounces the negative effect of production diseconomies and makes it exceed the negative effect of double marginalization on communicative supply chain, which may induce the information sharing equilibrium shift to *CC* to lower the average production cost. So as z_i increases, which means β_i increases, the information sharing strategy of supply chain i tends to shift from *N* to *C*, and also makes supply chain j prefer strategy *C* more, and thus information sharing equilibrium move towards *CN* or *NC* and finally *CC*. In each case, neither of the supply chains has incentive to switch to the other information sharing strategy so the *prisoner's dilemma* disappears in the presence of production diseconomies.

The billboard effect coefficient β_i magnifies both of the effects. Since z_i involves parameter c, the production cost c also affects the critical values of β_i. As the production cost c increases, the effect of production diseconomies is more likely to outweigh the effect of double marginalization, and thus the supply chain benefits from switching from being *non-communicative* to being *communicative*. The benefit increases in β_i. So the billboard effect coefficient β_i works in the same direction with production cost c on the information sharing equilibrium: increasing c or increasing β_i induces the supply chain to choose information sharing as the equilibrium strategy. Since $z_i = c/(1-\beta_i)$, z_i can be interpreted as the tolerance of billboard effect: when production cost is low, supply chain can tolerate relatively high billboard effect to remain non-communicative; as production cost increases, its tolerance of billboard effect decreases, so supply chain switch to be communicative after the billboard effect coefficient β_i exceeds certain critical value, and the critical value decreases in production cost c.

The information sharing equilibrium with respect to z_i stated in Theorem 2 is illustrated in Figure 5.1.

Figure 5.1: Information sharing equilibrium w.r.t. z_i



The dashed curve stands for t_i^C, which is a convex decreasing function of z_j. Since $0 < \beta < 1$, the domain of z_i is (c, ∞), and thus the relationship among c, t_i^C and 1.28 determines the set of potential information sharing equilibria. As c increases, the domains of equilibria *NC*, *CN NN* and *CC & NN* become smaller and may even disappear. As c increases, the areas of *I, II and III* decrease. When c increases to the point where $t_2^C(z_1 = 1.28) = c_1^*$ or $t_1^C(z_2 = 1.28) = c_1^*$, areas *II and III* both disappear. It can be shown that $c_1^* = 1.0115$. So when $c \geq 1.0115$, the information sharing equilibrium can only be *NN*, *CC & NN* or *CC*. When c increases to the intersection of t_1^C and t_2^C, $c_2^* = 1.0206$, area *I* disappears. So when $c \geq 1.0206$, the information sharing equilibria may only be *CC & NN* or *CC*. $c_2^* = 1.0206$ equals to the critical value of z for information sharing equilibrium in the case with common billboard coefficient. When c exceeds the point 1.28, area *IV* also disappears and there is only one information sharing equilibrium *CC*.

The impact of production cost c on potential information sharing equilibria is summarized below. The potential information sharing equilibria contains

$$\begin{cases} \text{Area I, II, III, IV and V} & \text{if} \quad 0 < c < 1.0115 \\ \text{Area I, IV and V} & \text{if} \quad 1.0115 \leq c < 1.0206 \\ \text{Area IV and V} & \text{if} \quad 1.0206 \leq c < 1.28 \\ \text{Area V} & \text{if} \quad c \geq 1.28 \end{cases}$$

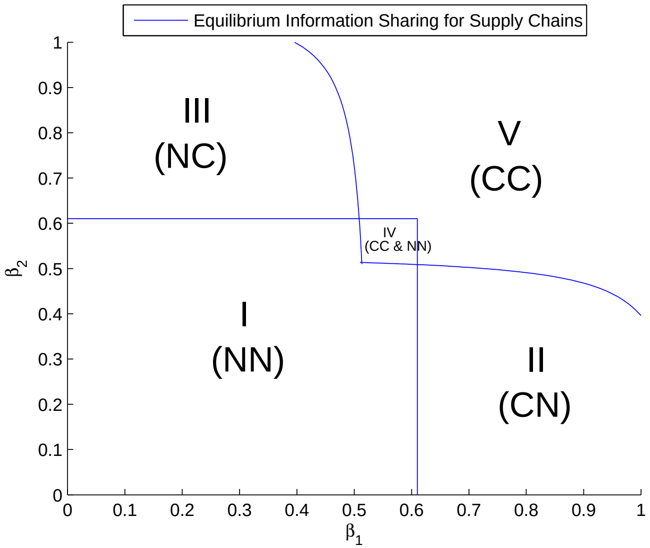
Area *I, II, III, IV and V* are as shown in Figure 5.1. When $0 < c < 1.0115$, all the equilibria are possible and thus there are five areas (as shown in Figure 5.1.). The information sharing equilibrium is determined by the value of β of the two supply chains. When $1.0115 \leq c < 1.0206$, area *II and III* disappear simultaneously, so there remain three areas *I, IV and V*. When $1.0206 \leq c < 1.28$, area *I* also disappears, the information sharing equilibrium can only be *CC & NN* or *CC*. After c exceeds 1.28, the information sharing equilibrium must be *CC*.

Next we convert the information sharing equilibrium with respect to z to that with respect to β_i. When $0 < c < 1.0115$, suppose $c = 0.5$, all the equilibrium strategies are possible, the equilibrium with respect to β_i is shown as Figure 5.2(a).

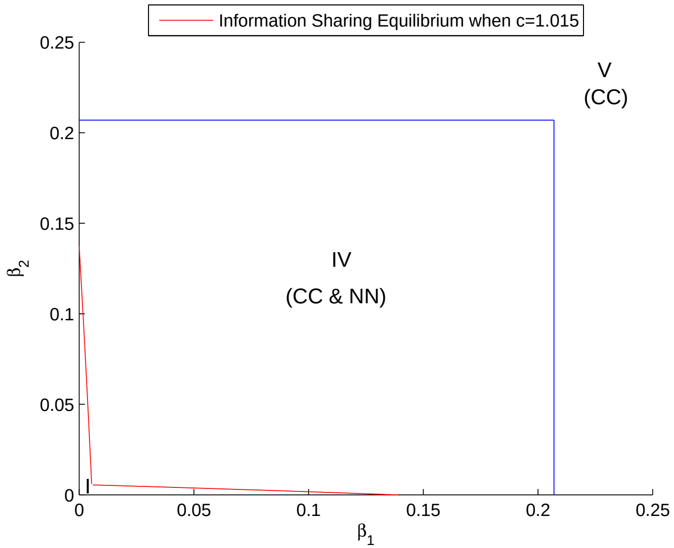
When $1.0115 \leq c < 1.0206$, equilibria *CN* and *NC* become impossible, the equilibrium with respect to β_i turns to be Figure 5.2(b), where *Area I* stands for equilibrium NN.

When $1.0206 \leq c < 1.28$, equilibrium *NN* also becomes impossible and there only exist two potential equilibria as shown in Figure 5.2(c).

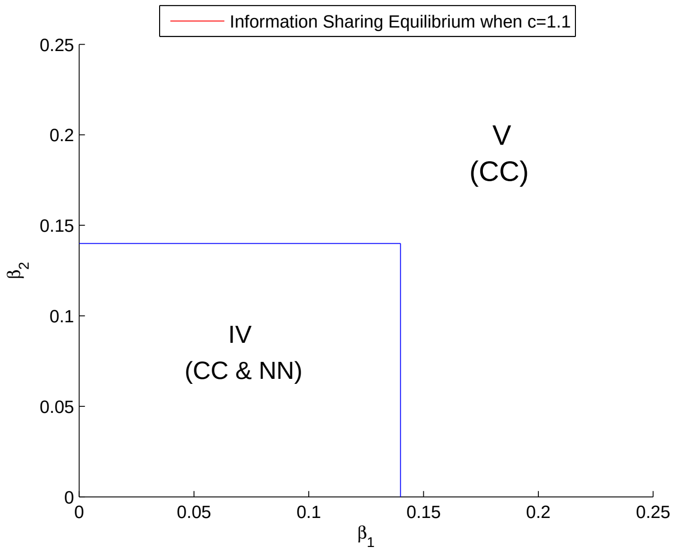
When $c \geq 1.28$, the information sharing equilibrium is always *CC* for any (β_1, β_2).



(a) Information sharing equilibrium w.r.t. β ($c = 0.5$)



(b) Information sharing equilibrium w.r.t. β ($c = 1.015$)



(c) Information sharing equilibrium w.r.t. β ($c = 1.1$)

Figure 5.2: Information sharing equilibrium w.r.t. β and impact of c

When $\beta_1 = \beta_2 = 0$ (no billboard effect), it is similar to the problem of Ha et al. (2011) with competition intensity $\gamma = 1$ and perfect information. We can analyze the impact of billboard effect on information sharing strategy by comparing our result with that of Ha et al. When their equilibrium is *NN*, the billboard effect in our problem makes it possible that one or both supply chains switch to be communicative and thus the equilibrium information sharing strategy changes to be *CN*, *NC* or *CC*. So we can say that the billboard effect makes supply chains favor information sharing more. For supply chain to benefit from information sharing, a smaller scale of diseconomy is required due to the billboard effect. It is because the billboard effect enlarges the retail quantity and the variance of that, and thus pronounces the value of information sharing. Therefore when their strategy is *CC*, the equilibrium information sharing strategy is also *CC*.

***Remark* 10.** The range of 'N-N' decreases in β_1 and β_2; when β_i is very small, supply chain j's decision isnot affected by β_i and is only determined by its own β_j and c.

***Remark* 11.** When $z_2 > 1.28$ & $z_1 > 1.28$, *C (information sharing)* is the dominant strategy; when $z_2 < t_2^C$ & $z_1 < t_1^C$, *N (no information sharing)* is the dominant strategy.

***Remark* 12.** When both *CC* and *NN* are information sharing equilibria, it is verified that *CC* dominates *NN*. So supply chains may make more profits when both of them choose to *communicative* (share information).

***Remark* 13.** It can be verified that the billboard effect increases the profits of retailers and informed manufacturers, while decreases the profits of uninformed manufacturers. For the perspective of the whole supply chain, for communicative supply chain, the ex ante profits increase in the billboard effect coefficient β_i. So even though the billboard effect increases the variance of retail quantity and thus incurs higher average production cost, its benefit in demand motivation outweighs the cost and makes the supply chain earns more profits. For non-communicative supply chain, since the supply chain cannot adjust the retail quantity effectively, whether the benefit of billboard effect can dominate the increased production cost is determined by the extent of the scale of diseconomy: when c is less than a certain value, the benefit of promotional demand dominates; when c is larger than a certain value, the effect on retail quantity variance dominates. The critical value depends on the information sharing strategy of the parallel supply chain.

To further analyze the billboard effect, we compare our results with those of Ha et al. (2011). For simplicity, we use the results of common billboard effect coefficient case. With competition intensity $\gamma = 1$, common production cost $c_1 = c_2 = c$, and perfect information (which means $s_1 = s_2 = 0$), the critical values for z in our problem are the same as the critical values for c in their problem. Since $0 < 1 - \beta < 1, z > c$. So when the equilibrium of their problem is *CC*, our equilibrium is also *CC*; when their equilibrium is *CC or NN*, our equilibrium may be either *CC or NN* or *CC*; when their equilibrium is *NN*, our equilibrium may be any of the three equilibria. This comparison result shows that the billboard effect makes supply chains prefer information sharing more.

Chapter 6

Extensions

In this chapter we extend the original model in two directions. We first consider the case with scarcity effect and compare the results with that of case with billboard effect. Then we study a more general and more practical problem where the demand information is imperfect and manufacturers need to make information sharing investment to be communicative.

6.1 Case with Scarcity Effect

When the product inventory displays a scarcity effect on sales, we model this problem with the coefficient $\beta < 0$ while most expressions remain the same. Since the results of different cases have the same structure, we take the case with common billboard effect as example to analyze the impact of scarcity effect on market price, retail quantity, profits and supply chain information sharing equilibrium.

6.1.1 Retail quantity

In the presence of scarcity effect, the equilibrium retail quantity of noncommunicative supply chain is still more responsive and thus more variable than that of communicative supply chain. In the meantime, for both communicative and noncommunicative supply chain, the scarcity effect decreases the expectation and variance of equilibrium retail quantity. We take CC as example. $q_i = \frac{\alpha+\theta}{5(1-\beta)+c}$. Denote the retail quantity without inventory-level-dependent demand as q_0, the retail quantity with billboard effect as q_b, the retail quantity with scarcity effect as q_s. When $0 < \beta < 1$ (billboard effect), $q_b > q_0$; when $\beta < 0$ (scarcity effect), $q_s < q_0$. So when the inventory displays scarcity effect on demand, retailers tends to hold less inventory to inspire greater demand.

6.1.2 Market price

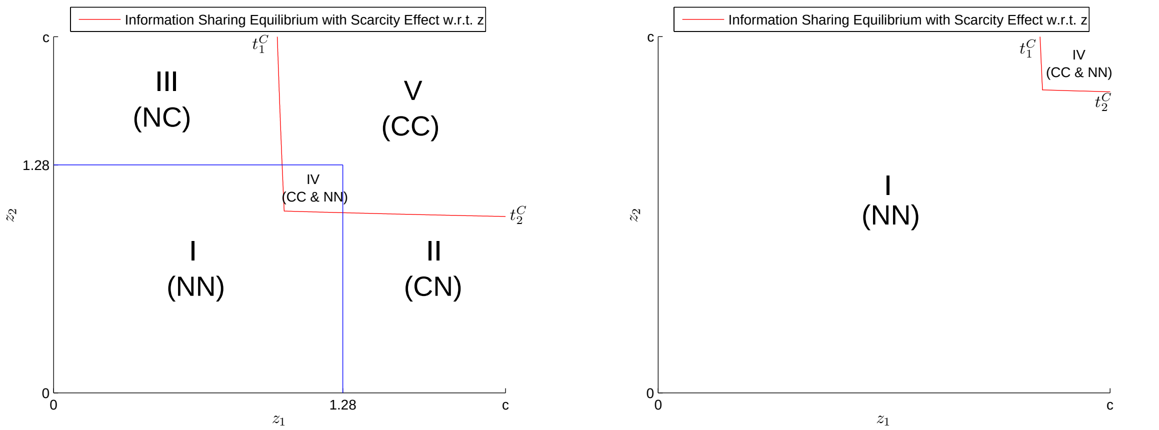
The realized market price is shown as Equation (3.3). Here $\alpha + \beta(q_i + q_j)$ is the market potential, which decreases in q_i and q_j when $\beta < 0$. Compared to the case without the consideration of inventory-level-dependent demand, the scarcity effect decreases both the market potential $\alpha + \beta(q_i + q_j)$ and the retail quantity $q_i + q_j$. The effect of scarcity effect on market price is determined by the term $(1-\beta)(q_i + q_j)$.

Given any information sharing strategy, the market price with scarcity effect is always less than that of traditional problem, $p_s < p_0$. While when $0 < \beta < 1$ (billboard effect), $p_b > p_0$. The subscript s, b and 0 stand for scarcity effect case, billboard effect case and traditional case. The reason is that, with $\beta < 0$ in our model, the demand decreases in the retail quantity, and thus the price also decreases.

6.1.3 Ex ante Profits & Information Sharing Equilibrium

Since both the market price and the retail quantity become smaller, the ex ante profits with scarcity effect is less than that without the consideration of the scarcity effect. In the ex ante profit analysis process, the equilibrium is determined by the coefficient only while not affected by the $1-\beta$ term, so the information sharing equilibrium remains the same as the case with billboard effect. But the scarcity effect makes supply chains earn less profits.

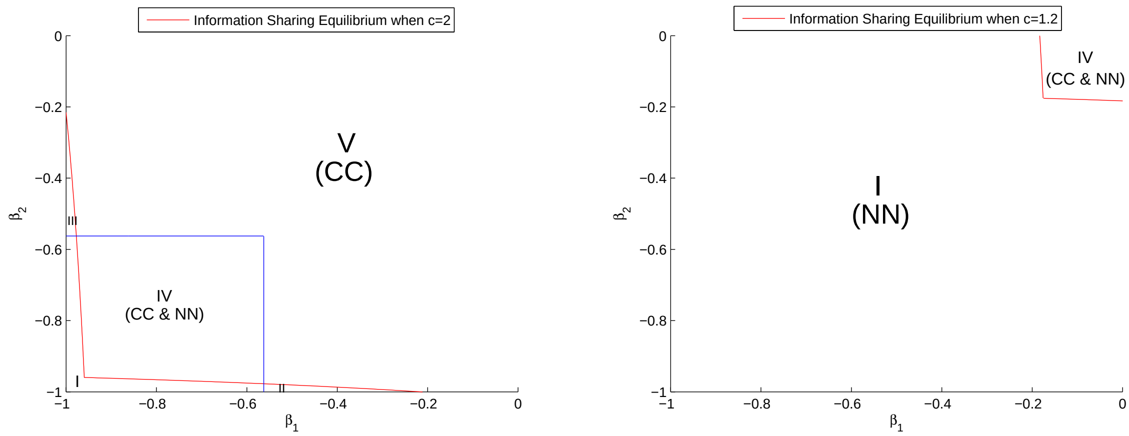
Since $\beta < 0$, $1-\beta > 1$, $z = c/(1-\beta) < c$, the domain of z is $(0,c)$. Thus the value of c affects the set of potential equilibrium strategy. When c gets smaller, some potential equilibrium strategies become impossible. The effect of c on information sharing equilibrium under scarcity effect is almost opposite to that under billboard effect. If $c > 1.28$, information sharing equilibrium *NN, CN, NC, CC & NN, and CC* are all possible, the equilibrium depends on the value of β; if $1.0206 < c \leq 1.28$, the information sharing equilibrium can only be *NN, or CC & NN*; if $0 < c \leq 1.0206$, the information sharing equilibrium is *NN* for sure, regardless of the value of β. This result can be illustrated by Figure 6.1. Suppose $c = 2$, the information sharing equilibrium is as shown in Figure 6.1(a). Suppose $c = 1.2$, there only exist two possible equilibria *NN* and *CC & NN*, and the equilibrium turns to be Figure 6.1(b).



(a) Information sharing equilibrium w.r.t. z ($c = 2$) (b) Information sharing equilibrium w.r.t. z ($c = 1.2$)

Figure 6.1: Information sharing equilibrium w.r.t. z under scarcity effect

The corresponding information sharing equilibrium w.r.t. β is shown in Figure 6.2. In Figure 6.2(a), Area *I, II* and *III* stand for equilibria *NN, CN* and *NC* respectively.



(a) Information sharing equilibrium w.r.t. β $(c = 2)$ (b) Information sharing equilibrium w.r.t. β $(c = 1.2)$

Figure 6.2: Information sharing equilibrium w.r.t. z under scarcity effect

The comparison of the effect of c on information sharing equilibrium under scarcity effect and that under billboard effect is shown below.

Table 6.1: Comparison of Information Sharing Equilibria

value of c	billboard effect $(0 < \beta < 1)$	scarcity effect $(\beta < 0)$
$0 < c < 1.0115$	NN, CN, NC, CC & NN and CC	NN
$1.0115 \leq c < 1.0206$	NN, CC & NN and CC	NN
$1.0206 \leq c < 1.28$	CC & NN and CC	CC & NN and NN
$c \geq 1.28$	CC	NN, CN, NC, CC & NN and CC

The information sharing equilibria set under billboard effect and that under scarcity effect are almost symmetric.

When the billboard effect exists$(0 < \beta < 1)$, equilibrium CC is always a potential equilibrium for any production c. It is because that the variability of retail quantity makes supply chain incur high production cost due to the production diseconomies, especially when c is large. Information sharing reduces the variability of retail quantity and thus it is always preferable. When c exceeds a certain value, supply chain will always choose to be communicative.

When the scarcity effect exists($\beta < 0$), the retail quantity becomes smaller since less inventory inspires more demand. In this case, the production cost is lower and may be compensated by the revenue. So even when production cost is high, *NN* is still a possible equilibrium.

6.1.4 Results Comparison

Sapra et al. (2010) study a similar problem with a different model. As they say, the key difference between the research on billboard effect and that on scarcity effect is the type of the product. The scarcity effect considers hyped products that are in the initial stage of their life cycle (e.g. fashion goods), while billboard effect is more common in the middle (and steady) phase of their life cycle. In the traditional setting where demand is deterministic and independent of the net ending inventory, the optimal policy is to stock an amount equal to the demand. In Sapra et al. (2010), the optimal order-up-to level in their model is strictly less than the demand. So same as our results, the scarcity effect makes the equilibrium retail quantity less than that without the consideration of scarcity effect.

They also show that the inventory-withholding strategy achieves profit improvement over the policy without inventory-withholding. But since some factors are not captured in our model, there is no profit increase in our results.

Another two similar results are: (1) As the variance grows, so does the profit differential. (2) The profit improvement of the withholding strategy declines as the price sensitivity of demand ($1-\beta$ in our model) increases.

6.2 Competition with Imperfect Information & Investment in Information Sharing

Now we generalize the problem with information of certain precision. In this section we focus on the impact of information accuracy and also the information sharing investment on the information sharing equilibrium, so we assume the two competing supply chains have the common billboard effect coefficient β for the analysis convenience. Retailer i has access to a demand signal Y_i, which is an unbiased estimator of demand uncertain θ, and chooses whether to share this information with Manufacturer i. Define

$$s_i = \frac{E\left[Var[Y_i|\theta]\right]}{\sigma^2}.$$

The reciprocal $1/s_i$ is an indicator of information accuracy. Manufacturers need to make investment for information sharing, denoted by K_i. In practice, the investment may refer to the cost of building physical infrastructure required for information sharing, such as information sharing platform. Investment K_i increases in the information accuracy, so $K_i = K(s_i)$ is a decreasing function of s_i. The production still displays production diseconomies.

It can be shown that $E[\theta|Y_i] = E[Y_j|Y_i] = Y_i/(1+s_i)$. Define $\delta_i = E\left[(E[\theta|Y_i])^2\right]$, then[1] $\delta_i = E\left[(E[\theta|Y_i])^2\right] = \sigma^2/(1+s_i)$. Given wholesale price w_i, retailer i maximizes his profit function

$$\Pi_{R_i} = \left[\alpha + \beta\left(q_i + E[q_j|Y_i]\right) + E[\theta|Y_i] - q_i - E[q_j|Y_i] - w_i\right] q_i \tag{6.1}$$

[1] $E\left[(E[\theta|Y_i])^2\right] = Var(E[\theta|Y_i]) + (E[E[\theta|Y_i]])^2 = Var(Y_i/(1+s_i)) = (E[Var[Y_i|\theta]] + Var[E[Y_i|\theta]])/(1+s_i)^2 = \left(s_i\sigma^2 + Var[\theta]\right)/(1+s_i)^2 = \sigma^2/(1+s_i)$

with the retail quantity

$$q_i(q_j, w_i) = \frac{\alpha + E[\theta|Y_i] - (1-\beta)E[q_j|Y_i] - w_i}{2(1-\beta)}. \tag{6.2}$$

If supply chain i is *communicative*, the retailer and the manufacturer have an agreement on information sharing and the manufacturer makes investment on it. Manufacturer i maximizes her profit

$$\Pi_{M_i} = w_i q_i(q_j, w_i) - K(s_i) - c q_i^2/2. \tag{6.3}$$

If supply chain i is *non-communicative*, the manufacturer needn't make investment but has no demand information. Manufacturer i maximizes her expected profit

$$\Pi_{M_i} = w_i E\left[q_i\right] - cE\left[q_i^2\right]/2. \tag{6.4}$$

Plugging the derived wholesale price into (9), we obtain the retail quantity of supply chain i in response to that of supply chain j. For any information sharing arrangement, the equilibrium is found by solving $q_1 = q_1(q_2)$ and $q_2 = q_2(q_1)$ simultaneously. Define the candidate linear strategies as

$$q_i^{(X_i,X_j)} = A_i^{(X_i,X_j)} + B_i^{(X_i,X_j)} Y_i, \tag{6.5}$$

where (X_i, X_j) stands for the information sharing strategy of two competing supply chains. In the following three sections, we analyze the equilibrium retail quantity, equilibrium wholesale price and ex ante profits for each case.

Lemma 6. *The equilibrium retail quantity under any information sharing strategy is* *linear in the demand information,*

$$q_i^{(X_i,X_j)} = A + B_i^{(X_i,X_j)} Y_i,$$

where $A = E[q_i]$ *and* $B_i^{(X_i,X_j)}$ *is given by*

$$\begin{cases} B_i^{CC} = \frac{(1+s_j)[4(1-\beta)+c]-(1-\beta)}{(1+s_i)(1+s_j)[4(1-\beta)+c]^2-(1-\beta)^2} \\ B_i^{NN} = \frac{2(1+s_j)-1}{\left[4(1+s_i)(1+s_j)-1\right](1-\beta)} \\ B_i^{CN} = \frac{2(1+s_j)-1}{2[4(1-\beta)+c](1+s_i)(1+s_j)-(1-\beta)} \\ B_i^{NC} = \frac{[4(1-\beta)+c](1+s_j)-(1-\beta)}{2[4(1-\beta)+c](1+s_i)(1+s_j)(1-\beta)-(1-\beta)^2} \end{cases}$$

6.2.1 Profits and Information Sharing Strategy of Retailers

Without the consideration of information payment from manufacturers, retailers' information sharing choice is determined by the term $B_i^{(X_i,X_j)}$. It can be shown that $B_1^{C,X_j} < B_1^{N,X_j}$ and $B_2^{X_i,C} < B_2^{X_i,N}$.

Lemma 7. *Both of the retailers prefer not to share the demand information with their manufacturers. Retailers would share the information only when the payment offered by manufactures can cover their profit loss resulted from information sharing.*

6.2.2 Profits and Information Sharing Strategy of Manufacturers

Similar to the previous chapters, the communicative manufacturer has more bargaining power and makes more profit than the non-communicative one. But with the consideration of investment, the information sharing strategy of manufacturers is determined by the tradeoff between incremental profit and investment in information sharing. Manufacturers' ex ante profits are given by

$$\begin{cases} \Pi_{M_i}^{C,X_j} = \left[2(1-\beta)+\frac{c}{2}\right]A^2 + \left[2(1-\beta)+\frac{c}{2}\right]\left(B_i^{C,X_j}\right)^2(1+s_i)\sigma^2 - K(s_i) \\ \Pi_{M_i}^{N,X_j} = \left[2(1-\beta)+\frac{c}{2}\right]A^2 - \frac{c}{2}\left(B_i^{N,X_j}\right)^2(1+s_i)\sigma^2 \end{cases}$$

Due to the complexity of the profit functions, it is difficult to solve the critical values of β analytically like what we did in previous chapters. So we try numerical examples to investigate how the information sharing strategy for manufacturers is affected by the billboard effect and the information accuracy. In the following numerical study, we assume the form of investment function is

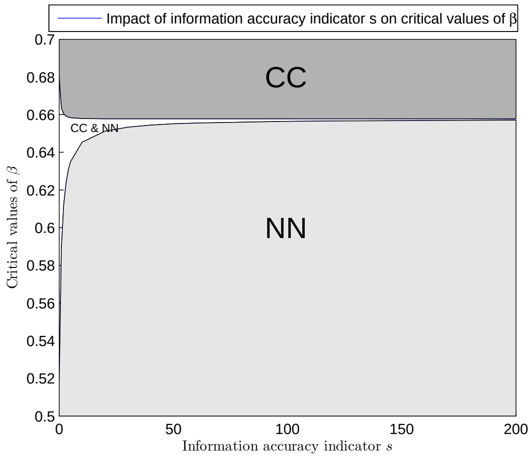
$$K(s) = \frac{1+s}{2\left[2(1+s)+1\right]^2}, \tag{6.6}$$

so $K(s)$ convex decreases in s. We also assume that the information accuracy is the same for the two supply chains, $s_1 = s_2 = s$.

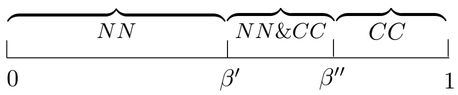
Lemma 8. *It can be analytically proved that neither* CN *nor* NC *is the information sharing equilibrium from the perspective of manufacturers.*

***Remark* 14.** From the proof of the above lemma, we observe that when manufacture i chooses N, manufacturer j earns more profit with the same strategy N. This means that when manufacture j chooses N, she hopes manufacture i also chooses N. And vice versa. So either manufacturer hopes the parallel manufacturer choose the same strategy with her.

Now we investigate the information sharing equilibrium for manufacturers and how it is affected by the information precision. Here $1/s$ is the indicator of information accuracy, β' and β'' are the two critical values of β. Assuming $\sigma^2 = 0.156$, the numerical results are shown below. In Figure 6.3(a), the lower curve stands for the critical value β' and the upper curve stands for the critical value β''. Both of them are affected by



(a) Impact of information accuracy indicator s on critical values of β and manufacturer's NE



(b) Information sharing equilibrium given certain information accuracy

Figure 6.3: Information sharing equilibrium for manufacturers and impact of information accuracy

the information accuracy indicator s. Given a certain value of s, we obtain the specific values of β' and β'', and the equilibrium is determined by the relationship among the actual billboard effect coefficient β and the two critical values β' and β'' in the way shown in Figure 6.3(b). From the above numerical results, we find that the structure of the information sharing equilibrium of case with imperfect information and investment is the same with that of section 5 (the special case with common β).

Theorem 3. *Manufacturer's information sharing strategy is determined by the two critical values of β. From the perspective of manufacturers, the information sharing equilibrium is*

$$\begin{cases} \text{NN} & \textit{if}\; 0 < \beta < \beta' \\ \text{NN \& CC} & \textit{if}\; \beta' \le \beta < \beta'' \\ \text{CC} & \textit{if}\; \beta'' \le \beta < 1 \end{cases}$$

Here β' and β'' are the two critical values of β and are affected by the information accuracy and variance of demand.

Since we have proven that *CN* and *NC* are not equilibrium, this equilibrium structure holds for any value of s. It means that regardless of the information accuracy, the information sharing equilibrium for manufacturers is determined by the relationship among specific β, critical β' and β''.

Lemma 9. *Information accuracy affects the critical values of β and thus affects manufacturers' information sharing strategies.*

As s increases, which means the information is less accurate, the area of equilibrium *NN* increases (decreasingly). The reason is that when the information is not that accurate, the value of demand information and then the value of information sharing decrease. And thus manufacturer's preference for strategy *N* increases.

As s increases, the area of equilibrium *CC* increases slightly. But on the whole, the area where *CC* is equilibrium (which is the combination of area *CC* and area *NN & CC*) decreases. The reason is similar to the previous result. The area of equilibrium *CC* gets smaller as manufacturers prefer strategy N more than before.

As s increases, the area of equilibrium *NN & CC* decreases and finally disappears. As the area of *NN* increases, the two critical values of β converges to a point, $\hat{\beta}$, and thus the area of *NN & CC* will finally disappear. In this example, $\hat{\beta} = 0.6578$.

After equilibrium *NN & CC* disappears, as s increases, the profit difference between *CC* and *NN* decreases and tends to be zero when s is large enough. The reason is that when the information is quite inaccurate, there is no difference for supply chains to choose information sharing or not.

Varying the value of σ^2, the above results still hold. Demand variance σ^2 also has impact on critical values of β and thus the information sharing equilibrium.

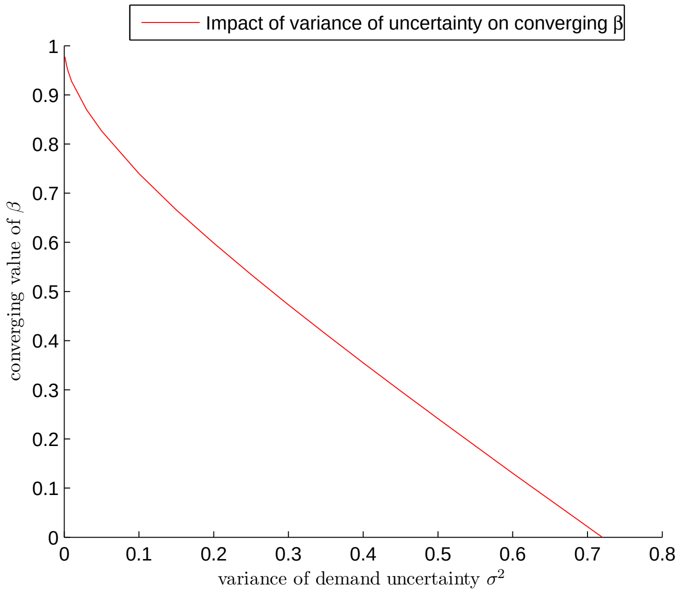
(1) The profit difference between *CC* and *NN* decreases in σ^2, the variance of demand uncertainty.

(2) Both β' and β'' decrease in σ^2, the variance of demand uncertainty. So with larger σ^2, the area of *NN* becomes smaller while the area of *CC* becomes larger. This

means that the manufacturer's preference for information sharing increases in σ^2, the variance of demand uncertainty.

(3) In this $\sigma^2 = 0.2$ case, the converging point $\hat{\beta}$ becomes smaller. This is consistent with result (2). The effect of σ^2 on the converging point $\hat{\beta}$ is shown in Figure 6.4.

Figure 6.4: Impact of variance of demand uncertainty on converging β



As σ^2 increases, the demand becomes more uncertain and thus the retail quantity may become more variable. Due to the production diseconomy, this will increases the production cost greatly. In this case, the information sharing can reduce the variability of retail quantity and thus reduce the production cost effectively.

6.2.3 Profits and Information Sharing Equilibrium for Supply Chains

Similar to the previous sections, retailers and manufacturers may have contradictory equilibrium. The manufacturer may prefer to make a payment to the retailer for information sharing. In that case, the condition for this payment agreement between retailer and manufacturer is that the supply chain earns more profit under information sharing strategy. So we need to consider this problem from the perspective of supply chain.

The ex ante profit of supply chain equals to the sum of the profits of retailer and manufacturer. So the ex ante profit of communicative supply chain is

$$\Pi_{S_i}^{C,X_j} = \left[3(1-\beta)+\frac{c}{2}\right]A^2 + \left[3(1-\beta)+\frac{c}{2}\right]\left(B_i^{C,X_j}\right)^2(1+s_i)\sigma^2 - K(s_i), \qquad (6.7)$$

and the ex ante profit of non-communicative supply chain is

$$\Pi_{S_i}^{N,X_j} = \left[3(1-\beta)+\frac{c}{2}\right]A^2 + \left(1-\beta-\frac{c}{2}\right)\left(B_i^{N,X_j}\right)^2(1+s_i)\sigma^2. \qquad (6.8)$$

The analysis procedure and equilibrium structure for supply chains are quite similar to those for manufacturer case.

Theorem 4. *The information sharing equilibrium for supply chains is determined by the two critical values of β, denoted by β_S' and β_S''. The information sharing equilibrium is*

$$\begin{cases} \text{NN} & \text{if } 0 < \beta < \beta_S' \\ \text{NN \& CC} & \text{if } \beta_S' \le \beta < \beta_S'' \\ \text{CC} & \text{if } \beta_S'' \le \beta < 1 \end{cases}$$

β_S' and β_S'' are affected by the information accuracy and variance of demand.

The impact of information accuracy and variance of demand on the converging value of β, and thus the information sharing equilibrium, is shown in Figure 6.5 and Figure 6.6 (derived from numerical examples).

Figure 6.5: Impact of information accuracy indicator s on critical values of β and supply chain's NE

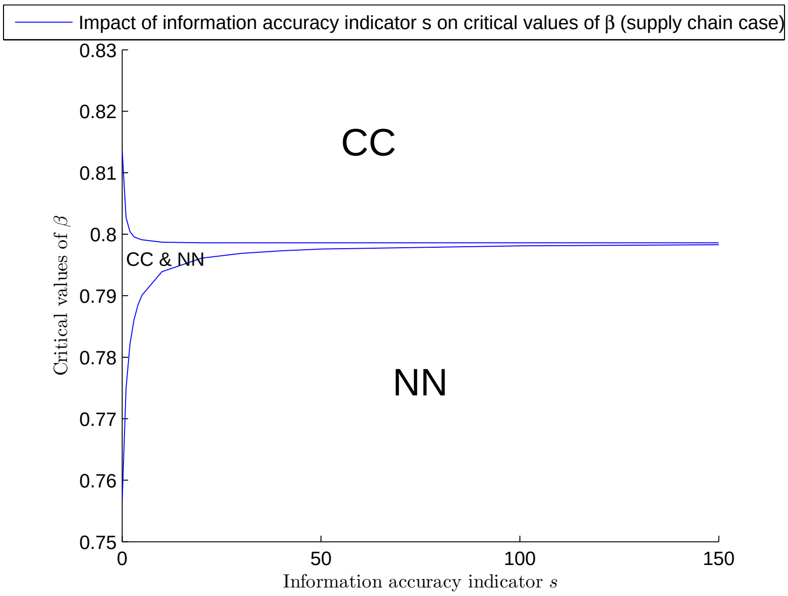
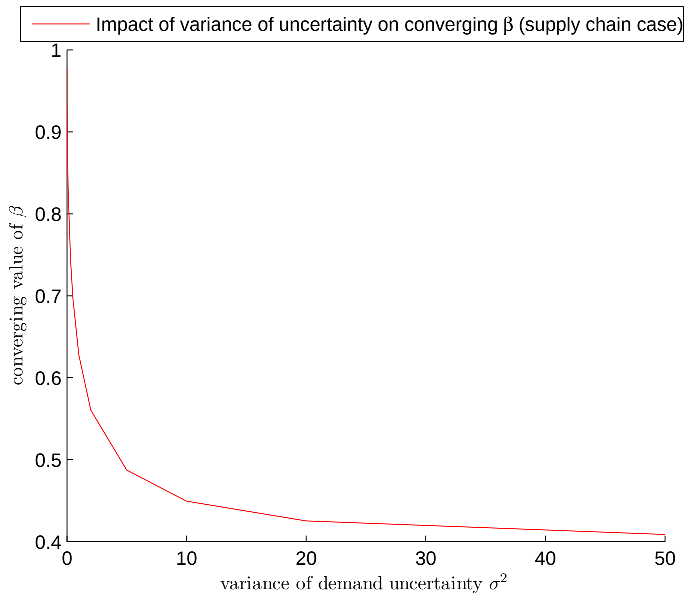


Figure 6.6: Impact of variance of demand uncertainty on converging β



The critical values of β for supply chains are larger than those for manufacturers.

This indicates that information sharing is less preferable from the perspective of supply chain compared to the perspective of manufacturer. This is due to the inclusion of retailers' profits. As we stated in Section 4, retailer's profit increases in the variability of retail quantity and thus retailers prefer non-communicative strategy. Therefore, combining the profits of retailer and manufacturer, the area of equilibrium *NN* for supply chains is larger than that for manufacturers.

Resulted from the same reason, $\hat{\beta}_S$, the converging value of β for supply chains, also differentiates from that for manufacturers. From the perspective of manufacturers, the converging value $\hat{\beta}$ decreases to zero after the variance of demand uncertainty σ^2 exceeds certain point. Added up with the profits of retailers, converging value $\hat{\beta}_S$ decreases in demand variance σ^2 but tends not to approach zero. So *NN* is always possible to be the information sharing equilibrium for supply chains, regardless of the production diseconomies or variance of demand uncertainty.

Chapter 7

Conclusion

The billboard effect in operations management indicates that the increasing shelf-space allocated to a product has a positive effect on the demand for it. This billboard effect can be explained from both the perspective of marketing and that of operations management. This promotional effect of inventory on demand has been verified by empirical evidence of previous research.

This paper studies the billboard effect on the vertical information sharing strategy in competing supply chains in an environment with production diseconomies. We consider a model of two competing supply chains, each consisting of one manufacturer and one retailer. The retailers engage in a Cournot (quantity) competition. We analyze how the equilibrium information sharing strategy, wholesale price and retail quantity are affected by the billboard effect coefficient.

We solve the problem in a backward procedure: (1) Given any information sharing strategy in Stage 1, we solve the equilibrium wholesale price and equilibrium retail quantity for each supply chain; (2) Based on the equilibrium wholesale price and retail quantity, each supply chain calculates the ex ante profits under different information sharing arrangements; (3) The information sharing equilibrium in the first stage is the information sharing strategy that optimizes the profits for both supply chains.

We first assume that both supply chains have access to the perfect demand information and focus on the billboard effect on information sharing strategy. Our results show that in the presence of production diseconomies, information sharing benefits the supply chain. The billboard effect makes the retail quantity larger and more variable, and thus incurs higher production cost due to the production diseconomies. While information sharing reduces the variability of retail quantity and thus lower the production cost. So the billboard effect coefficient increases the value of information sharing and thus pronounces supply chain's preference for information sharing. Comparing with the results of Ha et al. (2011), our study shows that the billboard effect makes information sharing preferable even when the production diseconomies is at a low level.

Then we extend the original problem in two directions. First we consider the case with scarcity effect. Contrary to the billboard effect, when the scarcity effect exists, the retail quantity becomes smaller since less inventory inspires more demand. In this case, the production cost is lower and the value of information sharing is less. So in the presence of scarcity effect, no information sharing *NN* is always a potential information sharing equilibrium.

Secondly we release the assumption of perfect demand information in the original problem to investigate the billboard effect on information sharing with imperfect demand information. At the meantime, we also take into account the investment in information sharing, which increases in the information accuracy. It is verified that information accuracy affects the critical values of billboard effect coefficient and therefore affects supply chains' information sharing equilibrium. The equilibrium is determined by the tradeoff between the benefit and the investment of information sharing. The critical values of billboard effect are also affected by the variance of demand. All the results are shown in numerical examples.

While we have explored the billboard effect on information sharing with imperfect information and investment in information, the more general case where competing supply chains have individual information accuracy is to be studied further. Also, we assume certain form of investment function and use numerical examples to show the impact of information accuracy on critical values of billboard effect coefficient. The more general and analytical relationship is worthy of further investigation.

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Appendix A

Technical Details

Proof of Lemma 1.

Given any wholesale price w_i set by the manufacture i, retailer i maximizes his profit

$$\begin{aligned}\max_{q_i} \Pi_{R_i} &= (p - w_i)q_i \\ &= \left[\alpha + \theta - w_i - (1-\beta)q_i - (1-\beta)q_j\right] q_i \\ &= \left[\alpha + \theta - w_i - (1-\beta)q_j\right] q_i - (1-\beta)q_i^2.\end{aligned}$$

The first-order derivative with respect to retail quantity q_i is

$$\frac{\partial \Pi_{R_i}}{\partial q_i} = \alpha + \theta - w_i - (1-\beta)q_j - 2(1-\beta)q_i.$$

The second-order derivative is

$$\frac{\partial^2 \Pi_{R_i}}{\partial q_i^2} = -2(1-\beta) < 0.$$

Since retailer i's profit function is concave in the retail quantity q_i, retailer i maximizes his profit by setting the retail quantity to

$$q_i(w_i, q_j) = \frac{\alpha + \theta - w_i - (1-\beta)q_j}{2(1-\beta)}. \tag{A.1}$$

Since there is no scale of diseconomy, the marginal production cost is a constant. Without loss of generality, we standardize the production cost to zero. If retailer *i* shares information with manufacturer *i* and thus the supply chain *i* is *communicative*, manufacturer *i* maximizes her profit

$$\begin{aligned}\Pi_{M_i} &= w_i q_i \\ &= \frac{\alpha w_i + \theta w_i - (1-\beta)q_j w_i - w_i^2}{2(1-\beta)}.\end{aligned}$$

The first-order derivative with respect to wholesale price w_i is

$$\frac{\partial \Pi_{M_i}}{\partial w_i} = \frac{\alpha + \theta - (1-\beta)q_j - 2w_i}{2(1-\beta)}.$$

The second-order derivative is

$$\frac{\partial^2 \Pi_{M_i}}{\partial w_i^2} = \frac{-1}{1-\beta} < 0.$$

Since manufacturer *i*'s profit function is concave in its wholesale price w_i, manufacturer maximizes her profit by setting the wholesale price to

$$w_i(q_j) = \frac{\alpha + \theta - (1-\beta)q_j}{2}.$$

Plugging the wholesale price into (1), the retail quantity of supply chain i in response to that of supply chain j is

$$
\begin{aligned}
q_i(q_j) &= \frac{\alpha+\theta-\frac{\alpha+\theta-(1-\beta)q_j}{2}-(1-\beta)q_j}{2(1-\beta)} \\
&= \frac{\alpha+\theta-(1-\beta)q_j}{4(1-\beta)}.
\end{aligned} \tag{A.2}
$$

If the supply chain i is *non-communicative*, manufacturer i anticipates the retailer's retail quantity and maximizes her expected profit

$$
\begin{aligned}
\Pi_{M_i} &= w_i E[q_i] \\
&= w_i \frac{\alpha-w_i-(1-\beta)E[q_j]}{2(1-\beta)} \\
&= \frac{\alpha w_i-(1-\beta)E[q_j]w_i-w_i^2}{2(1-\beta)}.
\end{aligned}
$$

The first-order derivative with respect to wholesale price is

$$
\frac{\partial \Pi_{M_i}}{\partial w_i} = \frac{\alpha-(1-\beta)E[q_j]-2w_i}{2(1-\beta)}.
$$

The second-order derivative is

$$
\frac{\partial^2 \Pi_{M_i}}{\partial w_i^2} = \frac{-1}{1-\beta} < 0.
$$

Since the manufacturer's profit function is concave in wholesale price, manufacturer i maximizes its expected profit by setting the wholesale price to

$$
w_i = \frac{\alpha-(1-\beta)E[q_j]}{2}.
$$

Plugging the wholesale price into retail quantity equation (1), we get the retail quantity of supply chain i in response to that of supply chain j,

$$\begin{aligned} q_i(q_j) &= \frac{\alpha+\theta-w_i-(1-\beta)q_j}{2(1-\beta)} \\ &= \frac{\alpha+\theta-\frac{\alpha-(1-\beta)E[q_j]}{2}-(1-\beta)q_j}{2(1-\beta)} \\ &= \frac{2\alpha+2\theta-\alpha+(1-\beta)E[q_j]-2(1-\beta)q_j}{4(1-\beta)} \\ &= \frac{\alpha+2\theta+(1-\beta)E[q_j]-2(1-\beta)q_j}{4(1-\beta)}. \end{aligned} \tag{A.3}$$

■

Proof of Lemma 3 .

We take the case *competition between two communicative supply chains* as example. If both of the retailers choose to share the demand information with their manufacturers and thus both supply chains are communicative, we denote the information sharing arrangement as CC. Both of the retailers will adopt the retail quantity shown as (2). The equilibrium retail quantities are found by solving the system of equations

$$\begin{cases} 4(1-\beta)q_1 = \alpha+\theta-(1-\beta)q_2 \\ 4(1-\beta)q_2 = \alpha+\theta-(1-\beta)q_1. \end{cases}$$

This is a symmetric competition and the equilibrium retail quantity is

$$q_1 = q_2 = q^{CC} = \frac{\alpha}{5(1-\beta)} + \frac{\theta}{5(1-\beta)}. \tag{A.4}$$

Plugging the equilibrium retail quantity to the manufacturer's wholesale price, we obtain the equilibrium wholesale price

$$w_1 = w_2 = w^{CC} = \frac{2}{5}\alpha + \frac{2}{5}\theta. \tag{A.5}$$

With the above equilibrium retail quantity and equilibrium wholesale price, the ex ante profits for each party and the whole supply chains are shown below:

$$E[\Pi_{R_1}] = E[\Pi_{R_2}] = E[\Pi_R] = E[(1-\beta)q^2] = \frac{\alpha^2}{25(1-\beta)} + \frac{\sigma^2}{25(1-\beta)} \tag{A.6}$$

$$E[\Pi_{M_1}] = E[\Pi_{M_2}] = E[\Pi_M] = E[wq] = \frac{2\alpha^2}{25(1-\beta)} + \frac{2\sigma^2}{25(1-\beta)} \tag{A.7}$$

$$E[\Pi_{S_1}] = E[\Pi_{S_2}] = E[\Pi_S] = \frac{3\alpha^2}{25(1-\beta)} + \frac{3\sigma^2}{25(1-\beta)}. \tag{A.8}$$

The other three cases are analyzed in the same way. ■

Proof of Lemma 4 .

The proof of Lemma 4 follows a procedure similar to that of Lemma 3.

Proof of Proposition 1 .

To obtain the equilibrium information sharing strategy, we need to analyze four scenarios: given supply chain i (j) is communicative (non-communicative), supply chain j's (i's) decision. Define

$$z_1 = \frac{c}{1-\beta_1}, \qquad z_2 = \frac{c}{1-\beta_2}.$$

Since $0 \le \beta_i < 1$, $z_i > 0$. Obviously z_i increases in c and β_i.

Scenario 1

Given supply chain 1 is communicative, supply chain 2 makes his decision by comparing $\left[3(1-\beta_2)+\frac{c}{2}\right]\left(B_2^{CC}\right)^2$ and $\left[1-\beta_2-\frac{c}{2}\right](B_2^{CN})^2$. Define

$$\Delta_1 = \left[3(1-\beta_2)+\frac{c}{2}\right]\left(B_2^{CC}\right)^2 - \left[1-\beta_2-\frac{c}{2}\right](B_2^{CN})^2.$$

Solving $\Delta_1 = 0$ w.r.t. z_2, we get the effective solution (the only positive solution), which is the critical value that determines supply chain 2's choice, is

$$t_2^C = f_1(z_1) = \frac{\sqrt{8z_1^2 + 64z_1 + 127} - 2z_1 - 7}{z_1 + 4}, \tag{A.9}$$

where the superscript of t_2^C stands for the parallel supply chain 1 is communicative. f_1 is a function of z_1 in the form shown above. It is easy to verify that t_2^C decreases in z_1. So given supply chain 1 is communicative, supply chain 2's optimal decision is

$$\begin{cases} N & \text{if } z_2 < t_2^C \quad (\Delta_1 < 0) \\ C & \text{if } z_2 > t_2^C \quad (\Delta_1 > 0) \end{cases}$$

Scenario 2

Given supply chain 1 is non-communicative, supply chain 2 compares $\left[3(1-\beta_2)+\frac{c}{2}\right]\left(B_2^{NC}\right)^2 = \frac{\left[3(1-\beta_2)+\frac{c}{2}\right]}{\left[7(1-\beta_2)+2c\right]^2}$ and $\frac{\left[1-\beta_2-\frac{c}{2}\right]}{9(1-\beta_2)^2}$ to make decision. Define

$$\Delta_2 = \left[3(1-\beta_2)+\frac{c}{2}\right]\left(B_2^{NC}\right)^2 - \frac{\left[1-\beta_2-\frac{c}{2}\right]}{9(1-\beta_2)^2}.$$

Solve $\Delta_2 = 0$ with respect to z_2. The results are $-2 < 0$, $-\frac{\sqrt{31}}{2} - \frac{3}{2} < 0$ and $\frac{\sqrt{31}}{2} - \frac{3}{2} \approx 1.28$. So the critical value for supply chain 2's decision is

$$t_2^N = 1.28.$$

So given supply chain 1 is non-communicative, supply chain 2's optimal decision is

$$\begin{cases} N & \text{if } z_2 < 1.28 \quad (\Delta_2 < 0) \\ C & \text{if } z_2 > 1.28 \quad (\Delta_2 > 0) \end{cases}$$

Scenario 3

Given supply chain 2 is communicative, supply chain 1 makes his choice by comparing $\left[3(1-\beta_1)+\frac{c}{2}\right]\left(B_1^{CC}\right)^2$ and $\left[1-\beta_1-\frac{c}{2}\right](B_1^{NC})^2$. The analysis process is similar to that in scenario 1. So given supply chain 2 is communicative, supply chain 1's optimal decision is

$$\begin{cases} N & \text{if } z_1 < t_1^C \quad (\Delta_3 < 0) \\ C & \text{if } z_1 > t_1^C \quad (\Delta_3 > 0) \end{cases}$$

where

$$t_1^C = f_2(z_2) = \frac{\sqrt{8z_2^2+64z_2+127}-2z_2-7}{z_2+4}.$$

Scenario 4

Given supply chain 2 is non-communicative, supply chain 1 makes his choice by comparing $\left[3(1-\beta_1)+\frac{c}{2}\right]\left(B_1^{CN}\right)^2$ and $\frac{1-\beta_1-\frac{c}{2}}{9(1-\beta_1)^2}$. The analysis process is similar to that of scenario 2. So given supply chain 2 is non-communicative,supply chain 1's optimal decision is

$$\begin{cases} N & \text{if } z_1 < 1.28 \quad (\Delta_4 < 0) \\ C & \text{if } z_1 > 1.28 \quad (\Delta_4 > 0) \end{cases}$$

It can also be verified that

$$t_1^C < t_1^N = 1.28.$$

Since $t_2^C < t_2^N = 1.28$ and $t_1^C < t_1^N = 1.28$, it holds that

$$\Delta_1 < 0 \Rightarrow \Delta_2 < 0,$$

$$\Delta_3 < 0 \Rightarrow \Delta_4 < 0.$$

■

Proof of condition for information sharing with payment in Section 6.2.3.

Based on the above equilibrium, voluntary information sharing between retailer and manufacturer is impossible. So it is natural to investigate the condition for information sharing with payment. For retailers to be willing to share information upon a certain amount of payment, denoted by G, it should be satisfied that

$$\Pi_{R_i}^{C,X_j} + G \geq \Pi_{R_i}^{N,X_j}.$$

For manufacturers to be willing to pay for the information, it should be satisfied that

$$\Pi_{M_i}^{C,X_j} - G \geq \Pi_{M_i}^{N,X_j}.$$

For both of the conditions hold, it is required that

$$\Pi_{R_i}^{C,X_j} + \Pi_{M_i}^{C,X_j} \geq \Pi_{R_i}^{N,X_j} + \Pi_{M_i}^{N,X_j},$$

or

$$\Pi_{S_i}^{C,X_j} \geq \Pi_{S_i}^{N,X_j}.$$

It can also be understood in this way: for manufacturer, the optimal payment for information is $G = \Pi_{R_i}^{N,X_j} - \Pi_{R_i}^{C,X_j} \geq 0$, so the condition for manufacturer to be willing to pay this amount is$\Pi_{M_i}^{C,X_j} - \Pi_{R_i}^{N,X_j} + \Pi_{R_i}^{C,X_j} \geq \Pi_{M_i}^{N,X_j}$, which is equivalent to $\Pi_{S_i}^{C,X_j} \geq \Pi_{S_i}^{N,X_j}$.
■

Proof of Lemma 6 .

Given wholesale price w_i, retailer i maximizes his profit function

$$\Pi_{R_i} = \left[\alpha + \beta\left(q_i + E[q_j|Y_i]\right) + E[\theta|Y_i] - q_i - E[q_j|Y_i] - w_i\right] q_i. \tag{A.10}$$

The first-order derivative w.r.t. retail quantity q_i is

$$\frac{\partial \Pi_{R_i}}{\partial q_i} = \alpha + 2\beta q_i + \beta E[q_j|Y_i] + E[\theta|Y_i] - 2q_i - E[q_j|Y_i] - w_i.$$

The second-order derivative is

$$\frac{\partial^2 \Pi_{R_i}}{\partial q_i^2} = -2(1-\beta) < 0.$$

Since Retailer i's profit function is concave in q_i, Retailer i maximizes his profit with the retail quantity

$$q_i(q_j, w_i) = \frac{\alpha + E[\theta|Y_i] - (1-\beta)E[q_j|Y_i] - w_i}{2(1-\beta)}. \tag{A.11}$$

If supply chain i is *communicative*, the retailer and the manufacturer have an agreement on information sharing and the manufacturer makes investment on it. Manufacturer i maximizes her profit,

$$\begin{aligned}
\Pi_{M_i} &= w_i q_i(q_j, w_i) - K(s_i) - \frac{c}{2} q_i^2 && \text{(A.12)}\\
&= \frac{\alpha w_i + E[\theta|Y_i] w_i - (1-\beta)E[q_j|Y_i] w_i - w_i^2}{2(1-\beta)} && \text{(A.13)}\\
&\quad - K(s_i) - \frac{c}{2} \frac{\left(\alpha + E[\theta|Y_i] - (1-\beta)E[q_j|Y_i] - w_i\right)^2}{4(1-\beta)^2}.
\end{aligned}$$

The first-order derivative w.r.t. wholesale price w_i is

$$\frac{\partial \Pi_{M_i}}{\partial w_i} = \frac{\alpha + E[\theta|Y_i] - (1-\beta)E[q_j|Y_i] - 2w_i}{2(1-\beta)} - c \frac{-\alpha - E[\theta|Y_i] + (1-\beta)E[q_j|Y_i] + w_i}{4(1-\beta)^2}.$$

The second-order derivative is

$$\frac{\partial^2 \Pi_{M_i}}{\partial w_i^2} = \frac{-1}{1-\beta} - \frac{c}{4(1-\beta)^2} < 0.$$

So Manufacturer i's profit function is concave in wholesale price w_i, and therefore Manufacturer i maximizes her profit by setting the wholesale price to

$$w_i(q_j) = \frac{[2(1-\beta)+c]\left(\alpha+E[\theta|Y_i]-(1-\beta)E[q_j|Y_i]\right)}{4(1-\beta)+c} = [2(1-\beta)+c]\,q_i. \quad \text{(A.14)}$$

Plugging (60) into (58), we get the retail quantity of supply chain i in response to that of supply chain j,

$$\begin{aligned} q_i(q_j) &= \frac{\alpha+E[\theta|Y_i]-(1-\beta)E[q_j|Y_i]-\frac{[2(1-\beta)+c]\left(\alpha+E[\theta|Y_i]-(1-\beta)E[q_j|Y_i]\right)}{4(1-\beta)+c}}{2(1-\beta)} \\ &= \frac{\alpha+E[\theta|Y_i]-(1-\beta)E[q_j|Y_i]}{4(1-\beta)+c}. \quad \text{(A.15)} \end{aligned}$$

If supply chain i is *non-communicative*, the manufacturer needn't make investment but has no demand information. Manufacturer i maximizes her expected profit,

$$\begin{aligned} \Pi_{M_i} &= w_i E\left[q_i\right]-\frac{c}{2}E\left[q_i^2\right] \\ &= \frac{\alpha w_i-(1-\beta)E[q_j]w_i-w_i^2}{2(1-\beta)} \\ &\quad -\frac{c}{2}E\left[\frac{\left(\alpha+E[\theta|Y_i]-(1-\beta)E[q_j|Y_i]\right)^2-2\left(\alpha+E[\theta|Y_i]-(1-\beta)E[q_j|Y_i]\right)w_i+w_i^2}{4(1-\beta)^2}\right]. \end{aligned}$$

The first-order derivative w.r.t. wholesale price w_i is

$$\begin{aligned} \frac{\partial \Pi_{M_i}}{\partial w_i} &= \frac{\alpha-(1-\beta)E[q_j]-2w_i}{2(1-\beta)}-cE\left[\frac{-\left(\alpha+E\left[\theta|Y_i\right]-(1-\beta)E[q_j|Y_i]\right)+w_i}{4(1-\beta)^2}\right] \\ &= \frac{\alpha-(1-\beta)E[q_j]-2w_i}{2(1-\beta)}-\frac{-\alpha+(1-\beta)E[q_j]+w_i}{4(1-\beta)^2}c. \end{aligned}$$

The second-order derivative is

$$\frac{\partial^2 \Pi_{M_i}}{\partial w_i^2} = \frac{-1}{1-\beta}-\frac{c}{4(1-\beta)^2} < 0.$$

So manufacturer i's profit function is concave in wholesale price w_i and thus the manufacturer maximizes her expected profit with the wholesale price

$$w_i(q_j) = \frac{[2(1-\beta)+c]\left[\alpha-(1-\beta)E[q_j]\right]}{4(1-\beta)+c}. \tag{A.16}$$

Plugging (63) into (58), we get the optimal retail quantity of supply chain i in response to that of supply chain j,

$$\begin{aligned} q_i(q_j) \quad = \quad & \frac{[4(1-\beta)+c]\left(\alpha+E[\theta|Y_i]-(1-\beta)E[q_j|Y_i]\right)}{2(1-\beta)\,[4(1-\beta)+c]} \\ & -\frac{[2(1-\beta)+c]\left[\alpha-(1-\beta)E[q_j]\right]}{2(1-\beta)\,[4(1-\beta)+c]}. \end{aligned} \tag{A.17}$$

For any information sharing arrangement, the equilibrium is found by solving $q_1 = q_1(q_2)$ and $q_2 = q_2(q_1)$ simultaneously. Define the candidate linear strategies as

$$q_i^{(X_i,X_j)} = A_i^{(X_i,X_j)} + B_i^{(X_i,X_j)} Y_i, \tag{A.18}$$

where (X_i, X_j) stands for the information sharing strategy of two competing supply chains. In the following three sections, we analyze the equilibrium retail quantity, equilibrium wholesale price and ex ante profits for each case.

We take the competition between two communicative supply chains as example. The equilibrium retail quantity can be obtained by solving the following system of equations

$$\begin{cases} [4(1-\beta)+c]\,q_1 = \alpha+E[\theta|Y_1]-(1-\beta)E[q_2|Y_1] \\ [4(1-\beta)+c]\,q_2 = \alpha+E[\theta|Y_2]-(1-\beta)E[q_1|Y_2]. \end{cases}$$

For retailer 1,

$$
\begin{aligned}
& [4(1-\beta)+c]\,q_1 = \alpha + E[\theta|Y_1] - (1-\beta)E[q_2|Y_1] \\
\Rightarrow \quad & [4(1-\beta)+c]A_1^{CC} + [4(1-\beta)+c]\,B_1^{CC}Y_1 \\
& = \alpha + \frac{Y_1}{1+s_1} - (1-\beta)E\left[A_2^{CC} + B_2^{CC}Y_2|Y_1\right] \\
& = \alpha + \frac{Y_1}{1+s_1} - (1-\beta)A_2^{CC} - (1-\beta)B_2^{CC}\frac{Y_1}{1+s_1} \\
\Rightarrow \quad & [4(1-\beta)+c]A_1^{CC} = \alpha - (1-\beta)A_2^{CC}, \qquad \text{(A.19)} \\
& [4(1-\beta)+c]\,B_1^{CC} = \left[1-(1-\beta)B_2^{CC}\right]\frac{1}{1+s_1}. \qquad \text{(A.20)}
\end{aligned}
$$

For retailer 2,

$$
\begin{aligned}
& [4(1-\beta)+c]\,q_2 = \alpha + E[\theta|Y_2] - (1-\beta)E[q_1|Y_2] \\
& \qquad = \alpha + \frac{Y_2}{1+s_2} - (1-\beta)E[A_1^{CC} + B_1^{CC}Y_1|Y_2] \\
\Rightarrow \quad & [4(1-\beta)+c]A_2^{CC} + [4(1-\beta)+c]\,B_2^{CC}Y_2 \\
& = \alpha - (1-\beta)A_1^{CC} + \left[1-(1-\beta)B_1^{CC}\right]\frac{Y_2}{1+s_2} \\
\Rightarrow \quad & [4(1-\beta)+c]A_2^{CC} = \alpha - (1-\beta)A_1^{CC} \qquad \text{(A.21)} \\
& [4(1-\beta)+c]\,B_2^{CC} = \left[1-(1-\beta)B_1^{CC}\right]\frac{1}{1+s_2}. \qquad \text{(A.22)}
\end{aligned}
$$

We need to solve two systems of equations: (66) & (68) and (67) & (69). For the constant term,

$$
\begin{cases}
[4(1-\beta)+c]A_1^{CC} = \alpha - (1-\beta)A_2^{CC} \\
[4(1-\beta)+c]A_2^{CC} = \alpha - (1-\beta)A_1^{CC} = \alpha - (1-\beta)\frac{\alpha-(1-\beta)A_2^{CC}}{4(1-\beta)+c}
\end{cases}
$$

$$
\begin{aligned}
\Rightarrow \quad & [4(1-\beta)+c]^2 A_2^{CC} = [4(1-\beta)+c]\,\alpha - (1-\beta)\left[\alpha - (1-\beta)A_2^{CC}\right] \\
\Rightarrow \quad & [4(1-\beta)+c]^2 A_2^{CC} = [3(1-\beta)+c]\,\alpha + (1-\beta)^2 A_2^{CC} \\
\Rightarrow \quad & [5(1-\beta)+c]\,[3(1-\beta)+c]\,A_2^{CC} = [3(1-\beta)+c]\,\alpha \\
\Rightarrow \quad & A_2^{CC} = \frac{\alpha}{5(1-\beta)+c} \\
\Rightarrow \quad & A_1^{CC} = \frac{\alpha - (1-\beta)\frac{\alpha}{5(1-\beta)+c}}{4(1-\beta)+c} = \frac{\alpha}{5(1-\beta)+c}.
\end{aligned}
$$

Denote this constant term as A. So

$$
A_1^{CC} = A_2^{CC} = A = \frac{\alpha}{5(1-\beta)} = E[q_1] = E[q_2]. \tag{A.23}
$$

For the coefficient of demand signal Y_i,

$$
\begin{cases}
[4(1-\beta)+c]\,B_1^{CC} = \frac{1-(1-\beta)B_2^{CC}}{1+s_1} \\
[4(1-\beta)+c]\,B_2^{CC} = \frac{1-(1-\beta)B_1^{CC}}{1+s_2}
\end{cases}
$$

$$
\begin{aligned}
\Rightarrow \quad & [4(1-\beta)+c]\,(1+s_2)B_2^{CC} \\
& = 1-(1-\beta)B_1^{CC} = 1-(1-\beta)\frac{1-(1-\beta)B_2^{CC}}{(1+s_1)\,[4(1-\beta)+c]} \\
\Rightarrow \quad & [4(1-\beta)+c]^2\,(1+s_1)(1+s_2)B_2^{CC} \\
& = (1+s_1)\,[4(1-\beta)+c]-(1-\beta)\left[1-(1-\beta)B_2^{CC}\right] \\
& = (1+s_1)\,[4(1-\beta)+c]-(1-\beta)+(1-\beta)^2B_2^{CC}
\end{aligned}
$$

$$
\Rightarrow \quad B_2^{CC} = \frac{(1+s_1)\,[4(1-\beta)+c]-(1-\beta)}{(1+s_1)(1+s_2)\,[4(1-\beta)+c]^2-(1-\beta)^2} \tag{A.24}
$$

$$
\begin{aligned}
\Rightarrow \quad B_1^{CC} &= \frac{1-(1-\beta)B_2^{CC}}{[4(1-\beta)+c]\,(1+s_1)} \\
&= \frac{1}{[4(1-\beta)+c]\,(1+s_1)}\left[1-(1-\beta)\frac{(1+s_1)\,[4(1-\beta)+c]-(1-\beta)}{(1+s_1)(1+s_2)\,[4(1-\beta)+c]^2-(1-\beta)^2}\right] \\
&= \frac{(1+s_2)\,[4(1-\beta)+c]-(1-\beta)}{(1+s_1)(1+s_2)\,[4(1-\beta)+c]^2-(1-\beta)^2}.
\end{aligned} \tag{A.25}
$$

So the equilibrium retail quantity is

$$
q_i^{CC} = A + B_i^{CC} Y_i, \tag{A.26}
$$

where

$$
\begin{cases}
B_1^{CC} = \frac{(1+s_2)[4(1-\beta)+c]-(1-\beta)}{(1+s_1)(1+s_2)[4(1-\beta)+c]^2-(1-\beta)^2} \\
B_2^{CC} = \frac{(1+s_1)[4(1-\beta)+c]-(1-\beta)}{(1+s_1)(1+s_2)[4(1-\beta)+c]^2-(1-\beta)^2}.
\end{cases}
$$

The equilibrium wholesale price is

$$
w_i = [2(1-\beta)+c]\,q_i. \tag{A.27}
$$

For the other three cases, the equilibrium retail quantity and wholesale price can be obtained in the same way. ■

Proof of Lemma 7 .

For retailers in communicative supply chains, the profit function is

$$\begin{aligned}
\Pi_{R_i}^{C,X_j} &= \left[\alpha + E[\theta|Y_i] - (1-\beta)\left(q_i + E[q_j|Y_i]\right) - w_i\right] q_i \\
&= \left([4(1-\beta)+c]q_i - (1-\beta)q_i - [2(1-\beta)+c]q_i\right) q_i \\
&= (1-\beta)q_i^2 \\
&= (1-\beta)\left(A + B_i^{C,X_j} Y_i\right)^2 \\
&= (1-\beta)\left[A^2 + 2AB_i^{C,X_j} Y_i + \left(B_i^{C,X_j}\right)^2 Y_i^2\right] \\
&= (1-\beta)\left[A^2 + 2AB_i^{C,X_j}(1+s_i)E[\theta|Y_i] + \left(B_i^{C,X_j}\right)^2 (1+s_i)^2 \left(E[\theta|Y_i]\right)^2\right].
\end{aligned}$$

The ex ante profit is

$$E\left[\Pi_{R_i}^{C,X_j}\right] = (1-\beta)\left[A^2 + \left(B_i^{C,X_j}\right)^2 (1+s_i)\sigma^2\right]. \tag{A.28}$$

For retailers in non-communicative supply chains, the profit function is

$$\begin{aligned}
\Pi_{R_i}^{N,X_j} &= \left[\alpha + E[\theta|Y_i] - (1-\beta)\left(q_i + E[q_j|Y_i]\right) - w_i\right] q_i \\
&= \left[2(1-\beta)q_i - (1-\beta)q_i\right] q_i \\
&= (1-\beta)q_i^2 \\
&= (1-\beta)\left(A + B_i^{N,X_j} Y_i\right)^2 \\
&= (1-\beta)\left[A^2 + 2AB_i^{N,X_j} Y_i + \left(B_i^{N,X_j}\right)^2 Y_i^2\right] \\
&= (1-\beta)\left[A^2 + 2AB_i^{N,X_j}(1+s_i)E[\theta|Y_i] + \left(B_i^{N,X_j}\right)^2 (1+s_i)^2 \left(E[\theta|Y_i]\right)^2\right].
\end{aligned}$$

The ex ante profit is

$$E\left[\Pi_{R_i}^{N,X_j}\right] = (1-\beta)\left[A^2 + \left(B_i^{N,X_j}\right)^2 (1+s_i)\sigma^2\right]. \tag{A.29}$$

Since the form of the ex ante profits are the same and the only difference lies in $B_i^{X_i,X_j}$, we need to compare B_1^{C,X_j} & B_1^{N,X_j}, and $B_2^{X_i,C}$ & $B_2^{X_i,N}$. It can be shown that

$$\begin{cases} B_1^{C,X_j} < B_1^{N,X_j} \\ B_2^{X_i,C} < B_2^{X_i,N}. \end{cases}$$

So retailers would choose *NN* without the payment from manufactures, which means that both of the retailers choose not to share demand information with their manufacturers voluntarily. ■

Proof of Theorem 3 .

For manufacturers in the communicative supply chains, they have access to demand information but also need to make investment for information sharing. Their profit function is

$$\begin{aligned} \Pi_{M_i}^{C,X_j} &= w_i q_i - \frac{c}{2} q_i^2 - K(s_i) \\ &= [2(1-\beta)+c]q_i^2 - \frac{c}{2} q_i^2 - K(s_i) \\ &= \left[2(1-\beta)+\frac{c}{2}\right]\left(A + B_i^{C,X_j} Y_i\right)^2 - K(s_i). \end{aligned}$$

The ex ante profit is

$$E\left[\Pi_{M_i}^{C,X_j}\right] = \left[2(1-\beta)+\frac{c}{2}\right] A^2 + \left[2(1-\beta)+\frac{c}{2}\right]\left(B_i^{C,X_j}\right)^2 (1+s_i)\sigma^2 - K(s_i). \quad \text{(A.30)}$$

For manufacturers in the non-communicative supply chains, they needn't make investment but also have no access to demand information. Their profit function is

$$\begin{aligned}
\Pi_{M_i}^{N,X_j} &= w_i E[q_i] - \frac{c}{2}E[q_i^2] \\
&= [2(1-\beta)+c]A^2 - \frac{c}{2}E\left[A^2 + 2AB_i^{N,X_j}Y_i + \left(B_i^{N,X_j}\right)^2 Y_i^2\right] \\
&= [2(1-\beta)+c]A^2 - \frac{c}{2}E\left[A^2 + 2AB_i^{N,X_j}(1+s_i)E[\theta|Y_i] + \left(B_i^{N,X_j}\right)^2 (1+s_i)^2\left(E[\theta|Y_i]\right)^2\right] \\
&= [2(1-\beta)+c]A^2 - \frac{c}{2}A^2 - \frac{c}{2}\left(B_i^{N,X_j}\right)^2(1+s_i)\sigma^2 \\
&= \left[2(1-\beta)+\frac{c}{2}\right]A^2 - \frac{c}{2}\left(B_i^{N,X_j}\right)^2(1+s_i)\sigma^2.
\end{aligned}$$

Since above is already the expected profit, the ex ante profit is

$$E\left[\Pi_{M_i}^{N,X_j}\right] = \left[2(1-\beta)+\frac{c}{2}\right]A^2 - \frac{c}{2}\left(B_i^{N,X_j}\right)^2(1+s_i)\sigma^2. \tag{A.31}$$

To obtain the equilibrium information sharing strategy, we need to compare $\Pi_{M_1}^{C,X_j}$ & $\Pi_{M_1}^{N,X_j}$, and $\Pi_{M_2}^{X_i,C}$ & $\Pi_{M_2}^{X_i,N}$. ■

Proof of Lemma 8 .

Now we analytically prove that *CN* and *NC* are impossible to be information sharing equilibrium . Information sharing arrangement *CN* and *NC* are symmetric, so we take *CN* as example. If *CN* is an equilibrium, two conditions should be satisfied simultaneously: (1) given supply chain 1 is communicative (*C*), supply chain 2 chooses *N*, (2) given supply chain 2 is communicative (*N*), supply chain 1 chooses *C*. These two conditions can be transformed to mathematical form

$$\begin{cases}
\Pi_{M_2}^{CN} > \Pi_{M_2}^{CC} & (I) \\
\Pi_{M_1}^{CN} > \Pi_{M_1}^{NN} & (II).
\end{cases}$$

Condition (I):

$$\Pi_{M_2}^{CN} = \left[2(1-\beta)+\frac{c}{2}\right]A^2 - \frac{c}{2}\left(\frac{[4(1-\beta)+c]\,(1+s)-(1-\beta)}{2\,[4(1-\beta)+c]\,(1+s)^2(1-\beta)-(1-\beta)^2}\right)^2(1+s)\sigma^2, \tag{A.32}$$

$$\begin{aligned}\Pi_{M_2}^{CC} &= \left[2(1-\beta)+\frac{c}{2}\right]\frac{1}{[(1+s)\,[4(1-\beta)+c]+(1-\beta)]^2}(1+s)\sigma^2 \\ &\quad + \left[2(1-\beta)+\frac{c}{2}\right]A^2 - K(s).\end{aligned} \tag{A.33}$$

Define

$$\mathscr{A} = \frac{[4(1-\beta)+c]\,(1+s)-(1-\beta)}{2\,[4(1-\beta)+c]\,(1+s)^2(1-\beta)-(1-\beta)^2},$$

$$\mathscr{B} = \frac{1}{(1+s)\,[4(1-\beta)+c]+(1-\beta)}.$$

Condition (II):

$$\begin{aligned}\Pi_{M_1}^{CN} &= \left[2(1-\beta)+\frac{c}{2}\right]\left(\frac{2(1+s)-1}{2\,[4(1-\beta)+c]\,(1+s)^2-(1-\beta)}\right)^2(1+s)\sigma^2 \\ &\quad + \left[2(1-\beta)+\frac{c}{2}\right]A^2 - K(s),\end{aligned} \tag{A.34}$$

$$\Pi_{M_1}^{NN} = \left[2(1-\beta)+\frac{c}{2}\right]A^2 - \frac{c}{2}\frac{(1+s)\sigma^2}{[2(1+s)+1]^2\,(1-\beta)^2}. \tag{A.35}$$

Define

$$\mathscr{C} = \frac{2(1+s)-1}{2\,[4(1-\beta)+c]\,(1+s)^2-(1-\beta)},$$

$$\mathscr{D} = \frac{1}{[2(1+s)+1]\,(1-\beta)}.$$

We first compare $\mathscr{B}$ and $\mathscr{C}$.

$$\begin{aligned}
\mathscr{B} &= \frac{1}{(1+s)\,[4(1-\beta)+c]+(1-\beta)} \\
&= \frac{2(1+s)-1}{((1+s)\,[4(1-\beta)+c]+(1-\beta))\,(2(1+s)-1)} \\
&= \frac{2(1+s)-1}{2\,[4(1-\beta)+c]\,(1+s)^2+2(1-\beta)(1+s)-[4(1-\beta)+c]\,(1+s)-(1-\beta)} \\
&= \frac{2(1+s)-1}{2\,[4(1-\beta)+c]\,(1+s)^2-[2(1-\beta)+c]\,(1+s)-(1-\beta)} \\
&> \frac{2(1+s)-1}{2\,[4(1-\beta)+c]\,(1+s)^2-(1-\beta)} = \mathscr{C} > 0,
\end{aligned}$$

so

$$\mathscr{B} > \mathscr{C}.$$

Then we compare $\mathscr{A}$ and $\mathscr{D}$.

$$\begin{aligned}
\mathscr{D} &= \frac{1}{[2(1+s)+1]\,(1-\beta)} \\
&= \frac{[4(1-\beta)+c]\,(1+s)-(1-\beta)}{2\,[4(1-\beta)+c]\,(1+s)^2(1-\beta)+[4(1-\beta)+c]\,(1+s)(1-\beta)-2(1+s)(1-\beta)^2-(1-\beta)^2} \\
&= \frac{[4(1-\beta)+c]\,(1+s)-(1-\beta)}{2\,[4(1-\beta)+c]\,(1+s)^2(1-\beta)+(1+s)\,[2(1-\beta)^2+c(1-\beta)]-(1-\beta)^2} \\
&< \frac{[4(1-\beta)+c]\,(1+s)-(1-\beta)}{2\,[4(1-\beta)+c]\,(1+s)^2(1-\beta)-(1-\beta)^2} = \mathscr{A},
\end{aligned}$$

so

$$\mathscr{A} > \mathscr{D}.$$

Because of the negative sign of term $\mathscr{A}$ and $\mathscr{D}$, in terms of the ex ante profits, we have the following results:

$$\begin{cases}
\Pi_{M_2}^{CN} < \Pi_{M_1}^{NN} \\
\Pi_{M_1}^{CN} < \Pi_{M_2}^{CC}.
\end{cases}$$

Back to the previous two conditions. Suppose condition (I) holds,

$$\Pi_{M_2}^{CN} > \Pi_{M_2}^{CC},$$

based on the above two inequalities, it can be verified that

$$\Pi_{M_1}^{NN} > \Pi_{M_2}^{CN} > \Pi_{M_2}^{CC} > \Pi_{M_1}^{CN},$$

and thus condition (II) doesn't hold; suppose condition (II) holds,

$$\Pi_{M_1}^{CN} > \Pi_{M_1}^{NN},$$

then based on the inequalities it can be verified that

$$\Pi_{M_2}^{CC} > \Pi_{M_1}^{CN} > \Pi_{M_1}^{NN} > \Pi_{M_2}^{CN},$$

and thus condition (I) doesn't hold.

In conclusion, condition (I) and (II) cannot be satisfied simultaneously, So *CN* is not an equilibrium. Symmetrically, *NC* is not an equilibrium.

Some remarks can be derived from the above inequalities.

(1) When supply chain 1 chooses *N*, manufacturer 2 earns more profit with the same strategy *N*. This means that when supply chain 2 chooses *N*, he hopes supply chain 1 also chooses *N*.

(2) Similarly, when supply chain 1 chooses *C*, he hopes supply chain 2 also chooses *C*.

(3) The other two symmetric results also hold.

The above three remarks can also explain the reason why *CN* and *NC* are not equilibria. ■