PARAMETRIC ESTIMATION OF MONTHLY VOLATILITY USING AUTOREGRESSIVE CONDITIONAL DURATION MODELS

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Parametric Estimation of Monthly Volatility Using Autoregressive Conditional Duration Models

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Abstract:

This paper employs a method to estimate monthly volatility by integrating the conditional return variance over a month using the autoregressive conditional duration (ACD) models. The ACD models fit the daily data surprisingly well. Maximum likelihood Estimation (MLE) method is used to estimate the conditional expected duration equation, which is assumed to follow the augmented ACD models. The estimated monthly stock volatility are adopted to investigate, if any, the link between macroeconomic variability and the stock market volatility. We find that, for the period 1944/01-1975/06, PPI inflation, monetary base growth and industrial production predict stock market volatility very well, which are estimated by ACD methods; the monthly stock volatility, estimated from ACD models, also helps predict the macroeconomic volatility in the period 1975/07-2008/12.

Keywords: AACD Models, Monthly Volatility, Daily data, Macroeconomics

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1 Introduction

The volatility of stock returns has been a hot issue for research since a very long time but still searching for the factors affecting them. The financial econometrics literature has been strikingly successful at measuring, modeling, and forecasting time-varying return volatility, contributing to improved asset pricing, portfolio management, and risk management, as surveyed for example in Andersen et al. (2006a) and Andersen et al. (2006b). Interestingly, the otherwise-massive financial econometric volatility literature is largely silent on the links, if any, between asset return volatility and its underlying determinants.

In the present value model such as Shiller's, a change in the volatility of either future cash flows or discount rates causes a change in the volatility of stock returns. If the volatility of real activity changes, the volatility of stock returns will change. From the point of aggregate level, the value of corporate equity clearly depends on the health of the economy. If macroeconomic data provide information about the volatility of either future expected cash flows or future discount rates, they can help explain why stock return volatility changes over time.

If the underlying business risk of the firm rises, the risk of both the stock and the bonds of the firm should increase. If leverage increases, both the stocks and the bonds of the firm become more risky. Thus, in many instances the risk of corporate stock and long-term corporate debt should change over time in a similar way.

Interest rates are determined by monetary policy of a country according to its economic situation. High interest rates induce the investors to keep their money deposited in saving accounts to get high interest rather to put it into risky stock market. As the risk free returns come down, investors switch their money from bank accounts to stock market investments. Consequently, demand of stocks increases and the stock markets go up as a result of interest rate cut. Thus, interest rates determined by monetary policy of economy has also been considered as an important factor to determine the stock return variance. However no unanimous viewpoint about the predictive power of interest rates to determine stock return variance has yet been observed.

The stock returns analyzed above all measure nominal payoff. When inflation of good's prices is uncertain, the volatility of normal asset returns should reflect inflation volatility. Monetary policy, as the most credible means to achieve sustainable economic growth, primarily focused on stability of the general level of prices of goods and services. Monetary policy is also a determinant of interest rate. Thus, there must exist a link between the stock return volatility and the monetary policy.

Movements in the stock market can also have a significant impact on the macroeconomy and are therefore likely to be an important factor in the determination of macroeconomic volatility. Stock market also plays a vital role in assessing its economic conditions in any economy. Improved stock returns means higher profitability of firms and thus overall growth of economic, and vice verse. Stock market basically serves as a channel to direct the funds from individuals to investors by mobilizing individually-owned resources. With this role of the stock market, volatility in stock price can significantly affect the performance of financial sector as well as the entire economy. Stock return volatility refers to the variation in stock price changes during a period of time. Normally investors and agents perceive this variation as a measure of risk. The policy makers use market estimates of volatility as a tool to measure the vulnerability of the stock market. An unexpected increase in volatility today leads to the upward revision of future expected volatility and risk premium, which further leads to discounting of future expected cash flows at an increased rate, which in turn results in lower stock prices or negative returns today (Pindyck (1984)).

Since the frequency of most obtainable macroeconomic factor data are monthly (such as, industrial production index), the estimation of monthly stock volatility is becoming increasing important in the macro-finance research field.

2 Literature Review

Schwert (1989) analyzes the relation of stock volatility with real and nominal macroeconomic volatility, also economic activity, financial leverage, and stock trading activity using monthly data and conclude a volatility puzzle, which is, the stock volatility is not more closely related to other measures of economic volatility. This is probably the first comprehensively research investigation in stock market volatility and macroeconomic factors. In his paper, he uses two methods to estimate the monthly stock market volatility. The first one is sum of the squared daily returns,

$$\hat{\sigma}_t^2 = \sum_{i=1}^{N_t} \gamma_{it}^2 \tag{2.1}$$

where γ_{it} is the daily return subtracting the average return in the month t. The second method is monthly volatility estimated from monthly data (thereafter we call it rolling volatility). The procedure for obtaining rolling monthly volatility is as follows:

(1) Estimate a 12th-order autoregression for the returns, including dummy variables to allow for different monthly mean returns, using all data available for the series,

$$R_t = \sum_{j=1}^{12} \alpha_j D_{jt} + \sum_{i=1}^{12} \beta_i R_{t-1} + \varepsilon_t \tag{2.2}$$

(2) Estimate a 12th-order autoregression for the absolute values of the errors form (2.2), including dummy variables to allow for different monthly standard deviations,

$$|\hat{\varepsilon}_t| = \sum_{j=1}^{12} \gamma_j D_{jt} + \sum_{i=1}^{12} \rho_i |\hat{\varepsilon_{t-i}}| + u_t \tag{2.3}$$

(3) The regressand $|\hat{\varepsilon}_t|$ is an estimate of the standard deviation of the stock market return for month t similar to $\hat{\sigma}_t$. The fitted values from (2.3) estimate the conditional standard deviation of R_t , given information available before month t.

The rolling volatility is similar to the autoregressive conditional heteroskedasticity (ARCH) model proposed by Engle (1982). Actually it is a generation of the 12-month rolling standard deviation estimator used by Officer (1973), Fama (1976), and Merton (1980). The monthly volatility series was obtained by estimating the standard deviation of the stock return for the first 12 months of data, then the first month was dropped and the thirteen month was added to obtain a new estimate. Each estimate was centered at its approximate midpoint, for example, 6 months. This procedure was followed until the last month of data was included in an estimate.

The rolling methods have given way to ARCH and GARCH models, since the first appearance of ARCH and GARCH models. ARCH model was proposed by Engle (1982), but often requires many parameters to adequately describe the volatility process of an asset return. Bollerslev (1986) proposed a useful extension known as the generalized ARCH (GARCH) model. For a long return series γ_t, let $a_t = \gamma_t - u_t$ be the innovation at time t, then a_t follows a GARCH(m,s) model if

$$a_t = \sigma_t \varepsilon_t, \qquad \sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i a_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2 \tag{2.4}$$

where ε_t is a sequence of iid random variables with mean 0 and variance 1, $\alpha_0 > 0, \alpha_i \geq 0, \beta_j \geq 0$ and $\sum_{i=1}^{max(m,s)}(\alpha_i + \beta_i) < 1$. The latter constraint on $\alpha_i + \beta_i$ implies that the unconditional variance of return series a_t is finite, whereas its conditional variance σ_t^2 evolves over time. ε_t is often assumed to be a standard normal or standardized Student-t distribution or generalized error distribution. To overcome some weaknesses of the GARCH model in handling financial time series analysis, many improved version are studied such as, the integrated GARCH model, the GARCH-M model, the exponential GARCH model etc.

Since the seminal work of Andersen et al. (2001b), realized volatility has been actively studied, both theoretically and empirically. To estimate a model for the volatility of the return on the stock market index, we first construct measures of ”actual” volatility using daily data. If the daily returns data do not exhibit any autocorrelation, the variance of the return in month t can be estimated as

$$\hat{\sigma}_t^2 = \sum_{i=1}^{N_t} (\gamma_{it} - \gamma_t)^2 \tag{2.5}$$

where N_t is the number of trading days in month t, γ_{it} is the return of days in month t, and γ_t denotes the average daily return in month t.

Akgiray (1989) shows that there exists linear dependence in daily return series of market indexes, and the presence of linear dependence can be attributed to various market phenomena and anomalies. The presence of a common market factor, the problem of thin trading in some stocks, the speed of information processing by market participants, and day-of-the-week effects could contribute partially to the observed first-order autocorrelation.

So, if daily returns are positively correlated, the estimation in (2.5) will underestimate the true volatility of monthly return. Therefore, we follow Akgiray (1989), and use an adjusted estimator, based upon the assumption that daily returns in month t are appropriately described by a first-order autoregressive process with coefficient ϕ_t. In particular, we use the following measure for realized monthly volatility.

$$RV_t = \sum_{i=1}^{N_t} (\gamma_{it} - \gamma_t)^2 [1 + 2N_t^{-1} \sum_{j=1}^{N_t - 1} (N_t - j)\hat{\phi}_t^j] \tag{2.6}$$

In this paper, we employ a method to estimate monthly volatility by integrating the conditional variance obtained from the ACD models. The ACD-ICV method was first proposed by Tse and Yang (2009) to estimate high-frequency volatility. They compared daily ACD-ICV estimate against several version of the realized volatility method, and find that ACD-ICV provide the smallest RMSE and ACD-ICV estimates perform well for intraday volatility estimation over 15-min and 1 hour intervals. In our work, we also investigate the relation between stock volatility and macroeconomic factors, and find that, in the early period of the sample, the macroeconomic volatility can predict the stock market volatility very well; whereas the stock market volatility play an vital role in predicting the macroeconomic volatility in later period.

3 ACD Models and Monthly Volatility

The ACD model was first proposed by Engle and Russell (1998) to analyze the durations of transaction data. A recent review of the literature on the ACD models and their applications to finance can be found in Pacurar (2008). Analogous to the generalized autoregressive conditional heteroscedasticity (GARCH) models, which capture the clustering of volatility, the ACD model analyzes the clustering of transaction durations. The latter phenomenon describes the stylized fact that short (long) transaction durations tends to be followed by short (long) transaction durations. Following Engle and Russell (1998), the research of Tse and Yang (2009) shows that the instantaneous variance per unit time derived from the ACD model can be used to calculate the integrated volatility over a time interval, thus providing a parametric daily (even intraday) estimate of volatility. In this section, we first review the ACD model and its augmented version, the empirical estimation of these models. We then outline our method for the estimation of monthly volatility using the ACD models.

3.1 ACD Models

Consider a sequence of times $t_0, t_1, \cdots, t_N$ in which t_i denotes the time of the ith transaction. Thus, $x_i = t_i - t_{i-1}$, for $i = 1, 2, \cdots, N$ are the intervals between consecutive transaction, called transaction duration. In this paper we consider the daily price index duration, which is defined as the time interval need to observe a cumulative change in the index of at least δ. Thus from day t_{i-1} to t_i, the index changes by at least an amount δ, whether upwards or downwards. The occurrence of this incident is called a price index change event.

Let Φ_i denote the information set upon the day t_i. We denote $\varphi_i = E(x_i|\Phi_{i-1})$ which is conditional expectation of the price index duration. We assume that

$$\varepsilon_i = \frac{x_i}{\varphi_i}, \qquad i = 1, 2, \cdots, N \tag{3.1}$$

are a sequence of $i.i.d.$ positive random variables with mean 1 and density function $f(\cdot)$. Thus, the hazard function of ε_i is

$$\lambda(\cdot) = \frac{f(\cdot)}{S(\cdot)} \tag{3.2}$$

where $S(\cdot)$is the survival function of ε_i. Assuming φ_i is known given Φ_{i-1}, the conditional hazard function (also called the conditional intensity) of x_i, denoted by $\lambda_x(x_i|\Phi_{i-1})$, is

$$\begin{aligned}\lambda_x(x_i|\Phi_{i-1}) &= \lambda(\frac{x_i}{\varphi_i})\frac{1}{\varphi_i} \\ &= \lambda(\frac{t_i - t_{i-1}}{\varphi_i})\frac{1}{\varphi_i}\end{aligned} \tag{3.3}$$

which is related to the base hazard function $\lambda(\cdot)$.

To model the conditional duration φ_i, Engle and Russell (1998) proposed the $ACD(p, q)$ defined by

$$\varphi_i = \omega + \sum_{i=1}^{p} \alpha_j x_{i-j} + \sum_{j=1}^{q} \beta_j \varphi_{i-j} \tag{3.4}$$

In particular, setting $p = q = 1$, we obtain the $ACD(1, 1)$ which is,

$$\varphi_i = \omega + \alpha\varphi_{i-1} + \beta x_{i-1} \tag{3.5}$$

where α, β and $\omega \geq 0$ with $\alpha + \beta \leq 1$.

Recently, Fernandes and Grammig (2006) proposed some extensions of the $ACD(1, 1)$ model, incorporating a Box-Cox type transformation with possible asymmetry in duration shocks. We shall consider their $AACD$ model, which is defined by

$$\varphi_i^{\lambda} = \omega + \alpha\varphi_{i-1}^{\lambda}[|\varepsilon_{i-1} - b| + c(\varepsilon_{i-1} - b)]^{\upsilon} + \beta\varphi_{i-1}^{\lambda} \tag{3.6}$$

The $AACD$ model nests the $ACD(1, 1)$ model as a special case and provides a more flexible model for the conditional expected duration. The Box-Cox transformation parameter $\lambda > 0$ determines the shape of the transformation, with $\lambda \geq 1$ representing a convex transformation and $\lambda \leq 1$ representing a concave transformation. Asymmetric responses in duration shocks are permitted through the shift parameter b and the rotation parameter c. In particular, a clockwise rotation is generated if $c < 0$ and a counter-clockwise rotation is obtained if $c > 0$. The shape parameter υ assumes a similar role as λ, with $\upsilon \leq 1$ including concavity and $\upsilon \geq 1$ inducing convexity. As in the case of the $ACD(1, 1)$ model, the parameters α, β and ω are assumed to be nonnegative.

3.2 Estimation of ACD Models

Given an assumed density function $f(\cdot)$, the maximum likelihood estimates (MLE) of the parameters of the ACD equation can be computed straightforwardly. In this model, ε_i are assumed to be standard exponential. Under this assumption the hazard function is constant and does not vary with the duration. Furthermore, the MLE computed using the exponential assumption is consistent (provided the conditional expected duration equation is correctly specified), regardless of the true distribution of the error.

3.3 Estimation of monthly volatility using ACD Models

Given the information Φ_i at time t_i, the conditional intensity function defined in equation (3.3) characterizes the probability that the next price event will occur at time $t > t_i$. Specifically, $\lambda_x(x_i|\Phi_{i-1})\triangle x$ is the probability that the next price event after time t_i occurs in the interval $(t_i + x, t_i + x + \triangle x)$ given the information at time t_i (which includes the fact that there is a price event at time t_i). The instantaneous return variance per unit time at time t_i is defined as

$$\sigma^2(t) = \lim_{\triangle t \to 0} \{\frac{1}{\triangle t} Var[\frac{p(t+\triangle t) - p(t)}{p(t)}]\} \tag{3.7}$$

where $p(t)$ is the price at time t. From Engle and Russell (1998), we see that the conditional instantaneous return variance per unit time give information Φ_i at time t_i, denoted by $\sigma^2(t|\Phi_i)$, is

$$\sigma^2(t|\Phi_i) = (\frac{\bar{\delta}}{p_i})^2 \lambda_x(x|\Phi_{i-1}) \tag{3.8}$$

where $x = t - t_i,\ t > t_i$ and Using equation (3.3), we have

$$\sigma^2(t|\Phi_i) = (\frac{\bar{\delta}}{p_i})^2 \lambda(\frac{t - t_i}{\varphi_{i+1}}) \frac{1}{\varphi_{i+1}}, \qquad t > t_i \tag{3.9}$$

where $\varphi_{i+1} = E(x_{i+1}|\Phi_i)$ is the conditional expected duration of the next price event given Φ_i, and $\lambda(\cdot)$is the base hazard function (of ε_i). Thus, the integrated variance IV over the interval (t_i, t_{i+1}) , denoted by IV$_i$, is

$$\begin{aligned} ICV_i &= \int_{t_i}^{t_{i+1}} \sigma^2(t|\Phi_i)dt \\ &= (\frac{\bar{\delta}}{p_i})^2 \frac{1}{\varphi_{i+1}} \int_{t_i}^{t_{i+1}} \lambda(\frac{t - t_i}{\varphi_{i+1}})dt \end{aligned} \tag{3.10}$$

if ε_i are $i.i.d.$ standard exponential distributions, then $\lambda(\cdot) \equiv 1$ and we have

$$ICV_i = (\frac{\bar{\delta}}{p_i})^2 [\frac{t_{i+1} - t_i}{\varphi_{i+1}}] \tag{3.11}$$

Thus, if $t_0 < t_1 < \cdots < t_N$ denotes the price events on one month, the integrated volatility IV of the month is

$$ICV = \bar{\delta}^2 \sum_{i=0}^{N-1} \frac{t_{i+1} - t_i}{\varphi_{i+1} p_i^2} \tag{3.12}$$

Since ε_i are assumed to be standard exponential and the QMLE method is adopted, the ACD-IV is,

$$ICV = \bar{\delta}^2 \sum_{i=0}^{N-1} \frac{t_{i+1} - t_i}{\hat{\varphi}_{i+1} p_i^2} \tag{3.13}$$

We can also calculate the return duration using the above method. For example, assume $s(t) = log(p(t))$, equation (3.7) can be rewritten into,

$$\sigma^2(t) = \lim_{\triangle t \to 0} \{\frac{1}{\triangle t} Var[s(t + \triangle t) - s(t)]\} \tag{3.14}$$

and we have,

$$\sigma^2(t|\Phi_i) = \delta^2 \lambda_x(x|\Phi_{i-1}) \tag{3.15}$$

Finally, the estimated volatility of one month is,

$$ICV = \delta^2 \sum_{i=0}^{N-1} \frac{t_{i+1} - t_i}{\hat{\varphi}_{i+1}} \tag{3.16}$$

In this paper, we will use the logarithm of the price index series instead of price index to model the duration series, so there is little difference from the research of Tse and Yang (2009). As a matter of fact, in our work, there is a price event when the accumulative index return excess δ, and there is a price event when the accumulative change of price exces $\bar{\delta}$ Tse and Yang (2009). The levels of δ and $\bar{\delta}$ are also very different because of the logarithm procedure we take.

4 Data

The daily data used in this paper are extracted from the Center for Research in Security Prices (CRSP) database and Datastream. I download and compile the daily stock market index data of U.S.A. from CRSP database. The data of U.K., Australia, Japan and Canada are from Datastream. Table 1 shows the data description of each country. For instance, within the index of Nikkei 225, there is 696 month in the whole sample period and the entire sample period is 1951/01-2008/12.

From Table 1, we can see that the minimum of the daily stock market index is 3.0863, the maximum of the daily index is 4477.9 in the USA stock market. As time passed by, there were rapid growth in Economic and Financial areas and so did the stock index. In order to make the index series a smooth time series, we take the logarithm of the index and equation (3.16) will be used to estimate monthly volatility.

Table 1: Data description of each country

Country	U.S.	U.K.	Japan	Australia	Canada
Begin of Date	1944-01-03	1969-01-01	1951-01-01	1980-01-01	1969-01-01
End of Date	2008-12-31	2008-12-31	2008-12-31	2008-12-31	2008-12-31
Name of Index	NYSE*	FTSE	Nikkei 225	ASX	S&P/TSX
Number of days	16645	10436	15133	7567	10436
Number of month	780	480	696	348	480
Average Days per Month	20.3	20.7	20.7	20.7	20.7
Minimum index	3.0863	61.92	101.91	443.1	800.29
Maximum index	4477.9	3479.0	38916	6853.6	15073

Notes: The price index of American stock market including dividends to the Value-weighted portfolio of all New York Stock Exchange (NYSE) stocks.

Table 2: The statistics of durations of each country

Return threshold (δ)	1 %	1.25 %	1.5 %	1.75 %	2 %	2.25 %	2.5 %	2.75 %	3.0 %	3.25 %
U.S.A.										
Sample size	5529	4101	3126	2492	2088	1719	1469	1288	1121	972
Mean	3.0103	4.0584	5.3242	6.6790	7.9711	9.6818	11.3297	12.9216	14.9669	17.1233
Median	1.9000	2.7000	3.6000	4.5000	5.4000	6.5000	7.9000	8.9000	10.5000	11.5000
Maximum	29.0000	41.0000	56.6000	61.9000	81.6000	90.3000	73.1000	86.0000	116.5000	169.3000
Minimum	0.1000	0.1000	0.1000	0.1000	0.1000	0.2000	0.2000	0.2000	0.2000	0.2000
U.K.										
Sample size	4437	3313	2588	2055	1691	1719	1220	1040	909	810
Mean	2.3516	3.1494	4.0319	5.0772	6.1699	7.2054	8.5521	10.0324	11.4617	12.8627
Median	1.4000	1.9000	2.4000	3.1000	3.9000	4.4000	5.4500	6.0000	7.1000	7.4500
Maximum	23.2000	25.1000	39.2000	56.2000	58.6000	70.8000	106.9000	107.3000	144.2000	137.8000
Minimum	0.1000	0.2000	0.2000	0.2000	0.2000	0.2000	0.3000	0.3000	0.3000	0.3000
Japan										
Sample size	6887	5140	4031	3259	2677	2282	1959	1691	1470	1314
Mean	2.1970	2.9438	3.7534	4.6422	5.6519	6.6303	7.7182	8.9468	10.2890	11.5067
Median	1.2000	1.7000	2.3000	2.9000	3.4000	4.1000	4.7000	5.4000	6.4000	7.3000
Maximum	32.7000	58.9000	70.2000	77.0000	77.0000	95.7000	105.9000	118.2000	120.7000	115.1000
Minimum	0.1000	0.1000	0.1000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.3000
Australia										
Sample size	2789	2053	1604	1282	1081	899	757	648	596	521
Mean	2.7127	3.6851	4.7168	5.9016	6.9987	8.4156	9.9937	11.6747	12.6935	14.5219
Median	1.7000	2.3000	2.9000	3.9000	4.6000	5.6000	6.8000	8.0000	9.0500	10.1000
Maximum	30.4000	45.1000	53.0000	63.3000	65.9000	73.6000	75.5000	89.8000	124.3000	190.0000
Minimum	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000	0.2000	0.2000
Canada										
Sample size	3499	2627	2068	1656	1371	1167	996	854	745	666
Mean	2.9822	3.9721	3.0000	6.3008	7.6108	8.9414	10.4761	12.2185	14.0047	15.6673
Median	1.7000	2.3000	2.9000	3.8000	4.7000	5.6000	6.5000	7.4000	8.9000	10.3000
Maximum	39.0000	49.4000	66.4000	68.2000	69.2000	81.8000	128.3000	154.2000	128.4000	125.7000
Minimum	0.1000	0.2000	0.2000	0.2000	0.2000	0.2000	0.3000	0.3000	0.3000	0.3000

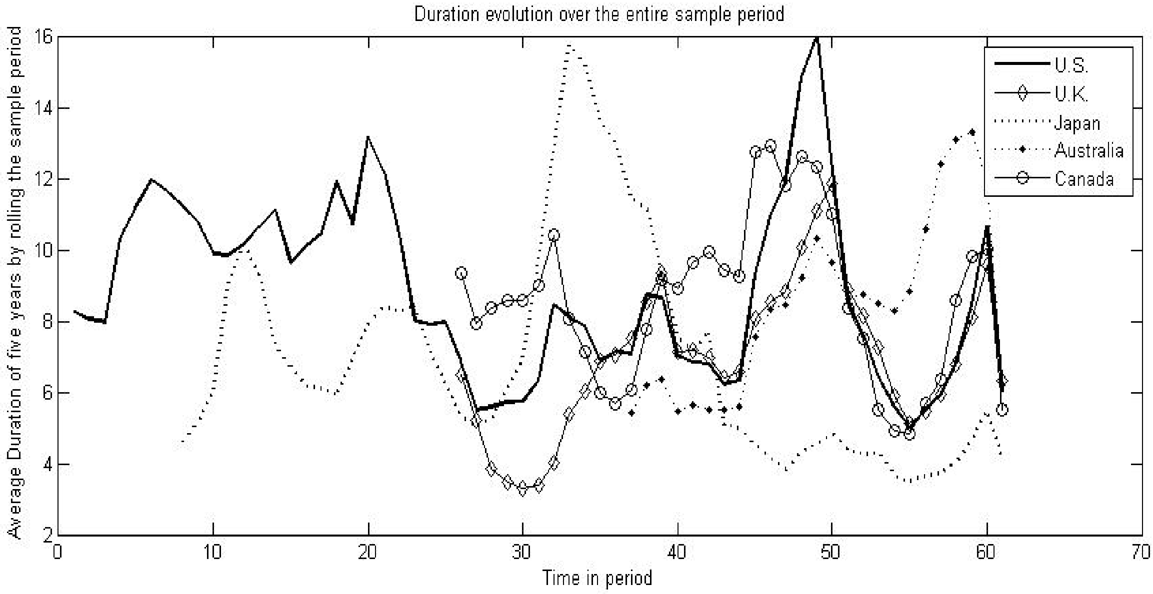

Fig 1: The average duration of five years is calculated when the price index return change is 2%. We then roll over the entire sample one year at a time by dropping the first one year and adding the next one year duration. This is done until the last year is included.

There are about 20 days excluding weekends and public holidays in each month. Firstly, we take the logarithm of the price index, then interpolate one day into 10 parts using linear interpolation in case there is some big index change within one day. There are more than 200 observations in one month, and the duration we have will not be integer any more.

There is little difference between the price index change in this paper and price change Engle and Russell (1998) has proposed. After taking the logarithm of the index series, we define the logarithm price index change as the logarithm index of the current observation subtracting the logarithm price index of previous observation(in another word, the difference between two consecutive observations is return). If the accumulate index return excess the threshold δ, we call it a return invent. We arrange the threshold δ from 1% to 3.25% by incremental 0.25%. Table 2 are the statistics of the duration series of each country. From Table 2 we can see that all the statistics (Mean, Median, Maximum and Minimum) increase as δ increases except the sample size.

Autoregressive Conditional Duration (ACD) models are widely used to model durations of intraday transactions. To manipulate the intraday data, we need to correct for the opening auction and time-of-day effects; we also have to take account of the diurnal factor. But for daily data, although the whole sample period is much longer than intraday data, we do not have so many observations as intraday data. Since manipulating a much longer period of daily data, we have to consider market liquidity. In the early part of the sample period, it took longer time for the market price index return to move by δ. The average value of the durations in the first five years is taken as the initial expected duration. Fig 1 is the plot of duration evolution of five years rolling frequency when the price index return threshold is 2%. The plots are pretty much the same when the price index return threshold differs from 2%. From Fig 1, we can see, there is a strong decreasing trend of the duration in the cases of the stock markets in U.S.A. and Japan. Since the sample period of the other countries are not as long as U.S.A. and Japan, the trend are not obvious. In order to model market liquidity, the time trend are considered in the AACD model,

$$\varphi_i^{\lambda} = \omega + \eta\, i + \alpha\varphi_{i-1}^{\lambda}[|\epsilon_{i-1} - b| + c(\epsilon_{i-1} - b)]^{v} + \beta\varphi_{i-1}^{\lambda} \tag{4.1}$$

where i is the time in the sample period. Equation (4.1) is exactly the same with equation (3.6) without time trend, and we expect η to be negative.

5 Empirical Results

In this part, we first estimate whether the AACD models fit the daily data well or not, and find the AACD models do fit the daily data very well. Then, we use the estimated parameters to calculate the monthly volatility by equation (3.16). After that, we select the optimal index return threshold δ^*, and use that model to investigate the link between stock market volatility and macroeconomic factors.

5.1 Results of AACD Model Estimation

Since we have a long time period of daily data, the AACD models with time trend (which is equation (4.1)) are consider to be estimated and to construct the monthly volatility. Tables 3a-3e (see 3b-3e in the Appendix) shows the results of estimation of U.S., U.K., Japan, Australia and Canada, respectively.

The asymptotic standard errors estimated here are the same with Fernandes and Grammig (2006). We employ the outer-product-of-the-gradient (OPG) estimator of the information matrix since the absolute value function in the shocks impact curve makes Hessian-based estimates tricky to compute due to numerical problems.

From table 3a - 3b, we can see that, all the estimated parameters are significantly different from zero except the rotation parameters c. The Box-Cox transformation parameter λ are all greater than zero although very close to zero in some cases (U.S.A 2.5%, U.K. 2$\%$, U.K. 2.75$\%$, Japan 1$\%$, and Japan 1.5$\%$). $\hat{\lambda}$ is smaller than one in all the cases and smaller than 0.1 in most cases, which represent concave transformation. Because durations are nonnegative, the shift parameters b are important to the identification of the asymmetric response implied by the shocks impact curve. The parameter c determines whether rotation is clockwise ($c < 0$) or counter-clockwise ($c > 0$). From these tables, we can see $c > 0$ in all the cases for the stock market of U.S.A and Canada. In some cases $c < 0$ (U.K. 1.5$\%$, U.K. 2$\%$, U.K. 2.25$\%$ etc). Indeed, the shift parameter affects mostly small shocks, whereas rotation parameter is dominant for large shocks. Despite the fact that b are significantly different from zero, the standard error of c are quite large, showing that the shocks impact curve features no rotation.

Table 3a: The parameter estimation of AACD models of American stock market

Return threshold	1 %	1.25 %	1.5 %	1.75 %	2 %	2.25 %	2.5 %	2.75 %	3.0 %	3.25 %
ω	0.0185	0.0275	0.0401	0.0431	0.0578	0.0463	0.0550	0.0660	0.0714	0.0769
	(0.0007)	(0.0046)	(0.0010)	(0.0085)	(0.0007)	(0.0005)	(0.0006)	(0.0023)	(0.0031)	(0.0002)
α	0.0222	0.0100	0.0146	0.0204	0.0146	0.0047	0.0216	0.0385	0.0216	0.0036
	(0.0009)	(0.0050)	(0.0007)	(0.0157)	(0.0009)	(0.0005)	(0.0018)	(0.0019)	(0.0029)	(0.0002)
β	0.9625	0.9635	0.9477	0.9402	0.9324	0.9503	0.9234	0.8976	0.9134	0.9206
	(0.0002)	(0.0000)	(0.0000)	(0.0004)	(0.0001)	(0.0000)	(0.0000)	(0.0005)	(0.0005)	(0.0000)
b	0.1267	0.0506	0.0710	0.0359	0.1390	0.0955	0.0201	0.0310	0.0651	0.1564
	(0.0095)	(0.0230)	(0.0168)	(0.0297)	(0.0173)	(0.0243)	(0.0309)	(0.0368)	(0.0223)	(0.0297)
c	0.3234	0.2818	0.1187	0.1249	0.0363	0.1706	0.1890	0.3060	0.1355	0.4365
	(0.1016)	(0.3771)	(0.1135)	(2.4645)	(0.0489)	(0.2811)	(44022)	(1.0360)	(0.2540)	(0.2274)
λ	0.0627	0.0145	0.0189	0.0284	0.0236	0.0083	0.0000	0.0098	0.0249	0.0037
	(0.0076)	(0.0088)	(0.0063)	(0.0194)	(0.0039)	(0.0026)	(0.0000)	(0.0160)	(0.0130)	(0.0006)
v	0.2661	0.1904	0.1886	0.2401	0.2749	0.2996	0.0000	0.0513	0.2220	0.2458
	(0.0377)	(0.1162)	(0.0617)	(0.1313)	(0.0422)	(0.0974)	(0.0000)	(0.0791)	(0.1103)	(0.0393)
η	-2.7e-07	-1.0e-07	-2.4e-07	-4.3e-07	-4.0e-07	-1.9e-07	-4.6e-12	-4.8e-07	-1.1e-06	-1.0e-07
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
$Q(12)$	12.0355	8.9832	7.9530	7.1293	7.5590	7.2446	6.1863	11.0330	17.6054	11.1874

Notes: The AACD equation is $\psi_i^\lambda = \omega + \eta\ i + \alpha\psi_{i-1}^\lambda[|\epsilon_{i-1} - b| + c(\epsilon_{i-1} - b)]^v + \beta\psi_{i-1}^\lambda$. This results are from American stock market with the index return change varys from 1% to 3.25%. The value in the Parentheses are the standard error calculated from MLE method(0.0000 is not exactly zero and represent smaller than 0.00005). Q(12) describes the Q-statistics of Ljung-Box Q-statistic lack-of-fit hypothesis test with lags up to 12 and the critical value is 21.0261.

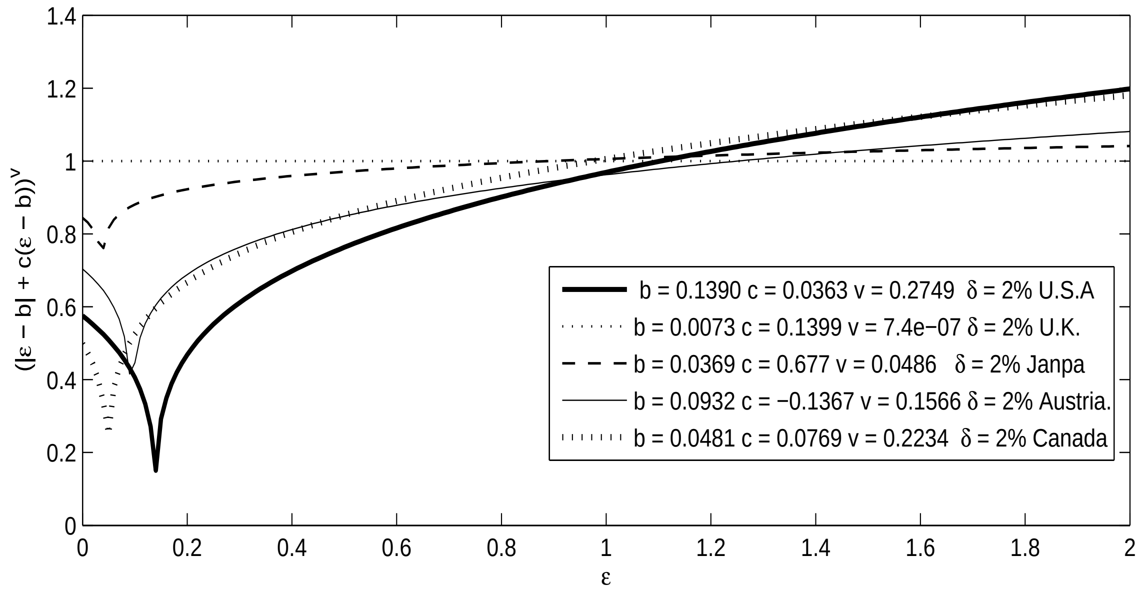

Fig 2: Shocks impact curves for the AACD model of each country when the price index return change is 0.02.

From Fig 2, we can see c = -0.1367 for Austria, compared to Canada of which the other parameter are very close to each other, the shock impact curve is more flat. The shape parameter v assumes a similar role as λ. For country U.K., v is close to zero and the shock impact curve turns out to be a flatter line. Time trend parameter η is negative for all the cases of U.S.A, Japan and Canada, which indicates that for these stock market, it took a longer time for the market price to move by δ in the early part of the period.

Q(12) describes Ljung-Box Q-statistic lack-of-fit hypothesis test of ε_i with lags up to 12 and the critical value is 21.0261. From the Q(12), we cannot reject that ϵ is a sequence of i.i.d. random variables (except Australia 1.5%). From this section, we conclude that AACD models fit the daily data very well.

5.2 Estimation of Monthly Volatility

In this section, we will calculate the monthly volatility by equation (3.16) using the parameters estimated in the last section. If a index return event spread across two (or more than two) month, the accumulated volatility during the index return event is divided into two (or more than two) parts according to their weight in each month. We then annualized the integrated volatility by $\sqrt{(vol * 12)}$.

Table 4: Statistics of monthly volatility from AACD methods

δ	1%	1.25%	1.5%	1.75%	2%	2.25%	2.5%	2.75%	3%	3.25%
U.S.A.										
Mean	0.0898	0.0964	0.1011	0.1049	0.1102	0.1118	0.1148	0.1178	0.1192	0.1196
STD	0.0221	0.0243	0.0257	0.0273	0.0285	0.0296	0.0312	0.0339	0.0338	0.0375
Minimum	0.0493	0.0563	0.0299	0.0264	0.0277	0.0289	0.0295	0.0290	0.0296	0.0254
Maximum	0.2410	0.2643	0.2827	0.3110	0.3076	0.3349	0.3308	0.3676	0.3497	0.3806
U.K.										
Mean	0.1019	0.1099	0.1166	0.1213	0.1253	0.1296	0.1316	0.1339	0.1376	0.1416
STD	0.0286	0.0291	0.0312	0.0334	0.0349	0.0362	0.0342	0.0407	0.0408	0.0458
Minimum	0.0541	0.0566	0.0628	0.0348	0.0339	0.0316	0.0330	0.0302	0.0272	0.0274
Maximum	0.2347	0.2529	0.2618	0.2817	0.2912	0.2938	0.2862	0.3204	0.3059	0.3760
Japan										
Mean	0.1054	0.1127	0.1205	0.1268	0.1309	0.1356	0.1389	0.1428	0.1458	0.1506
STD	0.0277	0.0301	0.0333	0.0342	0.0364	0.0387	0.0391	0.0405	0.0422	0.0442
Minimum	0.0518	0.0237	0.0269	0.0321	0.0315	0.0348	0.0344	0.0333	0.0334	0.0357
Maximum	0.2697	0.2833	0.3007	0.3288	0.3436	0.3558	0.3792	0.3804	0.3826	0.3996
Australia										
Mean	0.0957	0.1023	0.1082	0.1127	0.1191	0.1221	0.1233	0.1247	0.1306	0.1334
STD	0.0217	0.0242	0.0270	0.0274	0.0280	0.0354	0.0323	0.0398	0.0356	0.0367
Minimum	0.0537	0.0569	0.0263	0.0257	0.0264	0.0261	0.0273	0.0251	0.0298	0.0294
Maximum	0.2160	0.2358	0.2569	0.2554	0.2515	0.3372	0.2986	0.4003	0.3083	0.3308
Canada										
Mean	0.0907	0.0983	0.1043	0.1083	0.1130	0.1160	0.1197	0.1217	0.1250	0.1264
STD	0.0231	0.0265	0.0274	0.0296	0.0329	0.0368	0.0341	0.0369	0.0348	0.0409
Minimum	0.0554	0.0571	0.0275	0.0270	0.0289	0.0292	0.0315	0.0329	0.0327	0.0321
Maximum	0.2382	0.2740	0.2822	0.3088	0.3360	0.3677	0.3446	0.3694	0.3498	0.3912

Table 4 is the description of the monthly volatility calculated from AACD methods. From Table 4 we can see that when the index return threshold increases, the level of monthly volatility shifts up (the average of monthly volatility of each county increases); the variation of monthly volatility becomes bigger. Figure 3 presents the plots of monthly volatility when δ changes from 1% to 3% by an incremental of 1%. From the same figure, we can also see the level of volatility shifts up as δ increases.

We can see that, in this section, the monthly volatility estimated from AACD models are very sensitive to the index return threshold.

5.3 Model Selection

Since for different index return threshold, the estimated monthly volatility are different, the optimal index return threshold δ^* is the following task we will face.

For daily data, as mentioned earlier, daily returns are positively correlated; equation (2.5) will underestimate the true volatility of daily return. Therefore, we use an adjusted estimator based upon the assumption that daily returns within month t are appropriately described by a first-order autoregressive process. In particular, we use the measure of realized monthly volatility by equation (2.6) as the true monthly volatility, then, we employ the cross-validation, a data-driven selection method, to calculate the integrated square error between adjusted monthly realized volatility and monthly volatility estimated from AACD method. Consider the integrated square error,

$$ISE(\delta) = \int_{T_l}^{T_u} [\sigma_s^2 - \hat{\sigma}_s^2]^2 \qquad 0 < T_l < Tu < T \tag{5.1}$$

where σ_s^2 is the true monthly volatility and $\hat{\sigma}_s^2$ is the estimated monthly volatility, T_l and Tu are the lower and upper bound, respectively. In our case, the integrated square error will be

$$\widehat{ISE}(\delta) = \sum_{i=1}^{N} [RV(i) - ACDIV(i)]^2 \tag{5.2}$$

where N is the number of month in the sample period. We then define the cross-validated index return threshold as $\delta_{cv} = argmin_{\delta>0}\widehat{ISE}(\delta)$

In case that the realized volatility miss-estimates the true monthly volatility, we also compute the average standard deviation of monthly volatility series, using delta method, to select the optimal index return threshold. The parameters $\hat{\theta}$ of AACD model are estimated from Maximum likelihood estimation (MLE). Suppose that conditions for consistency of maximum likelihood are satisfied[①] , then the maximum likelihood estimator has asymptotically normal distribution,

$$\sqrt{n}(\hat{\theta} - \theta) \longrightarrow N(0, \Omega) \tag{5.3}$$

where θ is the true value of parameters. Since the monthly volatility is a function of the parameters estimated from the AACD model, we can employ delta method to estimate the standard deviation of monthly volatility.

$$\sqrt{n}(vol_t(\hat{\theta}) - vol_t(\theta)) \longrightarrow N(0, \bigtriangledown vol_t^T(\theta) \cdot \Omega \cdot \bigtriangledown vol_t(\theta)) \tag{5.4}$$

from equation (5.4), we can see that the monthly volatility also has an asymptotically normal distribution. After this is done, we expect a small average standard derivation of monthly volatility series with respect to the optimal index return threshold. We also calculate the correlation between the monthly volatility estimated from ACD method and realized volatility.

Although we use the integrated square error between adjusted monthly realized volatility and monthly volatility estimated from ACD method to select the optimal δ, we also should take into account the case that the realized volatility is not the true monthly volatility. In order to avoid miss-estimating of realized monthly volatility, we select several smaller δ instead of the smallest one compared to the others. From table 5, we can see that $\widehat{ISE}(\delta)$ is smaller when the price index return threshold δ are 2%, 2.25% , 2.5% and 2.75%. The correlations between monthly volatility estimated from ACD method and Realized method decreases when δ increases. The average standard deviation of monthly volatility is very small when δ are 1.5%, 1.75% and 2%. Compared with all the factors, $\delta = 2\%$ is the optimal index return threshold.

The same reason, the optimal δ is 2.25%, 2.5%, 2.5% and 2.25% for U.K., Japan, Australia and Canada respectively. From this section, we can see that there are 2-3 index return events with respect to the optimal δ.

① Newey & McFadden (1994, Theorem 3.3.)

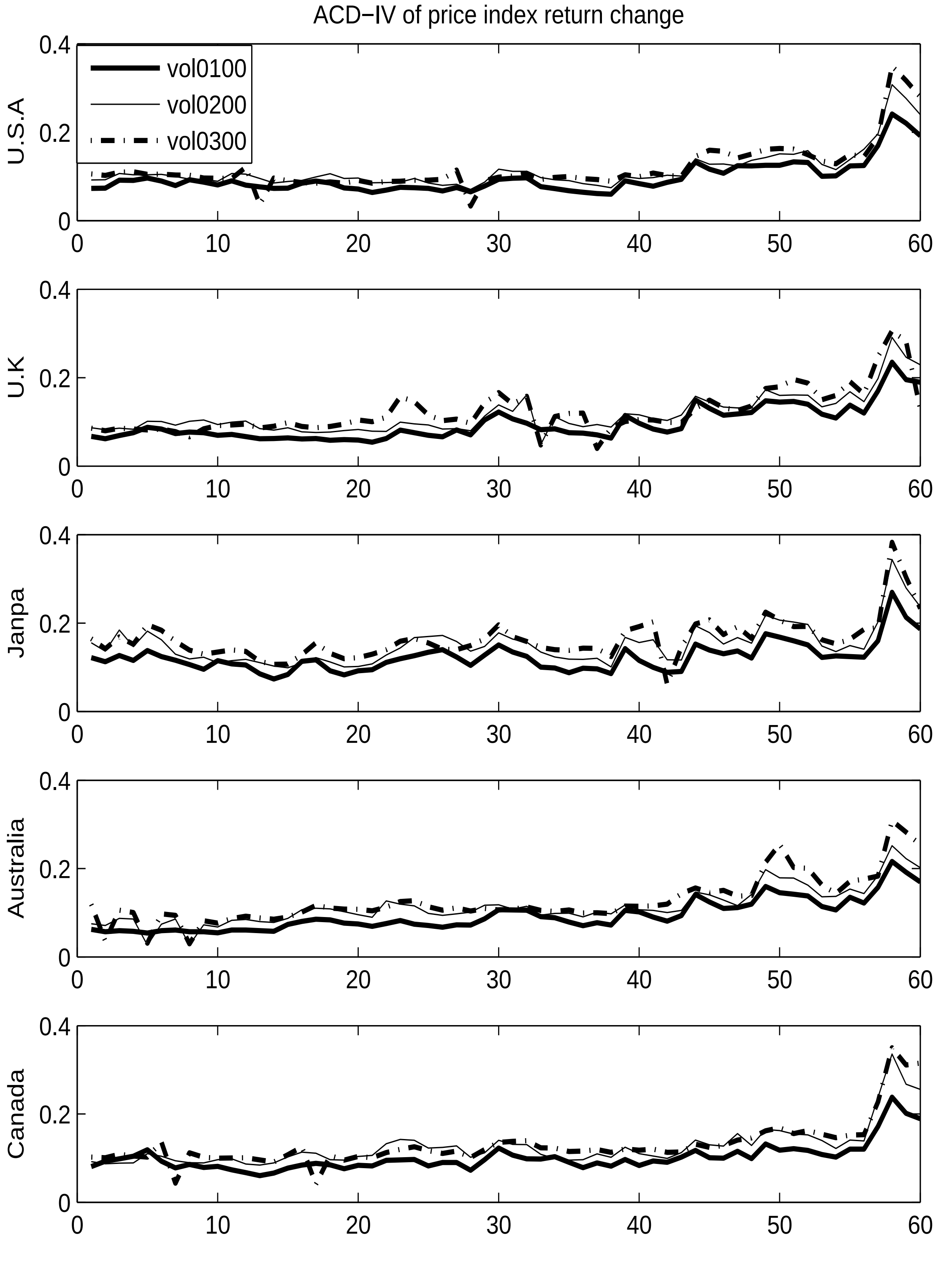

Fig 3: Monthly Volatility estimated from AACD models. Period: 2003/01-2008/12

Table 5: Optimal return change selection for different models

Rerurn Change (δ)	1%	1.25%	1.5%	1.75%	2%	2.25%	2.5%	2.75%	3%	3.25%
U.S.A										
$\widehat{ISE}(\delta)$	2.9459	2.5380	2.3730	**2.2360**	**2.0932**	**2.0969**	**2.0719**	**2.0810**	2.2050	2.1474
Corr(ACD-IV,RV)	0.8422	0.8430	0.8180	**0.8044**	**0.7989**	0.7759	0.7561	0.7253	0.6916	0.6870
STD(ACD-IV)	5.9e-005	2.5e-005	**5.6e-006**	**7.6e-006**	**9.6e-006**	9.2e-004	2.7e+004	0.0158	2.9e-005	1.2e-004
U.K.										
$\widehat{ISE}(\delta)$	2.4480	2.2173	1.9673	1.7630	1.6914	**1.6321**	1.7273	**1.5837**	**1.6695**	**1.5698**
Corr(ACD-IV,RV)	0.8951	0.8612	0.8503	0.8552	0.8343	**0.8175**	0.7952	0.7788	0.7426	0.7421
STD(ACD-IV)	0.0184	**4.7e-006**	**6.6e-006**	**3.5e-005**	168	**8.4e-005**	**9.4e-005**	0.0058	**3.3e-005**	**5.4e-005**
Japan										
$\widehat{ISE}(\delta)$	5.1671	4.5802	3.9610	3.6402	3.4666	3.2596	**3.1675**	**3.1575**	**3.1028**	**3.0978**
Corr(ACD-IV,RV)	0.8801	0.8613	0.8558	0.8496	0.8252	0.8115	**0.8087**	0.7810	0.7681	0.7653
STD(ACD-IV)	62.7135	0.0080	183.7	**2.6e-005**	0.6064	0.0239	**1.7e-005**	0.0040	**7.1e-005**	7.5e-004
Australia										
$\widehat{ISE}(\delta)$	1.9491	1.7095	1.5866	1.5462	1.4778	**1.2890**	**1.2108**	**1.3940**	**1.3787**	1.4539
Corr(ACD-IV,RV)	0.7370	0.7610	0.7289	0.7087	0.7045	0.7236	**0.6920**	0.7245	0.6661	0.6245
STD(ACD-IV)	**2.1e-005**	0.2122	0.0637	**3.7e-005**	2.3e-004	0.0018	**1.8e-004**	1.5e-004	**4.1e-005**	1.1e-004
Canada										
$\widehat{ISE}(\delta)$	2.0306	1.7454	1.6447	1.5923	1.4862	**1.3947**	1.4917	1.4948	1.6529	1.5942
Corr(ACD-IV,RV)	0.8739	0.8605	0.8437	0.8110	0.7992	**0.7914**	0.7793	0.7549	0.7178	0.7057
STD(ACD-IV)	**2.9e-006**	**5.3e-006**	0.0073	**4.4e-005**	1.3e-004	**8.6e-005**	2.2e-005	0.0173	2.0e-004	6.5e-005

Notes: $\widehat{ISE}(\delta)$ is the integrated square error. Corr(ACD-IV,RV) is the correlation between monthly volatility estimated from ACD methods and realized monthly volatility. STD(ACD-IV) is the average of the stand deviation of the monthly volatility in the entire sample.

6 Stock Market and Macroeconomy

The link between the macroeconomy and the stock market has been intuitive appeal, as macroeconomic variables affect both expected cash flows accruing to stockholders and discount rates. A common theoretical framework connecting stock prices to fundamentals is the dividend discount model. According to this model, new macroeconomic information will affect stock prices if it impacts on either expectations about future dividends, discount rates, or both.

Empirically, the evidence linking macroeconomic factors to the stock market is mixed at best. Chen et al. (1986) were one of the first to explore the link between macroeconomic variables and stock prices. Using a multifactor model, they found evidence that macroeconomic factors are priced. Some researchers also conclude that stock prices respond to macroeconomic news. Subsequent studies have produced more mixed results.

Moving from first to second moments, Veronesi (1999) presents a theoretical model that formalizes the link between economic uncertainty and stock market volatility. He shows that investors are more sensitive to news during periods of high uncertainty, which in turn increases asset price volatility. Yet establishing the empirical link between the second moments of stock returns and macroeconomic variables has proven to be even more challenging than that between their first moments.

Schwert (1989) analyzes the relation of stock volatility with real and nominal macroeconomic volatility, also economic activity, financial leverage, and stock trading activity using monthly data and conclude there is a volatility puzzle, which is, the stock volatility is not more closely related to other measures of economic volatility.

In this section, we will follow the analysis of Schwert (1989) using monthly volatility estimated from ACD models to find whether there is a link between macroeconomic factors and stock market volatility.

As Schwert (1989) described, it is useful to think of the stock price P_t as the discounted present value of expected future cash flows to stockholders:

$$E_{t-1}P_t = E_{t-1}\sum_{k=1}^{\infty}\frac{D_{t+k}}{[1+R_{t+k}]^k} \tag{6.1}$$

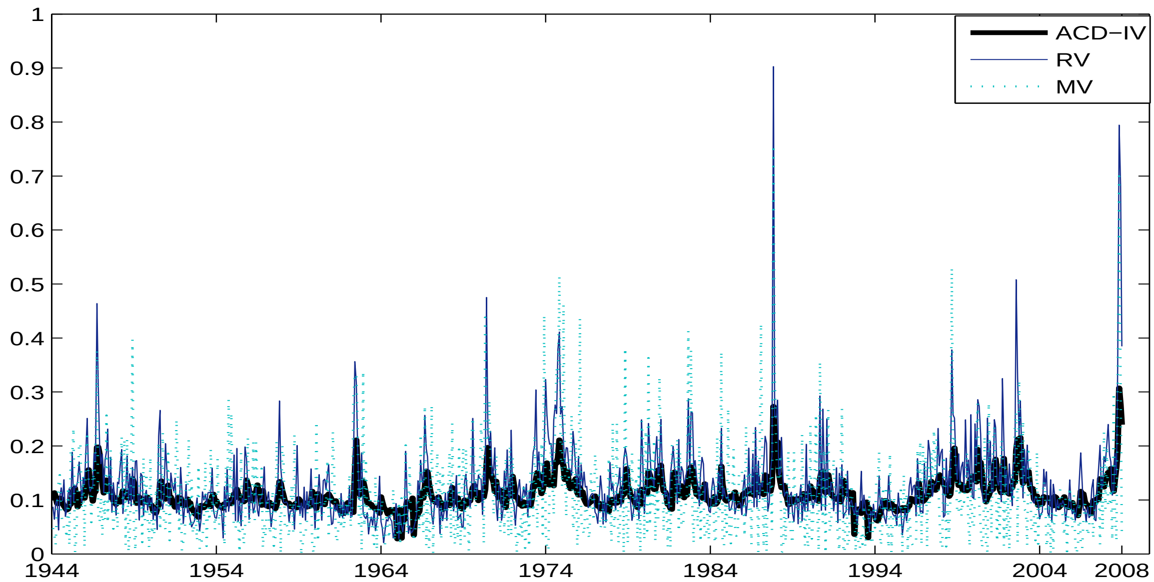

Fig 4: Predictions of the monthly annualized monthly volatility based on ACD-IV, Realized volatility and Rolling Volatility (MV for convenience).

where D_{t+k} is the dividends paid to stockholders in period $t + k$ and $1/[1 + R_{t+k}]$ is the discount rate for period $t + k$ based on the information available at time $t - 1$.

Since common stocks reflect claims on future profits of corporations, it is plausible that the volatility of real economic activity is a major determinant of stock return volatility. In the present value model, the volatility of future expected cash flows, as well as discount rates changes, if the volatility of real activity changes. As mentioned in the introduction of this paper, the bond return volatility, interest rate, PPI inflation, monetary base growth and industrial production have a plausible link with the stock market. Also, movements in the stock market can also have a significant impact on the macroeconomy and are therefore likely to be an important factor in the determination of macroeconomic volatility. In our analysis, since we can only get the monthly data of the economic variables, the macroeconomic variable volatilities are estimated using equation (2.2)and equation (2.3).

For the stock market return volatility, we use the rolling volatility (MV for convenience), realized volatility and conditional monthly volatility from AACD method to estimate the monthly volatility. Fig 4 plots the predicted annualized monthly volatility of three measures. From the figure we can see that the rolling volatility is more fluctuant than the other two measures. The monthly volatility estimated by ACD model traces the Realized volatility extremely well, but is more smooth than

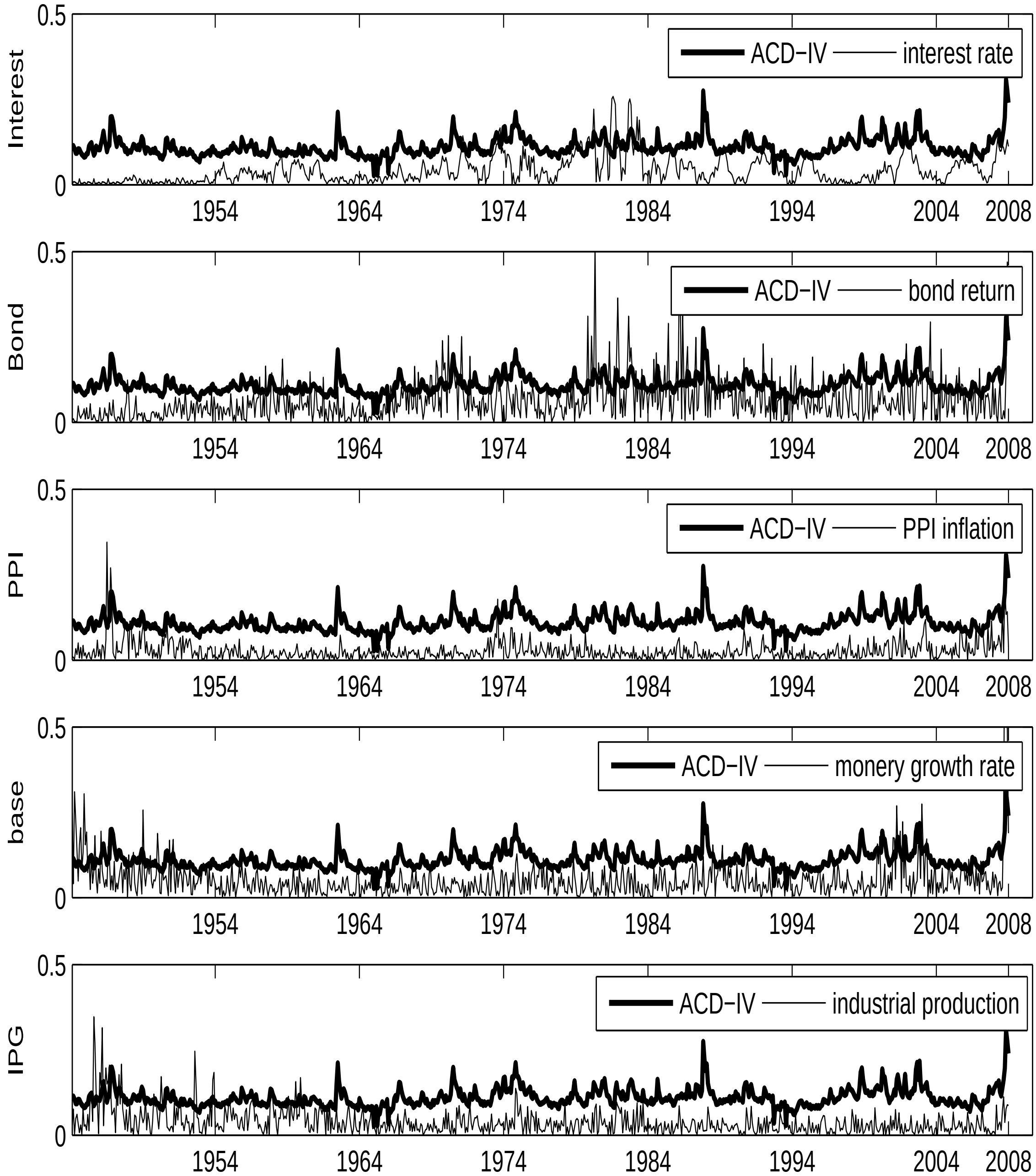

Fig 5: Predictions of the monthly annualized monthly stock market volatility based on ACD-IV, and annualized monthly macroeconomic volatility based on Rolling Volatility.

realized volatility when realized volatility jump up. All the monthly volatility measures share the same evolution trend.

Table 5 contains means, standard deviations, skewness, and kurtosis coefficients and autocorrelations of the estimates of stock volatility based on the rolling volatility, realized volatility, AACD measure, and also the monthly volatility of short-interest rate, bond return, PPI inflation, money growth and industrial production based on rolling volatility. From Table 5, we can see the level of stock market volatility based on ACD methods is much bigger than the macroeconomic volatility, but the standard deviation of stock market volatility based on the ACD methods is smaller than each macroeconomic volatility based on rolling volatility. This can also can be seen from Fig 5. Sometimes the macroeconomic volatility based on rolling volatility is bigger than stock market volatility based on ACD methods. Table 5 also contains the unit root test of the volatility series. From the T-statistic and P value, we conclude that the volatility series are stationary.

Table 6a contains tests of the incremental predictive power of 12 lags of PPI inflation volatility in a 12th-order vector autoregressive (VAR) system for stock volatility, bond return volatility, and short-term interest volatility that allows for different monthly intercepts. The VAR model uses both the monthly measure (Rolling Volatility) and daily measure (Realized Volatility and ACD-IV) of stock market volatility. The F-statistics measure the significance of the lagged values of the column variable in predicting the row variable, given the other variables in the model.

The largest F-statistics are on the main diagonal of these matrices, and the size of the statistics decrease away from the diagonal. For example, lagged stock volatility is the most important variable in predicting current stock volatility. Lagged PPI inflation volatility also helps in the period 1944/01-1975/06. Lagged bond return volatility and short-term interest rate volatility contribute less. Likewise, stock volatility estimated by realized volatility helps predict PPI inflation volatility in period 1944/01-1975/06; stock volatility estimated by Rolling method and ACD method help to predict the PPI inflation volatility in period 1975/07-2008/12. There is a strong evidence that bond return volatility helps predict short interest rate volatility; Interest rate helps predict bond return volatility in period 1975/07-2008/12.

Table 6b contains tests of the incremental predictive power of 12 lags of monetary base growth volatility in a 12th-order VAR system similar to table 6a. There is strong evidence that monetary base growth volatility predicts the stock return volatility estimated by both ACD method and rolling method. Also, the stock return volatility estimated by ACD methods can predict monetary base growth volatility in both periods 1944/01-1975/06 and 1975/07-2008/12. The link between interest rate and bond return is the same with Table 6a.

Table 5

Summary Statistics for Monthly Volatility of Stock Returns, Bond returns, Short Interest Rate, and Growth Rates of the Producer Price Index,the Monetary Base, and Industrial Production 1944-2008

							Autocorrelations						Unit Root Test	
Volatility series	Sample period	Sample size	mean	std.	skewness	kurtosis	r_1	r_2	r_3	r_{11}	r_{12}	Q(24)	T-statistic	P value
Stock market volatility series														
MV	1944-2008	780	0.1079	0.0912	1.9046	9.8501	0.0737	0.0706	0.1072	0.0458	0.0192	56.2059	-5.4993	(0.00)
RV	1944-2008	780	0.1287	0.0755	3.7431	29.6011	0.5137	0.3646	0.2501	0.1315	0.1110	615.5171	-3.9288	(0.00)
ACD-IV	1944-2008	780	0.1102	0.0285	1.7984	10.0412	0.7952	0.6235	0.4858	0.2475	0.2160	1794.1	-4.0246	(0.00)
Macroeconomic volatility series(based on rolling volatility measure)														
Interest	1944-2008	780	0.0417	0.0406	1.9153	8.4381	0.8498	0.7315	0.6237	0.2703	0.2435	3000.5	-4.1239	(0.00)
Bond	1944-2008	780	0.0644	0.0621	2.2915	12.0820	0.2599	0.1712	0.2185	0.1370	0.1938	571.3632	-3.7933	(0.00)
PPI	1944-2008	780	0.0272	0.0294	3.8359	29.8660	0.3672	0.2910	0.2776	0.1503	0.1856	551.9808	-3.8725	(0.00)
Base	1944-2008	780	0.0514	0.0621	6.8848	81.3675	0.5421	0.3286	0.1890	0.0435	0.0758	424.4934	-4.2492	(0.00)
IPG	1944-2008	780	0.0388	0.0386	2.6792	15.2454	0.3875	0.3226	0.2826	0.1647	0.2429	846.1941	-3.7889	(0.00)

Notes: The summary statistics are the means, standard deviations, skewness, kurtosis, and autocorrelations at lags 1, 2, 11, and 12 of the monthly annualized volatility and Ljung-Box Q-statistic lack-of-fit hypothesis test with lags up to 12 and the critical value is 21.0261. The unit root test panel are results of Augmented Dickey-Fuller unit root test for AR model with drift with lags up to 12.

Table 6a

F-Tests from Vector Autoregressive Models for Stock, Bond, Interest Rate Volatility, Including PPI Inflation Volatility,1944-2008.

	F-Tests with MV				F-Tests with RV				F-Tests with ACD-IV			
Dependent Variable	Stock	Bond	Interest	PPI	Stock	Bond	Interest	PPI	Stock	Bond	Interest	PPI
					1944/01-1975/06							
Stock	**1.59**	**1.95**	0.72	**1.93**	**7.81**	**1.80**	**1.59**	**1.95**	**64.27**	1.16	1.02	**1.88**
	(0.09)	(0.03)	(0.73)	(0.03)	(0.00)	(0.05)	(0.09)	(0.03)	(0.00)	(0.31)	(0.43)	(0.04)
Bond	0.55	**6.74**	0.95	1.13	1.53	**7.60**	1.16	1.23	1.12	**6.85**	1.25	0.99
	(0.88)	(0.00)	(0.49)	(0.33)	(0.11)	(0.00)	(0.31)	(0.26)	(0.34)	(0.00)	(0.25)	(0.46)
Interest	0.99	**2.61**	**45.26**	0.52	0.95	**2.65**	**43.80**	0.64	0.86	**2.53**	**44.41**	0.65
	(0.46)	(0.00)	(0.00)	(0.90)	(0.50)	(0.00)	(0.00)	(0.80)	(0.59)	(0.00)	(0.00)	(0.80)
PPI	0.82	0.89	0.99	**8.17**	**2.17**	1.15	1.21	**6.98**	1.28	0.99	1.10	**7.23**
	(0.63)	(0.56)	(0.46)	(0.00)	(0.01)	(0.32)	(0.28)	(0.00)	(0.23)	(0.46)	(0.36)	(0.00)
					1975/07-2008/12							
Stock	**2.37**	0.64	0.96	1.16	**9.40**	0.41	0.59	1.18	**64.69**	0.42	0.77	1.25
	(0.01)	(0.81)	(0.48)	(0.31)	(0.00)	(0.96)	(0.85)	(0.30)	(0.00)	(0.96)	(0.69)	(0.25)
Bond	1.10	1.24	**1.63**	1.35	**1.68**	**1.75**	**1.68**	1.12	1.44	1.44	**1.65**	1.24
	(0.36)	(0.26)	(0.08)	(0.19)	(0.07)	(0.06)	(0.07)	(0.34)	(0.15)	(0.14)	(0.08)	(0.25)
Interest	**1.68**	**2.21**	**72.00**	0.67	0.66	**2.04**	**69.91**	0.65	0.83	**1.97**	**70.30**	0.51
	(0.07)	(0.01)	(0.00)	(0.78)	(0.79)	(0.02)	(0.00)	(0.80)	(0.62)	(0.03)	(0.00)	(0.91)
PPI	**1.71**	1.13	0.57	**7.59**	1.43	1.16	0.77	**6.66**	**1.88**	1.48	0.92	**4.54**
	(0.06)	(0.33)	(0.87)	(0.00)	(0.15)	(0.31)	(0.68)	(0.00)	(0.04)	(0.13)	(0.53)	(0.00)

Notes: A four variable, 12th-order vector autogressive (VAR) model is estimated for stock, bond, interest rate and PPI inflation volatility, including dummy variables for monthly intercepts. The F-tests reflect the incremental ability of the column variable to predict the respective row variables, given the other variables in the model. Measures of stock return volatility based on monthly data are used in the second panel; measures from realized volatility base on daily data are used in the first panel; and the ACD-IV we introduced in our paper based on daily data are used in the third panel.

Table 6b

F-Tests from Vector Autoregressive Models for Stock, Bond, Interest Rate Volatility, Including Monetary Base Growth Volatility,1944-2008.

	F-Tests with MV				F-Tests with RV				F-Tests with ACD-IV			
Dependent Variable	Stock	Bond	Interest	Base	Stock	Bond	Interest	Base	Stock	Bond	Interest	Base
						1944/01-1975/06						
Stock	1.44	**1.66**	1.07	**2.47**	**10.52**	**1.90**	1.52	1.51	**64.22**	1.19	1.19	**2.44**
	(0.15)	(0.07)	(0.38)	(0.00)	(0.00)	(0.03)	(0.12)	(0.12)	(0.00)	(0.29)	(0.29)	(0.00)
Bond	0.51	**7.44**	0.96	0.68	1.43	**8.52**	1.06	0.70	1.20	**7.85**	1.21	0.65
	(0.91)	(0.00)	(0.48)	(0.77)	(0.15)	(0.00)	(0.39)	(0.75)	(0.28)	(0.00)	(0.28)	(0.80)
Interest	1.15	**2.44**	46.29	0.63	0.99	**2.74**	**42.76**	0.65	1.03	**2.56**	**43.03**	0.78
	(0.32)	(0.00)	(0.00)	(0.81)	(0.46)	(0.00)	(0.00)	(0.80)	(0.42)	(0.00)	(0.00)	(0.67)
Base	0.59	0.82	1.09	**9.71**	1.25	0.61	0.98	**8.63**	**1.90**	0.86	0.97	**6.67**
	(0.85)	(0.63)	(0.37)	(0.00)	(0.25)	(0.84)	(0.47)	(0.00)	(0.03)	(0.59)	(0.48)	(0.00)
						1975/07-2008/12						
Stock	**3.30**	0.83	1.26	**2.95**	**8.11**	0.69	0.49	**5.72**	**70.10**	0.67	0.70	**1.88**
	(0.00)	(0.62)	(0.24)	(0.00)	(0.00)	(0.76)	(0.92)	(0.00)	(0.00)	(0.78)	(0.75)	(0.04)
Bond	0.54	1.25	**1.65**	1.56	1.02	**1.77**	**1.77**	1.25	0.86	1.46	**1.66**	1.45
	(0.89)	(0.25)	(0.08)	(0.10)	(0.43)	(0.05)	(0.05)	(0.25)	(0.59)	(0.14)	(0.07)	(0.14)
Interest	**1.67**	**2.44**	**74.49**	1.40	0.95	**2.36**	**72.95**	**1.68**	1.13	**2.17**	**72.80**	1.54
	(0.07)	(0.00)	(0.00)	(0.17)	(0.50)	(0.01)	(0.00)	(0.07)	(0.34)	(0.01)	(0.00)	(0.11)
Base	0.95	1.21	0.73	**17.76**	**2.13**	1.36	0.76	**16.52**	**2.21**	**1.70**	0.81	**14.45**
	(0.50)	(0.27)	(0.72)	(0.00)	(0.01)	(0.18)	(0.70)	(0.00)	(0.01)	(0.07)	(0.64)	(0.00)

Notes: A four variable, 12th-order vector autogressive (VAR) model is estimated for stock, bond, interest rate and money base growth volatility, including dummy variables for monthly intercepts. The F-tests reflect the incremental ability of the column variable to predict the respective row variables, given the other variables in the model. Measures of stock return volatility based on monthly data are used in the second panel; measures from realized volatility base on daily data are used in the first panel; and the ACD-IV we introduced in our paper based on daily data are used in the third panel.

Table 6c

F-Tests from Vector Autoregressive Models for Stock, Bond, Interest Rate Volatility, Including Industrial Production Growth Volatility,1944-2008.

	F-Tests with MV				F-Tests with RV				F-Tests with ACD-IV			
Dependent Variable	Stock	Bond	Interest	IPG	Stock	Bond	Interest	IPG	Stock	Bond	Interest	IPG
					1944/01-1975/06							
Stock	**2.16**	**1.81**	0.79	1.50	**11.97**	**1.99**	**1.61**	1.55	**71.82**	**1.59**	1.26	**1.90**
	(0.01)	(0.05)	(0.66)	(0.12)	(0.00)	(0.02)	(0.09)	(0.10)	(0.00)	(0.09)	(0.24)	(0.03)
Bond	0.45	**6.92**	1.06	0.35	1.35	**7.82**	1.20	0.36	1.15	**7.01**	1.32	0.33
	(0.94)	(0.00)	(0.39)	(0.98)	(0.19)	(0.00)	(0.28)	(0.98)	(0.32)	(0.00)	(0.20)	(0.98)
Interest	1.08	**2.61**	**46.30**	0.40	0.92	**2.82**	**43.30**	0.41	0.79	**2.60**	**43.33**	0.38
	(0.37)	(0.00)	(0.00)	(0.96)	(0.53)	(0.00)	(0.00)	(0.96)	(0.66)	(0.00)	(0.00)	(0.97)
IPG	1.14	0.98	0.58	**9.13**	0.95	0.66	0.73	**7.93**	1.45	0.98	0.73	**7.13**
	(0.32)	(0.47)	(0.86)	(0.00)	(0.49)	(0.79)	(0.72)	(0.00)	(0.14)	(0.46)	(0.72)	(0.00)
					1975/07-2008/12							
Stock	**2.36**	0.68	0.90	1.31	**11.60**	0.52	0.56	**1.67**	**83.48**	0.55	0.89	0.68
	(0.01)	(0.77)	(0.55)	(0.21)	(0.00)	(0.90)	(0.88)	(0.07)	(0.00)	(0.88)	(0.56)	(0.77)
Bond	0.89	1.19	1.39	0.95	**2.04**	**1.59**	1.44	1.30	1.46	1.43	1.42	1.08
	(0.56)	(0.29)	(0.17)	(0.50)	(0.02)	(0.09)	(0.15)	(0.22)	(0.14)	(0.15)	(0.16)	(0.37)
Interest	1.28	**1.72**	**65.56**	2.70	0.39	**1.63**	**64.53**	**2.80**	0.74	**1.59**	**65.07**	**2.85**
	(0.23)	(0.06)	(0.00)	(0.00)	(0.97)	(0.08)	(0.00)	(0.00)	(0.71)	(0.09)	(0.00)	(0.97)
IPG	**1.85**	1.17	**3.02**	1.10	1.52	0.94	**2.98**	1.01	**1.64**	1.06	**3.08**	1.14
	(0.04)	(0.31)	(0.00)	(0.36)	(0.11)	(0.50)	(0.00)	(0.44)	(0.08)	(0.39)	(0.00)	(0.33)

Notes: A four variable, 12th-order vector autogressive (VAR) model is estimated for stock, bond, interest rate and industrial production growth volatility, including dummy variables for monthly intercepts. The F-tests reflect the incremental ability of the column variable to predict the respective row variables, given the other variables in the model. Measures of stock return volatility based on monthly data are used in the second panel; measures from realized volatility base on daily data are used in the first panel; and the ACD-IV we introduced in our paper based on daily data are used in the third panel.

Table 6c contains tests of the incremental predictive power of 12 lags of industrial production volatility in a 12th-order VAR system similar to Table 6a and Table 6b. From table 6c industrial production volatility predicts stock volatility estimated by ACD method very well in period 1944/01-1975/06. Stock volatility estimated by ACD method can predict industrial production volatility in period 1975/07-2008/12. Bond return volatility still predicts interest rate very well; interest rate contribute less in predicting bond return in this case.

To sum up, bond return volatility help a lot in predicting stock volatility estimated by both rolling method and realized method in period 1944/01-1975/06 but not in period 1975/07-2008/12. Interest volatility contributes less. PPI inflation volatility predicts stock volatility in period 1944/01-1975/06. Monetary base growth volatility helps much in prediction of stock volatility estimated by rolling method and ACD method in the entire sample period. Industrial production volatility help much to predict stock volatility estimated by ACD method in period 1944/01-1975/06. There is also strong evidence that stock volatility estimated by ACD method can predict macroeconomic volatility in period 1975/07-2008/12.

There are also strong evidence that bond return volatility can predict interest rate volatility using all three measures in the entire sample period.

From the table 6a-6c, PPI inflation, monetary base growth and industrial production predict stock market volatility using ACD methods very well in the period 1944/01-1975/06; the stock market volatility estimated from ACD models also predict PPI inflation, monetary base growth and industrial production very well in the period 1975/07-2008/12. It is big improvement compared to the performance of realized volatility and rolling volatility. In the early period, the stock market was not developed as the recent decades, the transactions in one day are very thin. The individual and the firm perform their transaction according to their economic situations, no wonder the macroeconomic volatility predict the stock market volatility well. As the economic develops, stock transactions become more and more frequently, there are not only investor but also speculators in the market.The stock market volatility become an predictor of the macroeconomic factors.

In this section we employ stock volatility estimated from ACD method, compared with rolling volatility and realized volatility, to test the link between stock market volatility and macroeconomic factor volatilities. We find that sometime macroeconomic factor volatility can predict stock market volatility estimated by ACD method but not realized volatility, and sometime macroeconomic factor volatility can predict volatility of realized volatility measure but not ACD method. Based on our methods, in the early period , the macroeconomic factor predict the stock market volatility very well, and the stock market volatility predicts the macroeconomic volatility very well in the recent decades. This is different from the research of Schwert (1989).

7 Conclusions

I employ a method to estimate monthly volatility by integrating the variance per unit time obtained from the ACD models. Time trend is inserted into AACD model to model the market liquidity. The ACD equations are estimated by MLE method. In our empirical test, we find when the index return threshold increases, the level and variance of monthly volatility increases.

We also compare our method to realized volatility and find that the monthly volatility series estimated from ACD models are smoother than those realized volatility. In order to find an optimal price index return threshold, we use integrated square error between ACD-ICV and realized volatility to select the one makes the smallest integrated square.

We also employ the monthly volatility estimated from ACD methods to test the link between stock market volatility and macroeconomic factors volatility. We find that sometime macroeconomic factors volatility can predict volatility estimated from ACD methods but not realized volatility, and sometime macroeconomic factor volatility can predict volatility of realized volatility measure but not ACD-ICV measure. ACD-ICV performs better than rolling volatility. The stock volatility of ACD-ICV measure can also predict macroeconomic factor volatility well.

8 Appendix

Data series used in Stock Market and Macroeconomy

A. Stock Returns, 1944-2008

I use the daily stock return index from the Center for Research in Security Prices (CRSP). The index return including dividends to the Value-weighted portfolio of all New York Stock Exchange (NYSE) stocks.

B. Short-Term Interest Rates, 1944-2008

I use the monthly yields on the shortest term U.S. Government security (90 days T-bills) which matures after the end of the month from the Government Bond File constructed by CRSP.

C. Long-Term Interest Rates, 1944-2008

I use the high-grade corporate bond yield for 1944-2008, from Federal Reserve Bank.

D. Returns to Long-Term Corporate Bonds, 1944-2008

I use the monthly yields on the long-term U.S. Government security (20 years bond) from the Government Bond File constructed by CRSP.

E. Inflation Rates, 1944-2008

For the period 1926-2009, I use the Industrial Production Index from Board of Governors of the Federal Reserve System, not seasonally adjusted.

F. Industrial Production, 1926-2008

I use the index of industrial production from the Federal Reserve Board.

G. Money Supply, 1926-2008

I use the seasonally adjusted monetary base reported by the Federal Reserve Board.

Table 3b: The parameter estimation of AACD models of British stock market

Return threshold	1 %	1.25 %	1.5 %	1.75 %	2 %	2.25 %	2.5 %	2.75 %	3 %	3.25 %
ω	0.0069	0.0234	0.0355	0.0344	0.0418	0.0701	0.1109	0.0444	0.1517	0.0822
	(0.0001)	(0.0027)	(0.0014)	(0.0007)	(0.0017)	(0.0010)	(0.0001)	(0.0000)	(0.0142)	(0.0015)
α	0.0124	0.0301	0.0143	0.0163	0.0317	0.0223	0.0060	0.0015	0.0246	0.1142
	(0.0001)	(0.0023)	(0.0018)	(0.0014)	(0.0023)	(0.0007)	(0.0001)	(0.0000)	(0.0252)	(0.0022)
β	0.9807	0.9476	0.9533	0.9497	0.9265	0.9106	0.8833	0.9541	0.8368	0.8254
	(0.0000)	(0.0000)	(0.0001)	(0.0000)	(0.0000)	(0.0001)	(0.0003)	(0.0000)	(0.0022)	(0.0023)
b	0.0450	0.0618	0.0982	0.0460	0.0073	0.0865	0.0273	0.0674	0.1001	0.1456
	(0.0322)	(0.0198)	(0.0320)	(0.0293)	(0.0504)	(0.0166)	(0.0440)	(0.0160)	(0.0316)	(0.0104)
c	0.0974	0.0685	0.0591	0.1603	0.1399	0.1974	0.0496	0.0715	-0.0919	0.2931
	(0.1480)	(1.2872)	(0.4746)	(0.7188)	(282.9)	(0.1144)	(0.3501)	(0.6791)	(0.7949)	(0.0762)
λ	0.0013	0.0153	0.0299	0.0047	0.0000	0.0152	0.0005	0.0000	0.0268	0.0805
	(0.0008)	(0.0305)	(0.0115)	(0.0045)	(0.0000)	(0.0036)	(0.0011)	(0.0000)	(0.0230)	(0.0054)
v	0.0127	0.0675	0.3157	0.0473	0.0000	0.1375	0.0194	0.0000	0.3710	0.1416
	(0.0092)	(0.1296)	(0.1135)	(0.0420)	(0.0000)	(0.0319)	(0.0460)	(0.0000)	(0.1368)	(0.0112)
η	-1.4e-09	-3.7e-10	-8.0e-09	-6.2e-09	1.5e-13	-4.1e-08	5.3e-09	6.8e-14	1.7e-07	-8.8e-07
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
$Q(12)$	19.5380	16.2152	18.5202	14.2145	8.1214	15.4469	17.3851	10.2399	11.4617	12.8627

Notes: The AACD equation is $\psi_i^\lambda = \omega + \eta\, trend + \alpha\psi_{i-1}^\lambda[|\epsilon_{i-1} - b| + c(\epsilon_{i-1} - b)]^v + \beta\psi_{i-1}^\lambda$. This results are from American stock market with the index return change varys from 1% to 3.25%. The value in the Parentheses are the standard error calculated from MLE method (0.0000 is not exactly zero and represent smaller than 0.00005). The Mean duration represents the mean of the durations when the index return change varys from 1% to 3.25%. Q(12) describes the Q-statistics of Ljung-Box Q-statistic lack-of-fit hypothesis test with lags up to 12 and the critical value is 21.0261.

Table 2c: The parameter estimation of AACD models of Japanese stock market

Return threshold	1 %	1.25 %	1.5 %	1.75 %	2 %	2.25 %	2.5 %	2.75 %	3 %	3.25 %
ω	0.0299	0.0299	0.0418	0.0558	0.0668	0.0752	0.0946	0.0763	0.1122	0.0719
	(0.0000)	(0.0018)	(0.0000)	(0.0005)	(0.0000)	(0.0000)	(0.0103)	(0.0001)	(0.0003)	(0.0001)
α	0.0086	0.0295	0.0243	0.0172	0.0106	0.0171	0.0546	0.0129	0.0274	0.0221
	(0.0000)	(0.0013)	(0.0000)	(0.0012)	(0.0002)	(0.0002)	(0.0105)	(0.0007)	(0.0014)	(0.0007)
β	0.9615	0.9415	0.9339	0.9288	0.9231	0.9085	0.8725	0.9124	0.8628	0.9087
	(0.0000)	(0.0001)	(0.0000)	(0.0000)	(0.0087)	(0.0020)	(0.0002)	(0.0016)	(0.0001)	(0.0000)
b	0.0090	0.0452	0.0232	0.0599	0.0369	0.0328	0.0466	0.0395	0.0163	0.0642
	(0.0305)	(0.0231)	(0.0283)	(0.0154)	(0.0093)	(0.0064)	(0.0237)	(0.0155)	(0.0189)	(0.0103)
c	0.2115	0.3990	0.0960	0.6080	0.1677	0.0615	-0.0962	0.1632	0.1605	0.1020
	(188.0173)	(0.3556)	(317.1)	(0.4755)	(20.48)	(2.9217)	(0.1184)	(1.1402)	(0.2270)	(0.1881)
λ	0.0000	0.0134	0.0000	0.0162	0.0028	0.0029	0.0679	0.0064	0.0069	0.0099
	(0.0000)	(0.0149)	(0.0000)	(0.0038)	(0.0021)	(0.0020)	(0.0158)	(0.0027)	(0.0024)	(0.0020)
v	0.0000	0.0563	0.0000	0.1483	0.0487	0.0336	0.2960	0.1001	0.0609	0.0826
	(0.0008)	(0.0619)	(0.0000)	(0.0281)	(0.0402)	(0.0253)	(0.0655)	(0.0496)	(0.0222)	(0.0174)
η	-6.7e-13	-8.4e-08	-2.4e-12	-2.9e-07	-6.8e-08	-9.7e-08	-3.4e-06	-2.6e-07	-3.3e-07	-5.1e-07
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
$Q(12)$	19.0764	9.6744	3.9503	10.4149	6.4708	19.0167	17.3424	8.5988	16.3847	18.0908

Notes: The AACD equation is $\psi_i^\lambda = \omega + \eta\, i + \alpha\psi_{i-1}^\lambda[|\epsilon_{i-1} - b| + c(\epsilon_{i-1} - b)]^v + \beta\psi_{i-1}^\lambda$. This results are from American stock market with the index return change varys from 0.5% to 2.75%. The value in the Parentheses are the standard error calculated from MLE method (0.0000 is not exactly zero and represent smaller than 0.00005). The Mean duration represents the mean of the durations when the index return change varys from 0.5% to 2.75%. Q(12) describes the Q-statistics of Ljung-Box Q-statistic lack-of-fit hypothesis test with lags up to 12 and the critical value is 21.0261.

Table 2d: The parameter estimation of AACD models of Australian stock market

Return threshold	1 %	1.25 %	1.5 %	1.75 %	2 %	2.25 %	2.5 %	2.75 %	3 %	3.25 %
ω	0.0182	0.0219	0.0272	0.0368	0.0870	0.0470	0.0609	0.0434	0.0602	0.0591
	(0.0031)	(0.0077)	(0.0079)	(0.0024)	(0.0008)	(0.0026)	(0.0015)	(0.0007)	(0.0036)	(0.0015)
α	0.0444	0.0357	0.0475	0.0190	0.0328	0.0770	0.0184	0.0330	0.0247	0.0033
	(0.0032)	(0.0100)	(0.0167)	(0.0048)	(0.0019)	(0.0022)	(0.0047)	(0.0009)	(0.0055)	(0.0012)
β	0.9455	0.9452	0.9365	0.9478	0.8879	0.8828	0.9251	0.9268	0.9221	0.9384
	(0.0000)	(0.1093)	(0.1088)	(0.0000)	(0.0001)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0004)
b	0.2319	0.0175	0.0106	0.0422	0.0932	0.0974	0.0552	0.0861	0.0607	0.0770
	(0.0263)	(0.1558)	(0.1903)	(0.0189)	(0.0219)	(0.0170)	(0.0390)	(0.0126)	(0.0279)	(0.0299)
c	0.6554	0.2464	-0.2166	0.0755	-0.1367	-0.1332	0.2229	-0.2105	0.0589	0.2039
	(0.2955)	(17.7819)	(7.4341)	(0.6079)	(0.3459)	(0.2975)	(1.7553)	(0.1554)	(0.2162)	(0.2054)
λ	0.1800	0.0636	0.0918	0.0335	0.0278	0.0345	0.0250	0.0129	0.0335	0.0039
	(0.0594)	(0.1572)	(0.2302)	(0.0278)	(0.0082)	(0.0103)	(0.0301)	(0.0020)	(0.0140)	(0.0013)
v	0.3016	0.2395	0.3249	0.2717	0.1566	0.0769	0.2384	0.0818	0.2908	0.2113
	(0.0838)	(0.2788)	(0.2570)	(0.1816)	(0.0311)	(0.0182)	(0.2134)	(0.0052)	(0.0937)	(0.1436)
η	-1.0e-06	-1.9e-07	-4.8e-07	-2.6e-07	-1.2e-07	1.8e-07	-9.0e-07	6.9e-07	-5.4e-07	2.1e-07
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
$Q(12)$	10.8104	8.2646	22.5976	10.9167	7.3524	14.1549	7.4671	10.2759	13.3945	6.0649

Notes: The AACD equation is $\psi_i^\lambda = \omega + \eta\, i + \alpha\psi_{i-1}^\lambda[|\epsilon_{i-1} - b| + c(\epsilon_{i-1} - b)]^v + \beta\psi_{i-1}^\lambda$. This results are from American stock market with the index return change varys from 0.5% to 2.75%. The value in the Parentheses are the standard error calculated from MLE method (0.0000 is not exactly zero and represent smaller than 0.00005). The Mean duration represents the mean of the durations when the index return change varys from 0.5% to 2.75%. Q(12) describes the Q-statistics of Ljung-Box Q-statistic lack-of-fit hypothesis test with lags up to 12 and the critical value is 21.0261.

Table 2e: The parameter estimation of AACD models of Canadian stock market

Return threshold	1 %	1.25 %	1.5 %	1.75 %	2 %	2.25 %	2.5 %	2.75 %	3 %	3.25 %
ω	0.0288	0.0482	0.0504	0.0604	0.0569	0.0341	0.0849	0.0485	0.0802	0.0260
	(0.0003)	(0.0076)	(0.0000)	(0.0149)	(0.0023)	(0.0068)	(0.0009)	(0.0006)	(0.0366)	(0.0013)
α	0.0071	0.1017	0.0000	0.0757	0.0226	0.0259	0.0168	0.0074	0.0837	0.0102
	(0.0011)	(0.0072)	(0.0000)	(0.0166)	(0.0060)	(0.0065)	(0.0020)	(0.0029)	(0.0229)	(0.0024)
β	0.9653	0.8682	0.9496	0.8855	0.9256	0.9451	0.9063	0.9461	0.8615	0.9668
	(0.0000)	(0.0020)	(0.0000)	(0.0002)	(0.0001)	(0.0000)	(0.0000)	(0.0062)	(0.0020)	(0.0001)
b	0.0755	0.1259	0.0093	0.0550	0.0481	0.0974	0.1252	0.0299	0.0331	0.0924
	(0.0301)	(0.0107)	(0.0806)	(0.0323)	(0.0324)	(0.0307)	(0.0243)	(0.0558)	(0.0502)	(0.0359)
c	0.2722	1.0000	0.1113	0.3408	0.0769	1.0000	0.1088	0.2544	0.7294	0.3186
	(0.5381)	(0.0000)	(3.2392)	(0.8028)	(1.0738)	(0.0000)	(0.1489)	(3.5623)	(1.2427)	(0.5710)
λ	0.0160	0.2886	0.0000	0.1592	0.0244	0.0696	0.0257	0.0121	0.1445	0.0238
	(0.0067)	(0.0651)	(0.0000)	(0.0547)	(0.0198)	(0.0225)	(0.0045)	(0.0135)	(0.1376)	(0.0101)
v	0.2850	0.3099	0.1523	0.3148	0.2234	0.3414	0.3081	0.2989	0.2662	0.3990
	(0.0927)	(0.0610)	(0.1191)	(0.1110)	(0.1376)	(0.0803)	(0.0480)	(0.1846)	(0.2183)	(0.1254)
η	-1.7e-07	-7.5e-06	-1.9e-12	-7.7e-06	-1.5e-06	-4.8e-06	-2.6e-06	-9.9e-07	-1.9e-05	-2.7e-06
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0001)	(0.0000)
$Q(12)$	11.3635	8.5146	14.1228	5.2742	12.5167	13.0942	10.7440	6.2607	9.2884	8.2179

Notes: The AACD equation is $\psi_i^\lambda = \omega + \eta\, trend + \alpha\psi_{i-1}^\lambda[|\epsilon_{i-1} - b| + c(\epsilon_{i-1} - b)]^v + \beta\psi_{i-1}^\lambda$. This results are from American stock market with the index return change varys from 0.5% to 2.75%. The value in the Parentheses are the standard error calculated from MLE method (0.0000 is not exactly zero and represent smaller than 0.00005). The Mean duration represents the mean of the durations when the index return change varys from 0.5% to 2.75%. Q(12) describes the Q-statistics of Ljung-Box Q-statistic lack-of-fit hypothesis test with lags up to 12 and the critical value is 21.0261.

Bibliography

AKGIRAY V. 1989. Conditional heteroscedasticity in time series of stock returns: Evidence and forecasts[J]. Journal of Business, 62(1):55–80.

ANDERSEN T, BOLLERSLEV T, DIEBOLD F, et al. 2001a. The distribution of exchange rate volatility [J]. Journal of American Statistical Assoction, 96(1):42–55.

ANDERSEN T, BOLLERSLEV T, DIEBOLD F, et al. 2001b. The distribution of realized stock return volatility* 1[J]. Journal of Financial Economics, 61(1):43–76.

ANDERSEN T, BOLLERSLEV T, CHRISTOFFERSEN P, et al. 2006a. Practical volatility and correlation modeling for financial market risk management[J].

ANDERSEN T, BOLLERSLEV T, CHRISTOFFERSEN P, et al. 2006b. Volatility and correlation forecasting[J]. Handbook of Economic Forecasting, 1:778–878.

BOLLERSLEV T. 1986. Generalized autoregressive conditional heteroskedasticity[J]. Journal of econometrics, 31(3):307–327.

CHEN N, ROLL R, ROSS S. 1986. Economic forces and the stock market[J]. Journal of Business, 59(3).

ENGLE R. 1982. Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation[J]. Econometrica: Journal of the Econometric Society, 50(4):987–1007.

ENGLE R, RUSSELL J. 1998. Autoregressive conditional duration: A new model for irregularly spaced transaction data[J]. Econometrica, 66(5):1127–1162.

FAMA E. 1976. Inflation uncertainty and expected returns on Treasury bills[J]. The Journal of Political Economy, 84(3):427–448.

FERNANDES M, GRAMMIG J. 2006. A family of autoregressive conditional duration models[J]. Journal of Econometrics, 130(1):1–23.

MERTON R. 1980. On estimating the expected return on the market* 1:: An exploratory investigation[J]. Journal of Financial Economics, 8(4):323–361.

OFFICER R. 1973. The variability of the market factor of the New York Stock Exchange[J]. Journal of Business, 46(3):434–453.

PACURAR M. 2008. Autoregressive conditional duration (acd) models in finance: A survey of the theoretical and empirical literature[J].

PINDYCK R. 1984. Risk, inflation, and the stock market[J]. The American Economic Review, 74(3):335–351.

SCHWERT G. 1989. Why does stock market volatility change over time?[J]. Journal of finance, 44(5):1115–1153.

TSE Y, YANG T. 2009. Estimation of High-Frequency Volatility: Autoregressive Conditional Duration Model Versus Realized volatility Methods[J]. working paper: Singapore Management University.

VERONESI P. 1999. Stock market overreactions to bad news in good times: a rational expectations equilibrium model[J]. Review of Financial Studies, 12(5):975.