Effects of Immigration in a Blanchardian Model

By

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ABSTRACT

In this thesis, the focus is on immigration as a tool to sustain fiscal debt in government policies. We used a model that captures demographic parameters like birth rate and death rates to model households in addition to the government's use of tax receipts from immigrants to finance government consumption and transfers to the households. We put the model in the context of a small open economy with one homogenous good and a small open economy with a tradable and a non-tradable sector. We further endogenized labour supply and studied the effect of tax transfers as well as obtained the equilibria under immigration. We found that tax transfers do not affect the supply of labour. For the last section, we studied the effectiveness of foreign worker levies as a tool to control the influx of foreign workers and increase the employment of the native workers. We found that the effectiveness of the foreign worker levies depends on whether the wage cost impact on firm entries and exits dominates over the substitution effect as firms use more of one type of labour over the other.

Contents

1. Introduction

Immigration is a topic that has been gathering interest in recent years. This is especially so in the context of increasing fiscal burden induced by increasing government spending during recessions and reducing tax receipts over the recent years especially under the circumstance of a greying or declining population. Having to solve the issue of maintaining fiscal balance, governments are faced with cuts in government expenditure and tax hikes. Reducing government expenditure requires certain hardship measures like the reduction of government payrolls or pension plans or increasing the retirement age. The measures are generally not viable in a politically sensitive context. Immigration provides an additional source of tax receipts which helps mitigate the fiscal burden by allowing a deficit that cannot otherwise be sustained to be sustainable without having to resort to harsh measures. It is not uncommon for governments to take the last option. There are several research papers on the topic of fiscal benefits due to immigrants.

In analysing the benefits and costs due to immigration, it would be necessary to work with a model that accounts for consumption spending, wealth and capital accumulation in an overlapping generation with finite horizon. One key consideration is that demographic parameters like birth rate, death rate are captured in the model as opposed to using an infinitely lived representative household model.

The general framework that is used in this study is one that is modified from the Buiter 1989/Philippe Weil 1988 version of the Blanchardian perpetual youth model. The framework captures the time evolution of consumption and wealth for an overlapping generation with a finite horizon in relation to a pertubation which in our case is immigration in addition to fiscal policy effect. The original version of the Blanchardian model captures the changes in wealth and consumption of a constant population with a constant probability of death. Philippe Weil (1988) further enhanced the model with the introduction of disconnected families in the context of a growth economy. Buitter enhanced it with the introduction of productivity growth, birth rates and death rates.

This thesis consists of 7 parts including the introduction and conclusion as the first and last section respectively. The main body consists of 5 parts and covers different aspects of immigration.

We started in section 2 with the introduction of the houshold model incorporating immigration and the usage of immigration as a fiscal tool to mitigate adjustment to fiscal spending and taxation. The dynamics of the economy's per capita consumption and wealth are captured within the model. In this part, we extended the work of Buitter 1989 with immigraton.

In section 3, we applied the dynamics of a small open economy with a single homogenous good sector to the household model. We derived the steady states of the economy to analyse the effects of an immigration rate increase and the use of the immigration surplus to finance government spending and alternatively, tax transfers to the population.

In section 4, the analysis was performed on an economy with a tradable and a non-tradable sector. The effects of immigration has an impact on per capita consumption and per capita net asset holdings. The combination of changes in the household and government demand for nontradables and tradables affects the relative size of the sectors. In the absence of costly capital adjustment, there is no impact on the price of non-tradable goods, wage level and interest rate. In the light of this, we evaluate the capital stock per capita in the economy and obtain insights into the net foreign asset position of the economy. In this section, we applied elements from Obtsfeld and Rogoff 1996.

In section 5, we examine the impact of transfers in the form of payroll tax reduction on endogenous labour supply by extending the model with Hoon 2010. We have in section 3 and 4 evaluated the model using exogenous wage or labour income. When subsidies are given to the households on the back of higher fiscal debt financed through immigration growth, a relevant question to ask is if the subsidies would result in a reduction in the hours worked as workers value leisure more over work with the transfers. Since tax receipts are in the form of payroll taxes and is a percentage of the hours workers work, we may encounter an increase or a reduction in tax receipts through the number of hours the workers are willing to work. Obviously, the answer to the question is key to whether we experience a bigger or smaller budget deficit than planned.

In section 6, we deviated from the concept of immigrants who integrate fully into the economy to the study of temporary migrant workers in a small open economy. We looked at a small open economy that controls the influx of immigrants using tax levies imposed on firms when they hire foreign workers. We extended elements from Ghironi 2000 on small open economies and firms heterogenity in productivity levels from Melitz 2003 to derive results to analyse the effect of adjustments in the foreign workers levies imposed on firms hiring foreign workers. In doing so, we are able to conduct a richer analysis with the use of some simplifications. We found that in the case of foreign workers, there are two effects that work to the opposite effects of each other. The lowering of the cost of an overall wage index with cheaper foreign workers would help firm entries and reduce firm exits. This result is stronger if firms are more labour intensive and are primarily clustered towards in the lower productivity bands. The opposing force is that of labour substitution as firms favour cheaper foreign workers over the more expensive local labour. The results depend on which effect dominates.

In section 7, we conclude the thesis with insights from the different sections on the aspects of immigration that we covered.

2. Modelling Households

In this chapter, we derive the household consumption and asset dynamics in the presence of immigration. In the model, the country has a population of L(t) at time $t \geq 0$ where L(0) is normalised to 1. The representative agent faces a constant probability of death p. Birth rate, β is constant throughout time. Immigration is a flow variable, m as a percentage of the population at time t. As such population growth = β-p+m or n+m where n is defined as the natural growth rate of the population.

Before we start on the representative household consumption model, we briefly touch on fiscal debt and the motivation of tax receipts from immigration. The evolution of aggregate fiscal debt (B_t) with a given fiscal policy at time t of T_t(aggregate Tax) and G_t(aggregate government expenditure) is as follows.

$$\partial B_t = r_t B_t + G_t - T_t$$

On a per capita basis, this translates to the following time transition equation where b_t, g_t and τ_t are the per capita fiscal debt, government spending and tax and n and m are natural population growth rate and immigration rate.

$$\partial b_t / \partial t = (r_t - n - m) b_t + g_t - \tau_t \qquad (1)$$

or alternatively,

$$b_t = \int_t^{\infty} \{\tau(v) - g(v)\} e^{-\int_t^{v} \{r(s)-n-m\} ds} dv \qquad (1a)$$

From equation 1, holding all other parameters constant, a positive n or m has the effect of causing per capita fiscal debt to reduce or slow the increase in per capital debt. Equation 1a shows that a constant per capita fiscal debt can be achieved with a lower fiscal surplus if n+m is positive. It is usually an uphill task to manipulate birth rates of a country. On the other hand, immigration as a population tool is seen as a much easier tool as long as there exists an income or welfare differential between the countries.

2.1 The Representative Agent

The population consists of cohorts of agents born at different period of time prior to time t. The representative agent of a cohort born in time s (s ≤t) faces a constant probability of death,p and a time discount factor of θ. At any time t, he consumes an amount of consumption good, c(s,t) where s reflects his vintage (the cohort he belongs to). An alternative interpretation of c(s,t) is that of a basket of good or a consumption good index when we are addressing more than one good. The lifetime utility maximisation problem posed to the representative agent is as follows

$$\max_{c(s,v)} E_t \int_t^\infty log\big(c(s,v)\big)\, e^{-\theta(v-t)} dv = \int_t^\infty log\big(c(s,v)\big)\, e^{-(p+\theta)(v-t)} dv \qquad (2)$$

The utility maximisation problem is subject to the agent's budget constraint and his asset holding dynamics.

As is standard in the Blanchardian model, a perfect actuarial bond market exists and in its presence, every agent purchases bonds and hedges against the probability of death and accidental bequealth. Agents receive a per period payout in exchange for the actuarial firm taking over their entire asset holding upon their death. Further with free entry of into the actuarial market, profit is driven to zero such that the period payout equals to the probability of death x his asset holdings.

The evolution of asset holding and budget constraint is therefore as given in equation 3.

$$\frac{\partial a(s,t)}{\partial t} = (r+p)a(s,t) + y(s,t) - c(s,t) - \tau(t) \tag{3}$$

In equation 3, a(s,t) is the representative's wealth and y(s,t) is the per period income. Tax rate $\tau(t)$is the lump sum tax levied on the agent and is independent of the agents age or cohort.

The transversality condition for asset holding is as follows:

$$\lim_{v\to\infty} a(s,v)e^{-\int_t^v (r+p)du} = 0$$

The transversality condition imposes the requirement that agents do not over consume out of debt and has to die debt free.

Integrating forward, equation 3 becomes:

$$a(s,t) = \int_t^\infty c(s,v)e^{-\int_t^v (r+p)du}\, dv - \int_t^\infty (y(v,s) - \tau(v))e^{-\int_t^v (r+p)du}\, dv \tag{4}$$

We define h(s,t) as human lifetime earnings as human wealth :

$$\mathrm{h(s,t)} = \int_t^\infty (y(s,v) - \tau(v))e^{-\int_t^v (r+p)du}\, dv \tag{5}$$

As in the standard Blanchardian model, solving the utility maximisation problem in (2) and (3) and combining with equation (4) gives

$$c(s,t) = (p+\theta)\,(a(s,t) + h(s,t)) \tag{6}$$

The evolution of human wealth of a representative can be derived from equation (5) and (6) as follows:

$$\frac{\partial h(s,t)}{\partial t} = (r+p)h(s,t) - y(s,t) + \tau(t) \tag{7a}$$

$$\frac{\partial c(s,t)}{\partial t} = (p+\theta)(\partial a(s,t) + \partial h(s,t)) \tag{7b}$$

2.2 National Level Aggregation

C(t), H(t), Y(t) and A(t) are the aggregations of the respective representative entities ie c(s,t), h(s,t), y(s,t) and a(s,t) over all surviving members at time t of the cohorts and represent national aggregates. Aggregating the cohort's representative c(s,t), h(s,t) and a(s,t) over all cohorts s born to time t, equation 6 is transformed into its national aggregate equivalents

$$C(t) = (p+\theta)(A(t) + H(t)) \tag{8}$$

The evolution of non-human wealth is derived as follows here. The immigrants are fully integrated instantaneously as such the birth rate is taken as the same between natives natives. It is reasonable in that immigrants typically integrate into the country and adopt the same profile in a short time.

$$A(t) = \int_{-\infty}^{0} \beta L(s)a(s,t)e^{-p(t-s)}ds + \int_{0}^{t}(m+\beta)L(s)a(s,t)e^{-p(t-s)}ds$$

$$\frac{\partial A(t)}{dt} = -p\text{A(t)} - (m+\beta)L(t)a(t,t) + \int_{-\infty}^{0} \beta L(s)\partial a(s,t)e^{-p(t-s)}ds +$$

$$\int_{0}^{t}(m+\beta)L(s)\partial a(s,t)e^{-p(t-s)}ds$$

$$= rA(t) + Y(t) - C(t) - T(t) \tag{9}$$

The term $\int_{0}^{t}(m+\beta)L(s)\partial a(s,\text{t})e^{-p(t-s)}ds$ equates to zero as new born does not possess any wealth and assuming that immigrants carry zero wealth into the country being that the bias is towards relatively younger immigrants who contribute more to tax receipts. Note that we made use of equation 3 to arrive at equation 9.

The dynamics of human wealth is derived as follows. We impose the condition that there is no age discrimination in the market where the old are paid less. Additionally, we also assume the condition that labour supply of agents is fixed (exogenous). The representative' income y(s,t) is therefore uniform and constant across cohorts. We can then simplify h(s,t) to h(t):

$$\mathrm{H(t)} = \int_{-\infty}^{0} \beta L(s) h(s,t) e^{-p(t-s)} ds + \int_{0}^{t} (m+\beta) L(s) h(t,s) e^{-p(t-s)} ds$$

$$= h(t)[\int_{-\infty}^{0} \beta L(s) e^{-p(t-s)} ds + \int_{0}^{t} (m+\beta) L(s) e^{-p(t-s)} ds]$$

$$\frac{\partial H(t)}{\partial t} = \frac{\partial h(t)}{\partial t} [\int_{-\infty}^{0} \beta L(s) e^{-p(t-s)} ds + \int_{0}^{t} (m+\beta) L(s) e^{-p(t-s)} ds] +$$

$$+ h(t)[-p\left\{\int_{-\infty}^{0} \beta L(s) e^{-p(t-s)} ds + \int_{0}^{t} (m+\beta) L(s) e^{-p(t-s)} ds\right\}]$$

$$+ (m+\beta) h(t) L(t)$$

$$= (r+m+\beta) H(t) - Y(t) + T(t) \tag{10}$$

Note that equation 10 is derived by making use of equation 7a and substituting for $\frac{\partial h(t)}{\partial t}$.

2.3 Per capita consumption and asset dynamics

We reuse c(t), a(t), h(t), y(t), g(t) and τ(t) to denote per capita consumption, wealth, human wealth, income, government expenditure and tax (lump sum) respectively.

The derivation for the respective per capita identities is straight forward and proceeds as follows:

$$c(t)=(p+\theta)(a(t) + h(t)) \tag{11}$$

$$\frac{\partial a(t)}{\partial t} = (r(t) - n - m)a(t) + y(t) - c(t) - \tau(t) \tag{12}$$

$$\frac{\partial h(t)}{\partial t} = (r(t) + m + \beta - n - m)h(t) - y(t) + \tau(t)$$

$$= (r(t) + p)h(t) - y(t) + \tau(t) \tag{13}$$

$$\frac{\partial c(t)}{\partial t} = (p + \theta)\left(\frac{\partial a(t)}{\partial t} + \frac{\partial h(t)}{\partial t}\right)$$

$$= (p + \theta)\big((r(t) - n - m)a(t) + y(t) - c(t) - \tau(t) + (r(t) + p)h(t) - y(t) + \tau(t)\big)$$

$$= (p + \theta)\big((r(t) - n - m)a(t) - c(t) + (r(t) + p)h(t)\big)$$

$$= (r(t) - \theta)c(t) - (p + \theta)(\beta + m)a(t) \tag{14}$$

Equations (11-14) captures the dynamics of consumption and wealth (asset) of the households in the presence of shock in population through a migration rate of m introduced starting from the reference time 0.

2.4 Lowering of mortality and birth rates

With the enhancement of health care infrastructure and accessibility of medical technologies and drugs in the developed countries, there is a marked reduction in mortality rate resulting in the higher median age and longer life expectancy in the population of the developed world. This is complemented by advancement in birth control technologies and women gaining equal status and advancement in careers with men simultaneously driving down birth rates. The results is a population with both longer life expectancy and lower birth rates.

An evaluation of the effects is as follows. The population is characterised by an equal reduction in both mortality and birth rate and is constant in size. There is no immigration. We assume that the economy is open so that there is international lending and borrowing with r(t) being constant and that the economy can carry a net negative asset per capita.

The characteristic equations in steady state are as follows:

$$\frac{\partial a(t)}{\partial t} = r(t)a(t) + \ y(t) - c(t) - \tau(t)$$

$$\frac{\partial c(t)}{\partial t} = (\mathrm{r(t)} - \theta)c(t) - (\ p + \theta)(\beta)a(t)$$

With the assumption of a constant y(t) and $\tau(t)$ to simplify the analysis, there are 2 saddle path equilibria as shown in Figures 1 and 2 depending on whether $r > \theta$ or $r < \theta$.

Figure 1 : $r - \theta < 0$

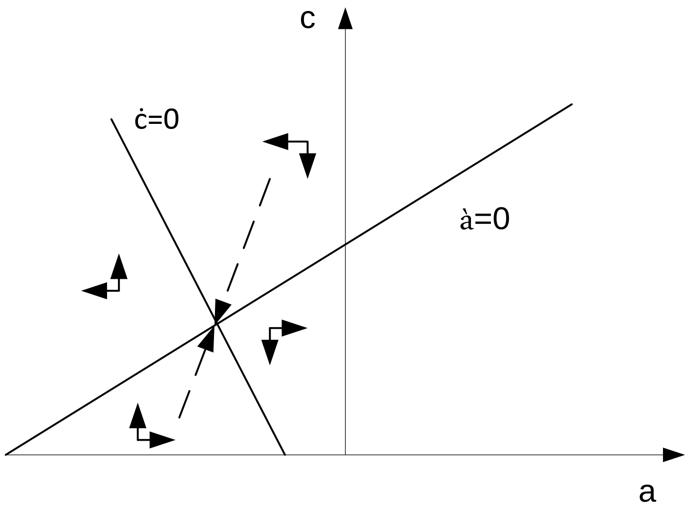
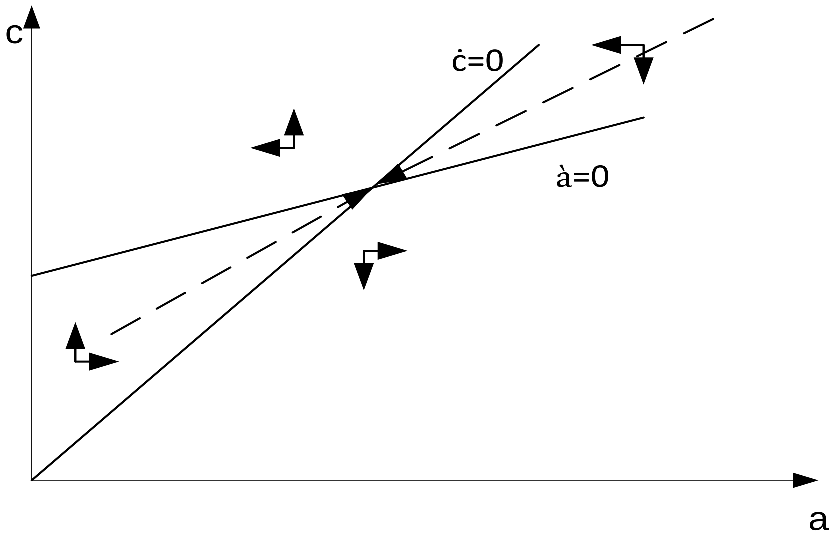


Figure 2: $r - \theta > 0$



We can check that for the saddle path equilibria in Figure 2, the required condition for a saddle path solution is $(p+\theta)(\beta+r) > (r+p)r$. In a constant greying population $\beta = p$, this is satisfied where $(p+\theta) > r$, otherwise both individual and aggregate consumption will increase without bounds for the constant population economy.

The steady state conditions imply:

$$\dot{a} = 0; \quad c = \frac{(p+\theta)\beta a}{r-\theta}$$

$$\dot{c} = 0; \quad c = ra + y - \tau$$

Combining the 2 equations gives :

$$a = (y-\tau)\frac{r-\theta}{(p+\theta)\beta - r(r-\theta)} \ ; c = \frac{(p+\theta)\beta}{(p+\theta)\beta - r(r-\theta)}(y-\tau)$$

If $r-\theta > 0$, a greying population accumulates more asset as the households propensity to spend out of asset holding and lifetime earnings declines resulting in increased savings dominating over the increase in spending due increase in spending due to increased lifetime earning. Consumption per capita increases driven by the increase in returns from savings. If $r-\theta < 0$, asset is decumulated, consumption drops. The increase in spending due to the increased lifetime earning dominates over the decline in spending propensity effect $p+\theta$.

3. Case of a small open single good economy

In this section, we apply the household model in section 2 to a small open economy in the context of immigration and fiscal debt/deficit financing. We describe the small open economy as follows. The production sector of the small open economy is characterised by a homogenous good sector. The production function per unit labour is denoted by f(k) where k is the capital intensity per labour unit. Population and labour force are by assumption the same. The size of the labour force and population is given at any point of time. Capital is internationally mobile and the actions of the small open economy do not affect the world wide interest rate. Additionally, there is free trade and perfect competition in the good produced and we normalise the price of the homogenous good to 1. There is a perfect market in international borrowing and lending. The domestic capital stock can be augmented instantaneously or decreased without adjustment cost by external borrowing or lending which allows the production sector to expand and contract.

3.1 Factor payments

With free international mobility in capital, the rate of capital rental is fixed to international interest rate, r. With factor compensated to the marginal product, capital intensity is fixed by the world interest rate as follows:

$$r = f^{'}(k) \Rightarrow \; k = f^{'-1}(r)$$

Similarly, labour is compensated to the marginal product of production and the wage rate (w) is also fixed as a function dependent on k as the only factor and indirectly to r as follows:

$$y(t) = f(k) - kf^{'}(k) = w(r)$$

Under such conditions, the inflow of immigrants does not change the income y(t). When labour increases, capital intensity reduces and the rate of return of capital increases which induces the inflow of capital until k is normalised and the rate of return of capital returns to the world interest rate. This happens almost instantaneously.

3.2 Government fiscal policies

In this section, we evaluate the actions of a government constraint with maintaining a per capita debt b but with immigration and tax receipts surplus from immigrants as measures of manoeuvres. Rewriting equation (1), a standard equation describing the evolution of financing, taxation and government consumption with a constant per capita debt is as follows:

$$\partial b_t/\partial t - rb_t = g_t - \tau_t - (n+m)b_t$$

Integrating and with the tranversality condition holding for an infinite life government and the condition of a constant b:

$$b = \int_t^{\infty}\{(n+m)b + \tau(v) - g(v)\}e^{-\int_t^v r(s)ds}dv$$

With a constant exogenous world interest rate, r, we obtain the following equation:

$$b = \frac{(n+m)b}{r} + \int_0^{\infty}\{\tau(v) - g(v)\}e^{-rv}dv \tag{15}$$

We impose the condition that the interest rate, r is greater than n+m where n+m is the effective population(labour) growth rate. Without this condition, the government would be issuing debt without limit, driving asset price to zero. The condition eliminates the possibility of the government running a ponzi scheme with transfers to current generation from future generations through debt issuance and also the existence of asset bubbles.

The value of immigration tax surplus $\frac{mb}{r}$ could be deployed in two ways, either a tax subsidy to the households (in the event of an existing high tax rate) or to sustain existing government expenditure.

We set n to zero as a simplification and assume a constant native population with zero growth. The action of the government can come in either of the following form:

a) a government expenditure of $g + mb$ per period
b) a tax rate of $t - mb$ per period

3.3 Funding government expenditure with immigration surplus

In this section, we evaluate the effect of immigration tax surplus used to fund government consumption. As can be seen from equations 12 -14, government consumption expenditure does not appear in the household consumption and asset dynamics when wage, interest rate and price are exogenously fixed by world interest rate, in the context of a small open economy.

3.3.1 Consumption and asset dynamics

To analyse the consumption and asset dynamics, we modify the consumption and asset evolution equations 12 and 14, using $a(t) = k(t) + e(t) + b(t)$ where $e(t)$ is the foreign asset position per capita and $k(t)$ is the amount of capital in the country's production sector per labour unit (per capita). Asset holding per capita essentially consists of capital holdings, net foreign asset holding and government bonds holding. In our context, $e(t)$ may be positive or negative. Positive indicates lending to abroad and negative indicates borrowing from abroad.

$$\frac{\partial k(t)}{\partial t} + \frac{\partial e(t)}{\partial t} = (r - n - m)(k(t) + e(t) + b(t)) + \ y - c(t) - \tau(t) \tag{18}$$

$$\frac{\partial c(t)}{\partial t} = (r - \theta)c(t) - (\ p + \theta)(\beta + m)\big(k(t) + e(t) + b(t)\big) \tag{19}$$

Steady state conditions for net asset holding per capita and consumption per capita are as follows after setting $\frac{\partial k(t)}{\partial t} + \frac{\partial e(t)}{\partial t}$ =0 and $\frac{\partial c(t)}{\partial t} = 0$.

$$k(t) + e(t) = \frac{(\mathrm{c(t)} - \mathrm{y(t)} + \tau(\mathrm{t}))}{(r-n-m)} - b \tag{18'}$$

$$c(t) = \frac{(\ p+\theta)(\beta+m)(k(t)+e(t)+b)}{(\mathrm{r}-\theta)} \tag{19'}$$

Inspection of 18' and 19' reveals two possible equilibria – a positive net asset position corresponding to $r - \theta > 0$ and the converse a negative net asset position (net debtor) corresponding to $r - \theta < 0$. The phase diagrams in Fig 3.1 and 3.2 show both equilibria to be saddle path equilibria.

Figure 3.1 : $r - \theta > 0$

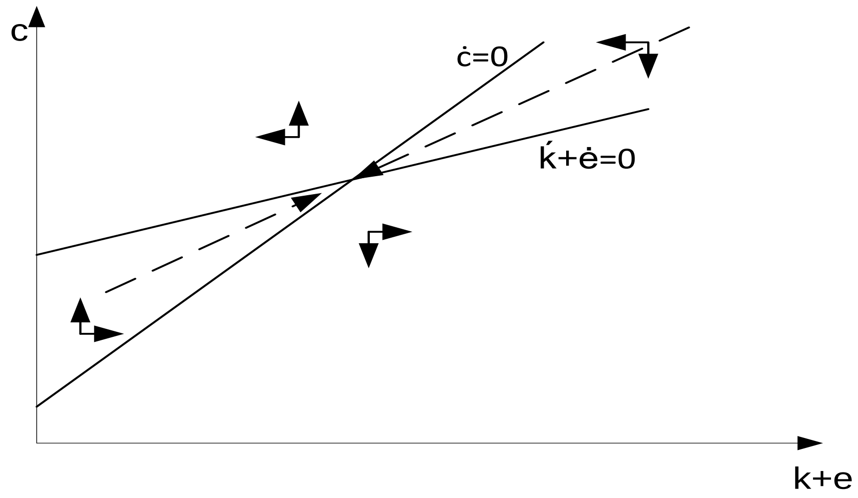
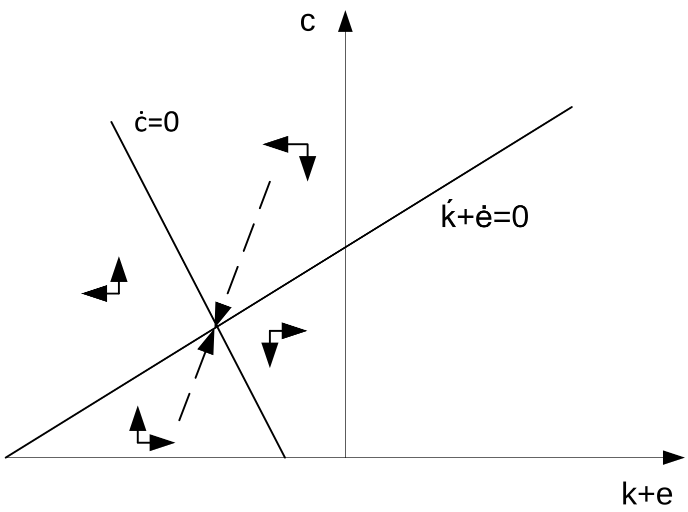


Figure 3.2: $r - \theta < 0$



3.3.2 Conditions for the existence of steady state equilibria

We need to validate the conditions for the existence of the saddle point equilibria for $r - \theta > 0$ which is not clear up front. On the other hand the saddle point equilibria for $r - \theta < 0$ is obviously guaranteed from the steady state equations.

The existence of a saddle path for $r-\theta>0$ requires that the an intersection occurs between the lines $\dot{c}=0$ and $\dot{k}+\dot{e}=0$. Note also that the gradient of $\dot{k}+\dot{e}=0$ is $(r(t)-n-m)>0$ is imposed by Non-ponzi-game condition of government debt financing and is therefore always upward sloping.

A saddle path equilibria requires $(r(t)-n-m)<\frac{(p+\theta)(\beta+m)}{(\mathrm{r(t)}-\theta)}$.

We can draw some insights into this prerequisite condition by manipulating the equation as follows:

$$\frac{(p+\theta)(\beta+m)}{(\mathrm{r(t)}-\theta)}>r(t)-n-m$$

$$(p+\theta)(\beta+m)>(r(t)-n-m)(\mathrm{r(t)}-\theta)=(r(t)+p-(\beta+m)(\mathrm{r(t)}-\theta)$$

$$=(r(t)+p-(\beta+m))\{(\mathrm{r(t)}+\mathrm{p})-(\mathrm{p}+\theta)\}$$

$$=(r(t)+p-(\beta+m))(\mathrm{r(t)}+\mathrm{p})-(r(t)+p)(\mathrm{p}+\theta)$$

$$+(\beta+m)(\mathrm{p}+\theta))$$

Further simplifications give the condition $\mathrm{r}-\theta-p-n-m<0$.

An interpretation of this gives the condition that at steady state, an increase in savings results in returns that is insufficient to cover consumption due to the wealth effect of the savings and dilution due to population growth. The converse of $\mathrm{r}-\theta-p-n-m>0$ would give a forever increasing per capita asset growth without steady state.

A further condition is for the intercepts so that $\dot{c} = 0$ and $\dot{k} + \dot{e} = 0$ will intersect with c>0 and k+e >0 . This imposes the following condition:

$$\frac{(p+\theta)(\beta+m)(b)}{(\mathrm{r}-\theta)} < b(r-n-m)+y-\tau$$

$$=> \frac{(p+\theta)(\beta+m)(b)}{(\mathrm{r}-\theta)} - b(r-n-m) < y-\tau$$

Simplifying, this yields

$$b < \frac{(p+\theta)(y-\tau)}{\frac{(p+\theta)(\beta+m)}{(r-\theta)} - (r-n-m)} = \frac{(r-\theta)(y-\tau)}{(r-p)(n+m+p+\theta-r)}$$

This condition implies that the government debt per capita has to be sufficiently small. This is consistent with a positive $r - \theta > 0$ and k+e steady state which we will see in equation 21 such that the steady state of k+e implies the above condition.

3.3.3 Phase diagram analysis

A. Case of $\boldsymbol{r} - \boldsymbol{\theta} > 0$

Figure 3.3 shows the phase diagram for the condition of $r - \theta > 0$. A saddlepath exists through the steady state as shown. The impact of immigration is to shift the $\dot{c}$=0 curve upwards as well as rotate upwards and to rotate $\dot{k} + \dot{e} = 0$ downwards as well as shift it downwards. The new lines are now $\dot{c}$=0' and $\dot{k} + \dot{e} = 0'$ respectively. The resulting effect is that both per capita consumption and net foreign asset position will decline.

B. Case of $\boldsymbol{r} - \boldsymbol{\theta} < 0$

The impact of immigration for this case is depicted in Figure 3.4. The impact of introducing an immigration rate at time 0 has the result of a clockwise rotation for ċ=0 line and a gentler slope for $\dot{k} + \dot{e} = 0$ and a downward shift of mb resulting in ċ=0' and $\dot{k} + \dot{e} = 0$' in Figure 3.4. The resulting steady state as opposed to the case of $r - \theta > 0$ is that consumption and net capital asset both increases.

Figure 3.3

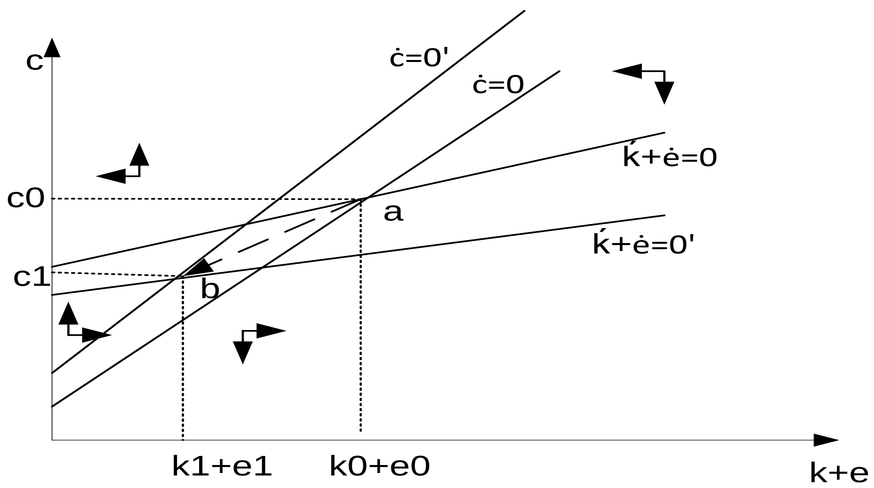
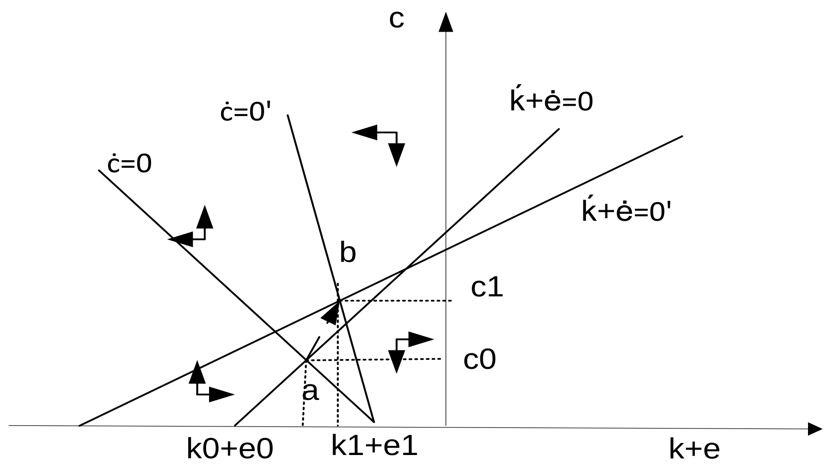


Figure 3.4



3.3.4 Steady State Analysis

The steady state equations 18' and 19' can be solved algebraically. Substituting for k+b+e of 19' into 18' gives the following:

$$c = \frac{(p+\theta)(\beta+m)(c-y+\tau)}{(\mathrm{r}-\theta)(r-n-m)}$$

$$c = \frac{(p+\theta)(\beta+m)(y-\tau)}{(\mathrm{r}+\mathrm{p})(\theta+\beta+m-r)}$$

$$= \frac{(p+\theta)(y-\tau)}{(\mathrm{r+p})\left(1-\frac{(r-\theta)}{(\beta+m)}\right)} \tag{20}$$

This shows that consumption per capita decreases with the immigration rate m where $(r-\theta) > 0$. In the case where $(r-\theta) < 0$, consumption per capita increases with the immigration rate m.

Substituting 20 into 19' gives the following

$$k+e = \frac{(\mathrm{c}-\mathrm{y}+\tau)}{(r-n-m)} - b$$

$$= \frac{(p+\theta)(y-\tau)}{(\mathrm{r+p})\left(1-\frac{(r-\theta)}{(\beta+m)}\right)(r-n-m)} - \frac{(y-\tau)}{(r-n-m)} - b$$

$$= \frac{(y-\tau)(r-\theta)}{(\mathrm{r+p})(\theta+\beta+m-r)} - b \tag{21}$$

The results are consistent with the phase diagrams. Where $(r-\theta) > 0$, when m increases, net asset holding per capita and consumption per capita decreases. Whereas for $(r-\theta) < 0$, net asset holding per capita and consumption per capita increases. The steady state however for net asset per capita will remain negative.

3.4 Funding government transfers with immigration surplus

This section describes government transfers in the form of a reduction in the lump sum tax per period that is feasible through a continuous augmentation of the population size from immigration and tax receipts. With tax reduction we need to modify equation 18 into equation 22 to reflect the immigration surplus transfer passed on to each generation. The steady state asset holding equation becomes equation 23.

$$\frac{\partial k(t)}{\partial t} + \frac{\partial e(t)}{\partial t} = (r-m)\big(k(t)+e(t)\big) + \; y - c(t) - \tau + rb \qquad (22)$$

$$k(t) + e(t) = \frac{(\mathrm{c(t)} - \mathrm{y} + \tau)}{(r-m)} - \frac{rb}{r-m} \qquad (23)$$

3.4.1 Phase diagram analysis

Referring to Figure 3.5 for the case of $r - \theta > 0$, the effect of the transfer is such that $\dot{k(t)} + \dot{e(t)} = 0$ line only rotates downwards without the shift. This reduces the decline in both consumption and asset holding per capita. On the other hand for the case of $r - \theta$ <0, the $\dot{k(t)} + \dot{e(t)} = 0$ line only rotates upwards without the shift downwards as previously. This results in higher per capita consumption and lower net asset holding than without the transfer. The net effect is still higher per capita consumption and higher net asset holding (less negative) than in the absence of tax transfer.

Figure 3.5

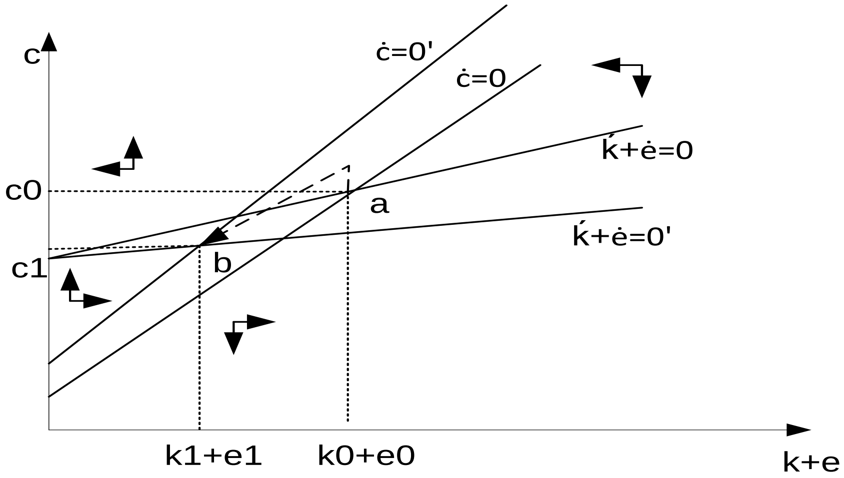
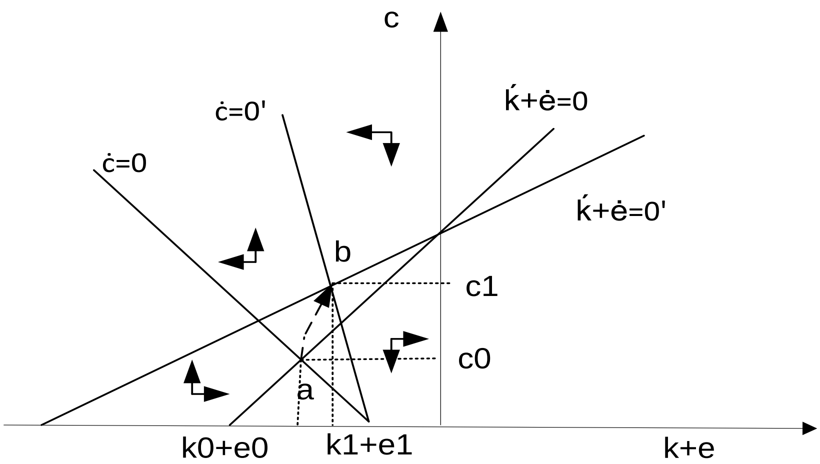


Figure 3.6



4. A two sector open economy with tradable and non tradable goods

In section 3, we analyse a small open economy with a single homogenous good sector. In this section, we extend the single homogenous good open economy to an open economy with a tradable and a non-tradable sector. The point of interest is to evaluate the impact of household spending and government spending funded through an increased fiscal debt allowed through immigration. There are conditions from section 3 applying to the mechanics of open economy that applies in this section as well. Capital is internationally mobile as well as between sectors. This condition means that capital can adjust instantaneously without any adjustment costs. Firms produce with a combination of labour and capital as the only factors of production. Additionally, labour supply at any point of time is given and equal to the population and is fully employed in the production of goods at all times. The clearing of labour is facilitated by the tradable sector, free international and intersector mobility of capital and free mobility of labour between the tradable and non-tradable sectors. Wage rate is equalised between the 2 sectors as a result of the last point.

4.1 Household consumption preferences

As opposed to a single good economy, the consumption good c(t) in the intertemporal consumption equation is now a basket of good. The representative household combines the traded good and the non traded good in a fixed proportion of their consumption expenditure. The household preference is thus best represented as a Cobb-Douglas function as follows:

$$U(c(s,t)) = \log\{(c(s,t)^{T})^{\gamma}\,(\mathrm{c(s,t)^{N}})^{1-\gamma}\}$$

where $c(s,t)^T$ represents the the consumption of tradables and $c(s,t)^N$represents the consumption of nontradables.

As in the single good open economy, we use the tradable good as the numeraire good and measure total consumption expenditure c in units of tradables. Denoting p^N as the price of the nontradable in terms of tradables, the consumption of the tradables and non-tradables are as follows:

$$c = c^T + p^N c^N$$

Optimising the intra-temporal utility to the budget constraint gives the following:

$$c^T = \gamma c$$

$$c^N = (1-\gamma)c$$

Note that we have dropped the cohort s and t, as the equations hold for all vintages and time. Household consumption and human and non-human wealth dynamics follows equations 18-19 in section 3.

4.2 Production sectors

The economy consists of two production sectors with the tradable sector being the more capital intensive. Although it is not necessary to specify the production function to achieve the results, we specify the production functions to be of Cobb-Douglas form for clarity and ease of reasoning. Per unit labour production functions are as follows:

$$f^T(k^T) = (k^T)^\alpha$$

$$f^N(k^N) = (k^N)^\beta \qquad 1 > \alpha > \beta$$

k^N, k^T are the capital per unit labour in the non-tradable and tradable sector respectively.

With free mobility of capital internationally and between the tradable and non-tradable sector, the rate of return of capital employed in the respective sectors is fixed to world interest rate. Therefore the marginal product in the 2 sectors are as follows:

$$r = \alpha(k^T)^{\alpha-1} = \beta p^N (k^N)^{\beta-1}$$

Further with free mobility of workers between the tradable and non-tradable sector, the wage rate is equalised inter-sector. In perfect competition, labour is paid the marginal product. Therefore the following holds:

$$w = (1-\alpha)(k^T)^\alpha = p^N(1-\beta)(k^N)^\beta$$

We have 4 equations and 4 endogenous variables with r being the only exogenous variable . Solving for the variables we have the following:

$$k^T = (\frac{r}{\alpha})^{\frac{1}{\alpha-1}} \tag{24}$$

$$k^N = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\beta}{1-\beta}\right)k^T = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\beta}{1-\beta}\right)(\frac{r}{\alpha})^{\frac{1}{\alpha-1}} \tag{25}$$

$$p^N = \frac{r}{\beta}(k^N)^{1-\beta} = \left(\frac{1-\alpha}{\alpha}\right)^{1-\beta}\left(\frac{\beta}{1-\beta}\right)^{1-\beta}\left(\frac{r}{\alpha}\right)^{\frac{1-\beta}{\alpha-1}}(\frac{r}{\beta}) \tag{26}$$

$$w = (1-\alpha)(k^T)^\alpha = (1-\alpha)(\frac{r}{\alpha})^{\frac{\alpha}{\alpha-1}} \tag{27}$$

We note k^T, k^N, p^N, w are locked in by the exogenous world interest rate. This is the case as a result of free mobility of the factors of production.

4.3 Equilibrium and Dynamics

The labour market clears every period such that labour demand for the non tradable sector $L^N(t)$ and labour demand for the tradable sector $L^T(t)$ is matched by the labour supply which is equal to the population size.

$$L(t) = L^N(t) + L^T(t)$$

For simplicity, all capital goods are supplied by the tradable sector. This is not essential but allows us to draw insight easily from the non-tradable goods market clearing.

The government expenditure consists of consumption of tradable goods and non-tradable goods measured in tradables:

$$g(t) = g^T(t) + g^N(t)$$

The non-tradable goods market such that demand from households and government consumption is matched by the supply of non-tradable goods:

$$p^N L^N(t) f^N\big(k^N(t)\big) = L(t)\{(1-\gamma)c + g^N(t)\} \tag{28}$$

Dividing equation 28 by $L^N(t)$ results in an equation which allows us to draw further insights easily.

$$\frac{L^N(t)}{L(t)} p^N f^N\big(k^N(t)\big) = \{(1-\gamma)c + g^N(t)\} \tag{29}$$

With free mobility of capital internationally and between the tradable and non-tradable sector, capital always clears. By walras law, the last market which is the tradable goods market also clears.

From equation 25 and 26, at a given world interest rate, $k^N(t)$ and p^N are constant as they are functions of r only. This is due to the instantaneous adjustment of capital. The proportion of labour employed in the non-tradable segment increases with per capita consumption and government expenditure in the non-tradable segment.

Further to the above, the amount of capital stock per capita k(t) is given as follows:

$$k(t) = \frac{L^N(t)}{L(t)} k^N(t) + \frac{L^T(t)}{L(t)} k^T(t) \tag{30}$$

It is obvious from equation (30) that an increase in the proportion of labour employed in the non-tradable sector reduces the capital stock per capita as $k^T(t) > k^N(t)$. Conversely, if $k^N(t) > k^T(t)$, the capital stock per capita will increase. However in general, the non-tradable sector representing the service sector and non-exportable sector is less capital intensive.

We combine equation 29 and equation 30 into equation 31 as follows :

$$k(t) = \frac{\{(1-\gamma)c + g^N(t)\}}{p^N f^N\left(k^N(t)\right)} k^N(t) + (1 - \frac{\{(1-\gamma)c + g^N(t)\}}{p^N f^N\left(k^N(t)\right)}) k^T(t) \tag{31}$$

Full differentiation of equation 31 gives equation 32

$$\partial k = \frac{(1-\gamma)\partial c}{p^N f^N(k^N)}(k^N - k^T) + \frac{(1-\gamma)\partial g^N}{p^N f^N(k^N)}(k^N - k^T) \tag{32}$$

In equation 32, noting that p^N, k^N, k^T are constants and that $(k^N - k^T) < 0$, an increase in per capita consumption,c or government expenditure in non-tradables, g^Nresults in a decline in k. The converse is also true.

4.4 Combining household dynamics and production dynamics

Section 4.3 provides insights into how the dynamics of capital stock per capita relate with the government and household's consumption. In this section, we integrate the households' consumption and asset holding dynamics with the production sector dynamics.

We proposition that the households' dynamics is the same as section 3's analysis of the small open single homogenous good economy's household's dynamics given that w and r are fixed together with p^N from equations 26 and 27. As such real wages and interest rate are taken as constant.

We now summarise the results from the section 4.3 with the results from section 3.

In the case of $(r - \theta) > 0$, per capita consumption decreases together with the net asset holding per capita. From equation 29, when the government consumption is in tradables, the decrease in per capita consumption results in a lower $\frac{L^N(t)}{L(t)}$ which results in an increase in capital stock per capita. The resulting net foreign asset position (e(t) = a(t) –k(t) –b) decreases. When the immigration surplus is spent on the non-tradables, the decline in per capita consumption is offset by the government expenditure. The result is that the effect on the capital stock per capita is ambiguous.

In the case of $(r - \theta) < 0$, consumption per capita increased with net asset holding. When government spending is also on non-tradables, the combined effect is a larger decrease in capital stock per capita as the less capital intensive non-tradable sector's relative size increases. On the other hand, when the government spending is on tradables, the effect is a lesser decrease in capital stock per capita. As the net asset holding increases, foreign asset position e(t) increases unambiguously as the relative size of the non-tradable sector increases and the capital stock per capita in the economy decreases.

5. Endogenous labour supply and tax subsidies

In this section, we study the effect of a tax subsidy arising from the tax surplus from an increased immigrant population if households have a preference for leisure as well. The implication of a reduction in taxation through the transfer from immigrants' tax receipt to the current generation could be a reduction in labour supply as the additional utility gain from leisure is valued over the gain in consumption. In turn, the reduction in labour supply will reduce the payroll tax receipts and thereby causing the per capita government debt to increase beyond the planned target. In the following section, we extended the work of Hoon 2010 on taxation effect on market hours to an economy with growth sustained by immigration.

5.1 Production sectors and firms

The production sector can be characterised by a single homogenous good small open economy or a small open economy with a tradable and a non- tradable sector. Both economies are characterised by free mobility of capital internationally and intersector. Labour is mobile between the sectors for the latter. In both scenarios, the characteristics of a freely tradable homogenous sectors as described in section 3 and 4 result in a similar set of firm production conditions for the analysis of households' behaviour so that we can generalise for both types of economy.

To summarise from section 3 and 4, in both a single homogenous good open economy and an open economy with a tradable and a non-tradable sector, the presence of trade, perfect capital mobility (international and inter-sector) and inter-sector labour mobility (for the 2 sector economy with tradable and nontradable goods), both wage rate and capital rental rate are underpinned by the world interest rate. Perfect competition ensures that the price of the tradable good is fixed. Additionally, the price of the non-tradable good is also underpinned by the world interest rate. Therefore real wage and interest rates are fixed to the exogenous world interest rate,r irrespective of the net asset position of the small economy.

As in the previous section, it is good for us to be reminded that we are looking at longer term equilibrium impact. As such, capital adjustment is instantaneous and without adjustment cost.

5.2 Household preferences

We introduce utility for leisure and non-market work into the utility preference of the representative households of vintage s and at time v. We do not distinguish between immigrants and natives in the model but take it that the immigrants integrate fully into the native population. As such there is only one representative household utility function given by:

$$U(s,v) = logc(s,v) + A^{'}\log\left[\bar{l} - l_m(s,v) - l_n(s,v)\right] + B' \qquad l_m(s,v) > 0 \qquad (33)$$

$$U(s,v) = logc(s,v) + A^{'}\log\left[\bar{l} - l_m(s,v) - l_n(s,v)\right] \qquad l_m(s,v) = 0$$

The model specified above is based on Benhabib, Rogerson and Wright(1991) and extended by Hoon(2010). We describe the model in this section in brief.

Households consume a basket of goods which is represented by $c(s,v)$. The components of the basket of goods are $c_m(s,v)$ and $c_n(s,v)$. $c_m(s,v)$ is the market good which requires to be purchased out of the income earned. $c_n(s,v)$ is the home produced non-market good. $l_m(s,v)$ is the time spent in the market producing the market good, $c_m(s,v)$ for which the household is compensated at the market wage rate. On the other hand, $l_n(s,v)$ is the time taken in the production of non market good and the household is not compensated for the amount of time spent. $\bar{l}$ is the time endowment per household. The model parameters are as follows: A', B' >0 and $c = {c_m}^{\mu}{c_n}^{1-\mu}$, $1 > \mu > 0$. The presence of a sufficiently high B' ensures that $l_m(s,v) > 0$ as working in the market gives positive direct utility for example social interactions and mental stimulation. This will be the condition imposed in the model such that every household spends a positive amount of time in the production of market good at any period of time.

The representative household then maximises the following utility function

$$\int_t^{\infty}[logc(s,v) + A' \log[\bar{l} - l_m(s,v) - l_n(s,v)] + B']e^{-(p+\theta)(v-t)}dv \qquad (34a)$$

The constraints are as follows:

$$\frac{\partial a(s,t)}{\partial t} = (r+p)a(s,t) + (1-\tau(t))w^f l_m(s,t) - c_m(s,t) \qquad (34b)$$

$$c_n(s,t) = s_n l_n \qquad (34c)$$

As per usual the transversality condition for asset holding holds. The non-market good production function does not require capital and is given by equation 34c. As opposed to section 1, 2 and 3 where y(s,t)=w(s,t) we have $w^f(s,t)$ which is a time rated wage rate paid by firms. The per period wage paid by the firm to the representative agent is $w^f(s,t)l_m$ for the work performed in the production of the market good. With a payroll tax in place, the take home pay is now $(1-$

$\tau(t))w^f(s,t)l_m$. As discussed in section 5.1, the wage rate is pinned down by the exogenous world interest rate, *r* which we assumed to be constant as it is not affected by the economic actions of the small open economy. As such, we drop the vintage and time factor off and refer only to w^f.

Maximisation of the representative's utility now involves the choice of l_m, l_n and c_m. For simplicity, we drop the notations (s,t) which denotes vintage (cohort) and time until the stage when we need to identify the different vintages.

To solve the optimisation, we form the hamiltonian:

$$J = \log(c_m{}^{\mu} c_n{}^{1-\mu}) + A' \log\left(\bar{l} - l_m - l_n\right) + B' + \lambda_n(s_n l_n - c_n) +$$

$$\lambda_m\left((r+p)a + (1-\tau)w^f l_m - c_m\right) \qquad (35)$$

where λ_n, λ_m are the lagrangian constraint multipliers; $\lambda_m = e^{(p+\theta)t}\lambda$

$$\frac{\partial J}{\partial c_m} = 0 => \frac{\mu}{c_m} = \lambda_m \qquad (35a)$$

$$\frac{\partial J}{\partial c_n} = 0 => \frac{1-\mu}{c_n} = \lambda_n \qquad (35b)$$

$$\frac{\partial J}{\partial l_m} = 0 => \frac{A'}{\bar{l}-l_m-l_n} = \lambda_m(1-\tau)w^f \qquad (35c)$$

$$\frac{\partial J}{\partial l_n} = 0 => \frac{A'}{\bar{l}-l_m-l_n} = \lambda_n s_n \qquad (35d)$$

Combining equations 35a and 35c by eliminating λ_m gives:

$$c_m A' = \mu(1-\tau)w^f(\bar{l} - l_m - l_n) \qquad (36a)$$

Combining equations 35b and 35d by eliminating λ_n gives:

$$c_n A' = (1-\mu)(\bar{l} - l_m - l_n)s_n \qquad (36b)$$

We have sufficient equations to work out c_n and l_n in terms of the rest of the parameters. Dividing 36a with 36b, we obtain

$$\frac{c_m}{c_n} = \frac{\mu(1-\tau)w^f}{(1-\mu)s_n} => \ c_n = \frac{s_n(1-\mu)c_m}{\mu(1-\tau)w^f}$$

Combining with 34c gives

$$l_n = \frac{(1-\mu)c_m}{\mu(1-\tau)w^f}$$

Substituting for c_m with equation 36a, we obtain the following identities:

$$l_n = \frac{(1-\mu)(\bar{l}-l_m-l_n)}{A^{'}}$$

$$= \frac{(1-\mu)(\bar{l}-l_m)}{A^{'}+(1-\mu)}$$

The above implies the following:

$$\bar{l} - l_m - l_n = \frac{A^{'}(\bar{l}-l_m)}{A^{'}+(1-\mu)} \tag{37a}$$

$$c_n = \frac{s_n(1-\mu)(\bar{l}-l_m)}{A^{'}+(1-\mu)} \tag{37b}$$

With equations 37a and 37b, we can rewrite the equation 34a (the household maximisation equation) as:

$$max \int_t^{\infty} \{\mu log c_m(s,v) + (1-\mu) log\left(\frac{s_n(1-\mu)}{A^{'}+(1-\mu)}\right)$$

$$+ \ ((1-\mu)+A^{'}) \log\left[\bar{l} - l_m(s,v)\right]$$

$$+ A^{'} \log\frac{A^{'}}{A^{'}+(1-\mu)} + B^{'}\} e^{-(p+\theta)(v-t)} dv$$

The maximisation solution for the above equation is synonymous to maximising the following equation:

$$max \int_t^{\infty}\{logc_m(s,v) + A\log\left[\bar{l} - l_m(s,v)\right] + B\}e^{-(p+\theta)(v-t)}dv$$

In the above equation, $A = \frac{((1-\mu)+A^{'})}{\mu}$ and B=$\frac{1}{\mu}\{(1-\mu)\log\left(\frac{s_n(1-\mu)}{A^{'}+(1-\mu)}\right) +$

$A^{'}\log\frac{A^{'}}{A^{'}+(1-\mu)} + B^{'}\}$

subject to the same constraint

$$\frac{\partial a(s,t)}{\partial t} = (r+p)a(s,t) + \left(1-\tau(t)\right)w^f l_m(s,t) - c_m(s,t) \tag{38}$$

The present value Hamiltonian is as follows:

$$J = \log c_m + A\log\left(\bar{l} - l_m\right) + \ B + \lambda_m\left((r+p)a + (1-\tau)w^f l_m - c_m\right)$$

Where $\lambda_m = e^{(p+\theta)t}\lambda$

Giving the following equations:

$$\frac{\partial J}{\partial c_m} = 0 => \ \frac{1}{c_m} = \lambda_m$$

$$\frac{\partial J}{\partial l_m} = 0 => \ \frac{A}{\bar{l}-l_m} = \lambda_m(1-\tau)w^f$$

Subsituting λ_m out by combining the 2 equations gives the following:

$$Ac_m = (1-\tau)w^f(\bar{l} - l_m) \tag{39}$$

$$\frac{\partial J}{\partial a} = (p+\theta)\lambda_m - \dot{\lambda_m} \Leftrightarrow$$

$$\lambda_m(r+p) = (p+\theta)\lambda_m - \dot{\lambda_m}$$

Substituting for λ_m with $\frac{1}{c_m}$ gives the familiar Euler equivalent equation in continuous time:

$$r - \theta = \frac{\dot{c_m}}{c_m} \tag{40}$$

5.3 Equilibria and Dynamics

In deriving the dynamics and equilibria condition, we cannot use the standard modified blanchardian equations 11 to 14 directly. Firstly, equations 11-14 have been derived using exogenous and constant labour income w(t). Wealth increases with the older vintages. When wealth increases, the labour market participation reduces and income decreases. Human wealth dynamics cannot be aggregated across different cohorts as in equation 10 where labour income is constant across vintages. The issue is briefly summarized as follows:

$$H(t) = \int_{-\infty}^{0} \beta L(s) h(s,t) e^{-p(t-s)} ds + \int_{0}^{t} (m+\beta) L(s) h(s,t) e^{-p(t-s)} ds$$

$$= \int_{-\infty}^{0} \beta L(s) \int_{t}^{\infty} (1-\tau) w^f l_m(s,t) e^{-(r+p)(v-t)} dv \, e^{-p(t-s)} ds$$

$$+ \int_{0}^{t} (m+\beta) L(s) \int_{t}^{\infty} (1-\tau) w^f l_m(s,t) e^{-(r+p)(v-t)} dv \, e^{-p(t-s)} ds$$

$$\frac{\partial H(t)}{\partial t} = rH(t) - \int_{-\infty}^{0} \beta L(s)(1-\tau) w^f l_m(s,t) e^{-p(t-s)} ds$$

$$+ \int_{-\infty}^{0} (\beta + m) L(s)(1-\tau) w^f l_m(s,t) e^{-p(t-s)} ds$$

$$+ (\beta + m) L(t) \int_{t}^{\infty} (1-\tau) \, w^f l_m(t,t) e^{-(r+p)(v-t)} dv$$

$$= rH(t) - (1-\tau) w^f L_m(t)$$

$$+ (\beta + m) L(t) \int_{t}^{\infty} (1-\tau) \, w^f l_m(t,t) e^{-(r+p)(v-t)} dv \tag{10'}$$

Contrasting equation 10' with 10, the last term in equation 10' is the human wealth of the new born and immigrants and could not be aggregated together with the first term as in the exogenous constant income case in equation 10.

Nevertheless, we can approach the solution from another angle by making use of the relationship between c_m and l_m in equation 39. The working is as follows:

We apply the usual integration on equation 38 with the transversality condition holding giving:

$$\int_t^{\infty} c_m(s,v)e^{-\int_t^v (r+p)ds}\,dv = a(s,t) + \int_t^{\infty} (1-\tau)w^f l_m(s,v)e^{-\int_t^v (r+p)ds}\,dv$$

$$= a(s,t) + h(s,t)$$

$$h(s,t) = \int_t^{\infty} (1-\tau)w^f l_m(s,v)e^{-\int_t^v (r+p)ds}\,dv$$

substituting for l_m using equation 39 gives the following :

$$\int_t^{\infty} c_m(s,v)e^{-\int_t^v (r+p)ds}\,dv = \frac{1}{1+A}\left(a(s,t) + \int_t^{\infty}(1-\tau)w^f \bar{l} e^{-\int_t^v (r+p)ds}\,dv\right) \quad (41)$$

Combining equation 40 and equation 41 and integrating gives the following:

$$c_m(s,t) = \frac{p+\theta}{1+A}(a(s,t) + \int_t^{\infty}(1-\tau)w^f \bar{l} e^{-\int_t^v (r+p)ds}\,dv) \quad (42)$$

The transversality condition for consumption assumption in working out equation 42 is as follows and holds since the transversality equation for asset holds:

$$\lim_{v\to\infty} c_m\, e^{-(r+p)(v-t)} = 0$$

In equation 42, we see a familiar propensity to consume equation. Consumption at time t is dependent on a propensity factor of the current asset holding as well as the future discounted earning of total labour endowment. We also see the disincentive to work, A, factored into the propensity to consume. The higher the disincentive to work , the lower the consumption rate. The normal demographic factors also applies. The higher mortality rate, p, and discount rate,θ, will increase the consumption propensity whereas a lower mortality rate and /or discount rate will reduce the consumption propensity.

Equation 42 can also be rewritten as follows using equation 39:

$$c_m(s,t) + (1-\tau)w^f\left(\bar{l} - l_m\right) = (p+\theta)(a(s,t) + \int_t^{\infty}(1-\tau)w^f\bar{l}e^{-\int_t^v(r+p)ds}\,dv) \quad (43)$$

We substitute equation 43 into equation 38 and arrive at an asset evolution equation in the following:

$$\frac{\partial a(s,t)}{\partial t} = (r-\theta)a(s,t) + \big(1-\tau(t)\big)w^f\bar{l}(s,t)$$

$$-(p+\theta)\int_t^{\infty}(1-\tau)w^f\bar{l}e^{-\int_t^v(r+p)ds}\,dv$$

Aggregation over the population at time t across cohorts gives :

$$A(t) = \int_{-\infty}^{0}\beta L(s)a(s,t)e^{-p(t-s)}ds + \int_0^t(m+\beta)L(s)a(s,t)e^{-p(t-s)}ds$$

$$\dot{A}(t) = -pA(t) + (m+\beta)L(t)a(t,t) + (r-\theta)A(s,t) + \big(1-\tau(t)\big)w^f\bar{L}(t)$$

$$-(p+\theta)\int_t^{\infty}(1-\tau)w^f\bar{L}(v)e^{-\int_t^v(r+p)ds}\,dv$$

$$= (r-p-\theta)A(s,t) + \big(1-\tau(t)\big)w^f\bar{L}(t)$$

$$-(p+\theta)\int_t^{\infty}(1-\tau)w^f\bar{L}(v)e^{-\int_t^{v}(r+p)ds}\,dv$$

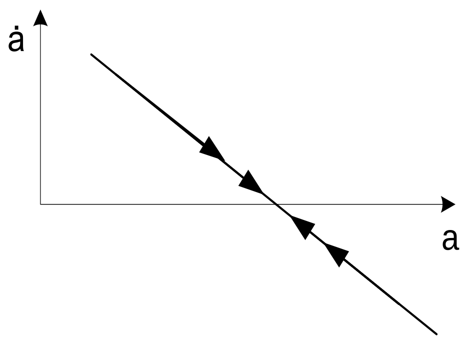
Per capita asset accumulation is thus as follows:

$$\dot{a}(t) = (r-\beta-m-\theta)a(s,t) + \big(1-\tau(t)\big)w^f\bar{l}(t)$$

$$-(p+\theta)\int_t^{\infty}(1-\tau)w^f\bar{l}(v)e^{-\int_t^{v}(r+p)ds}\,dv$$

Steady state per capita asset accumulation is guaranteed with $r-\beta-m-\theta<0$. The phase diagram is given in Figure 5.1. We observe in the phase diagram that per capita asset holding reaches a steady state from any starting position as long the above condition is statisfied. If $r-\beta-m-\theta>0$, asset per capita would be growing without bounds and therefore can be ruled out.

Figure 5.1 Phase diagram for per capita asset holding



As in exogenous labour supply, there are two steady states equilibria, a negative and a positive asset holding equilibria. The steady state is given by setting $\dot{a}(t)$ =0.

$$a(s,t) = \frac{(p+\theta)\int_t^{\infty}(1-\tau)w^f\bar{l}(v)e^{-\int_t^{v}(r+p)ds}\,dv - \big(1-\tau(t)\big)w^f\bar{l}(t)}{(r-\beta-m-\theta)}$$

Simplifying the equation for a constant exogenous r gives:

$$a(s,t) = \frac{\frac{(p+\theta)(1-\tau)w^f \bar{l}(t)}{r+p} - (1-\tau(t))w^f \bar{l}(t)}{(r-\beta-m-\theta)} = \frac{(\theta-r)(1-\tau(t))w^f \bar{l}(t)}{(r+p)(r-\beta-m-\theta)} \quad (44)$$

If $(r > \theta)$, we can see that net asset holding will be positive whereas if $(r < \theta)$, steady state net asset holding is negative. The effect of an increase in immigration rate on the net asset holding is to reduce the net asset holding if it is positive and increase the net asset holding(making it less negative) if the original net asset holding is negative ie it is a net debtor nation.

From equation 42, we have the steady state for per capita consumption:

$$c_m(s,t) = \frac{(p+\theta)}{(1+A)} \frac{(-\beta-m)(1-\tau(t))w^f \bar{l}(t)}{(r+p)(r-\beta-m-\theta)}$$

$$= \frac{p+\theta}{1+A}\left(\frac{(1-\tau(t))w^f \bar{l}(t)}{(r+p)\left(1-\frac{r-\theta}{\beta+m}\right)}\right) \quad (45)$$

If immigration rate is greater, per capita consumption decreases for a net creditor nation whereas for a net debtor nation, per capita consumption increases.

5.4 Tax and immigration effect on endogenous labour supply

Using equation 39 and the above expression for c_m, the following is obtained:

$$(\bar{l} - l_m) = \frac{A(p+\theta)}{1+A}\left(\frac{\bar{l}(t)}{(r+p)\left(1-\frac{r-\theta}{\beta+m}\right)}\right) \quad (46)$$

We see that labour worked is independent of taxation. The result is similar to Hoon 2010 but set in a growth economy sustained through immigration. We see that when taxes are reduced, the hours worked remain independent of the taxation effect. Instead, consumption and asset holdings increased by the same proportion as the reduction of taxation. Instead of reducing labour supply, households augment asset holdngs with the taxation subsidy in the long run (equation 44) and transform the increase in income into an increase in consumption (equation 45).

From equation 46, we see an increase in the hours worked with an increase in immigration when $r > \theta$. In this case, the households put in more labour hours per capita in market work as per capita consumption and asset drops. On the other hand where $r < \theta$, labour hours drop as per capita consumption and asset increases. The impact as such is that in the first case, the fiscal debt per capita would be lesser than planned as tax receipts are higher and in the latter case, the fiscal debt per capita would be greater than planned as tax receipts are lower.

6. Foreign worker levies in a small open economy

In this section we evaluate policy impacts from foreign worker levies which are taxes by governments on companies employing foreign workers. This is a policy instrument which the Singapore government employs as a lever to control the foreign worker population in Singapore. As reflecting of the scenario, we will build a model along the line of an open economy and in this case as opposed to earlier models, the foreign workers are not integrated into the population and are distinct as compared to the studies in sections 2-5 in which the immigrants are naturalised and fully integrated into the population. Further in this case, the foreign workers are characterised by a high rate of return or an expected time duration in the host country before they return to their home countries. The model below extends model elements from Zlate and Fredrico 2008, Ghironi 2000/Obstfeld and Rogoff(1996 Chapter 10)) and Melitz 2003.

The central elements of the model are small open economies models of Ghironi 2000/Obstfeld 1996, endogenous entries of firms with heterogenous firm productivities (Merlitz 2003) and immigration incentives (Zlate and Fredrico 2008)

6.1 Foreign Immigrants and Incentives

In the model context, there are two countries namely home and foreign. Both home and foreign are small open economies. Home hosts a population of immigrant workers from foreign as the wages are higher than in foreign. The size of the foreign immigrant population in home is controlled by policy settings on payroll tax on immigrant workers imposed on the hiring firms.

In this section, we describe the mechanics of foreign workers migration and determinants of the population of temporary foreign workers in home. In the model, we model foreign workers as (imperfect) substitutes for home's natives in home's production sector. Foreign workers demand a lower wage in their home country as foreign's firm productivity levels are lower than home's firms. Foreign workers are therefore motivated to migrate due to the wage differentials.

At every period, a worker in foreign has to make a decision on 2 options a)to work in foreign with an income w_f in the next period or b) to migrate and work in home where he earns a higher income of w_i. However, in order to migrate and work in home, he has to incur a cost of f_i to emigrate in the current period. The emigration cost would typically represent payment to work agencies, travel expenses and payment for any relocation services.

Additionally, the duration in host country is expected to be of a temporary nature and is susceptible to shocks. We model this as a probability of return rate (or shock arrival rate) per period of δ_i. The population of immigrant in home in period t, $L_{i,t}$ is a function of the new immigrants $L_{e,t-1}$ and the immigrants in period t-1, $L_{i,t-1}$ with the following dynamics:

$$L_{i,t} = (1-\delta)(L_{i,t-1} + L_{e,t-1}) \tag{47}$$

We therefore have the following aggregated inter-temporal utility function of foreign households as in equation (48). Foreign households maximise the utility function by deciding on consumption, labour effort in foreign and emigration rate, which is represented by $L_{e,t}$.

$$\sum_{t=0}^{\infty}(1-\theta)^t\{\log C_{f,t} + \psi \log(\tilde{L}_f - L_{f,t})\} \tag{48}$$

In equation 48, $C_{f,t}$ represents the aggregate consumption of foreign households. ψ measures the disincentive of work. $L_{f,t}$ is the total labour supply of foreign workers in foreign working in both the foreign and home production sectors and $L_{i,t}$ is the labour supply of foreign workers working in the home production sector. $\tilde{L}_f$ is the total labour endowment in Foreign. The budget constraint which the households are subjected to is as follows:

$$w_{f,t}L_{f,t} + w_{i,t}L_{i,t} - w_{f,t}L_{i,t} + \left(1 + i_{f,t}\right)A_{f,t} = P_{f,t}C_{f,t} + A_{f,t+1} + f_i w_{i,t}L_{e,t} \tag{49}$$

Optimizing the utility function using Lagrange yields the following equations where λ_t is t period's budget multiplier:

$$\frac{\partial L}{\partial C_{f,t}} = 0:$$

$$\frac{(1-\theta)^t}{C_{f,t}} = \lambda_t P_{f,t} \tag{50}$$

$$\frac{\partial L}{\partial L_{f,t}} = 0:$$

$$\frac{\psi(1-\theta)^t}{\tilde{L}_f - L_{f,t}} = \lambda_t w_{f,t} \tag{51}$$

$$\frac{\partial L}{\partial A_{f,t+1}} = 0:$$

$$\lambda_{t+1}(1 + i_{f,t}) = \lambda_t \tag{52}$$

$$\frac{\partial L}{\partial L_{e,t}} = 0:$$

$$-\sum_{n=1}^{\infty} \frac{(1-\theta)^{t+n}\psi(1-\delta)^n}{\tilde{L}_f - L_{f,t}} + \sum_{n=1}^{\infty} \lambda_{t+n}(1-\delta)^n(w_{i,t+n} - w_{f,t+n}) = \lambda_t f_i w_{f,t} \tag{53}$$

Equations 51 and 53 gives the following:

$$-\sum_{n=1}^{\infty} \lambda_{t+n} w_{f,t+n} (1-\delta)^n + \sum_{n=1}^{\infty} \lambda_{t+n} (1-\delta)^n (w_{i,t+n} - w_{f,t+n}) = \lambda_t f_i w_{f,t} \quad (54)$$

Using equation 50 to substitute for λ_t in equation 54 gives

$$\sum_{n=1}^{\infty} \frac{(1-\theta)^n (1-\delta)^n P_{f,t} C_{f,t} (w_{i,t+n} - w_{f,t+n})}{P_{f,t+n} C_{f,t+n}} = f_i w_t^f + \sum_{n=1}^{\infty} \frac{(1-\theta)^n (1-\delta)^n w_{f,t+n} P_{f,t} C_{f,t}}{P_{f,t+n} C_{f,t+n}} \quad (55)$$

An interpretation of equation is that the second term on the right hand side is the disincentive to work. When the foreign wage rate is higher, the labour supply decreases and the wage differential will have to be higher. Abstracting the disincentive to work ie setting ψ=0and therefore the second term to zero, the equation simplifies to

$$\sum_{n=1}^{\infty} \frac{(1-\theta)^n (1-\delta)^n P_{f,t} C_{f,t} (w_{i,t+n} - w_{f,t+n})}{P_{f,t+n} C_{f,t+n}} = f_i w_{f,t} \quad (56)$$

The explanation of the above is that the present value of the differential earnings streams to the future discounted by the likelihood of stay in the home and time preference factor must equate the sunk in cost of immigration. In line with the model of an small open economy that embraces open immigration policy and controlling immigration through levies on the firms, we impose that the supply of immigrant drives any surplus above the immigration cost to zero. As such equation (56) holds in equality.

The Euler equation for equation (56) is as follows:

$$f_i \frac{w_{f,t}}{P_{f,t}} = (1-\theta)(1-\delta) E_t \frac{C_{f,t}}{C_{f,t+1}} \frac{\{w_{i,t+1} - w_{f,t+1}\}}{P_{f,t+1}} \quad (57)$$

From equation 57 where we have abstracted disincentive to work away and from equation 55 where we maintain the discentive to work which is a functions of $w_{f,t}$, we note that $w_{i,t}$ in steady state is a function of $w_{f,t}$, $P_{f,t}$ and f_i. Additionally, we hold $w_{f,t}$ as exogenous and constant together with $P_{f,t}$ while we evaluate policies impact on home. As such, we see that $w_{i,t}$ is pinned down exogenously by $w_{f,t}$ and f_i.

6.2 Home Production

Home production consists of a differentiated good sector. Home firms compete with producers world wide in a monopolistic competitive market. As in Ghironi 2000, we abstracted export cost in a small open economy setting. We also apply the same implications on real interest rate equalisation and insignificant price index impacts from firm entries and exits in home.

There is a continuum of firms in the monopolistic sector, in home as well as abroad, with each firm producing a different variety i ($0 \leq i \leq 1$). The firms have pricing power and the elasticity of substitution between goods (σ) is constant where is $\sigma > 1$. The consumption preference of home for the differentiated good can be identified by a consumption index, C_t representing a basket of differentiated goods in home:

$$C_t = \int_0^1 c(i)_t^{\frac{\sigma}{\sigma-1}} di^{\frac{\sigma-1}{\sigma}}$$

$$= \int_0^a c(i)_t^{\frac{\sigma}{\sigma-1}} di^{\frac{\sigma-1}{\sigma}} + \int_a^1 c(i)_t^{\frac{\sigma}{\sigma-1}} di^{\frac{\sigma-1}{\sigma}} \qquad (58)$$

In equation 58, the first term represents the varieties of goods produced in home and the second term the varieties of goods produced abroad. The total number of varieties available in worldwide is normalised to 1. 0-a represents the varieties of goods produced in home and a-1 represents the varieties produced abroad. As with a small open economy, the variety of goods supplied by firms producing in home denoted by a is small in comparison to the global varieties, as in Ghironi 2000. The relative small weight of the home economy together implies that the economy's action has negligible effect on the rest of the world ie world interest rate and price indices.

The nominal price index for the basket goods in home is as follows where p(i) represents the nominal price of the good of variety i in home.

$$P_t = \int_0^a p(i)_t^{1-\sigma} di^{\frac{1}{1-\sigma}} + \int_a^1 p(i)_t^{1-\sigma} di^{\frac{1}{1-\sigma}} \tag{59}$$

We treat the rest of the world as one trading bloc. All consumers are assumed to have the same identical preferences for the differentiated good given by equation (58). This includes households and government consumptions.

The equivalent P_t* in the trading bloc is represented in equation (60) with $p^*(i)(p(i))$ as the nominal price in the trading bloc(home). Using Q_t as the nominal exchange rate (the cost of 1 unit of home currency in foreign bloc's currency), it can be expressed in terms of home nominal price for the goods produced in home.

$$P_t^* = \int_0^a p^*(i)_t^{1-\sigma} di^{\frac{1}{1-\sigma}} + \int_a^1 p^*(i)_t^{1-\sigma} di^{\frac{1}{1-\sigma}}$$

$$= \int_0^a Q_t p(i)_t^{1-\sigma} di^{\frac{1}{1-\sigma}} + \int_a^1 p^*(i)_t^{1-\sigma} di^{\frac{1}{1-\sigma}} \tag{60}$$

It is noted from the above that in the absence of trade cost and with the implication of no arbitrage being possible, that the law of one price holds for each variety of good such that for each variety the following holds:

$p(i)_t = p^*(i)_t / Q_t$

The relationship between the foreign and the home price indices is therefore as follows:

$Q_t P_t = P_t^*$

The world consumption can be taken as exogenous to the small open economy. This allows us to aggregate the demand and derive a simple expression for the demand facing a home firm producing variety i as follows:

$$d(i) = \frac{p(i)_t{}^{-\theta}}{P_t{}^{-\theta}} C_w \tag{61}$$

C_w denotes the aggregate world demand. Because the variety of goods produced in home is a small proportion of the total varieties, income effect of changes in the price of home production's varieties are not insignificant. However exchange rate dynamics will have an impact on the demand for home's goods.

6.3 Production Technologies and Productivities

The production function of home's firms producing the monopolistic competitive good is a CES production function mixing capital(equipment), immigrant and local labour. A firm producing a good variety i is characterised by its productivity, z_i which does not change upon inception. Immigrant and local labour are substitutes with a constant elasticity of substitution of $\varepsilon > 1$. Capital is neutral to both types of labour and as such, enters the production in the form of a Cobb Douglas function characterised by α. A representative home firm has a production function F_i of the following form:

$$F_i(K, L_n, L_i) = z_i K^{\alpha}\left(L_n^{\frac{\varepsilon-1}{\varepsilon}} + L_i^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon(1-\alpha)}{\varepsilon-1}} \quad (62)$$

The monopolistic good sector is characterised by firms with a spectrum of heterogenous productivities level as in Merlitz 2003. This section describes the behaviour of the firms.

A representative firm in home faces a startup cost which we term as the cost of entrepreneurial effort which is sunk in and has to be incurred prior to entry. This cost is represented by f_e. Upon entry, the firm draws from a cumulative productivity distribution of G(z). The productivity drawn by the firm is invariant during its operation until its exit.

During each period, the firms are subject to a constant shock arrival rate and exit at the rate of δ_f. The shocks affect equally all firms and the demise rate is independent of the firms productivity level as in Merlitz 2003.

The productivity draws which the firm achieves post entry is unknown exante. As such, the startup cost which includes entrepreneurial effort and product development cost needs to be compensated by the expected profit stream which is dependent on its productivity draws until the firm's exit. With free entry, any excess expected profit stream over the sunk in entrepreneurial cost is driven to zero such that the sunk in cost matches the expected profit stream. Firm exit and entry conditions are characterised as follows:

$$N_{d,t+1} = (1 - \delta_f)(N_{e,t} + N_{d,t}) \quad (63)$$

In equation 63, $N_{d,t}$ represents the number of domestic firms existing at period t and $N_{e,t}$ is the number of new firms entering at period t that will be operational the next period.

With home biasedness in firms ownership, the aggregate inter-temporal household utility which the households face is as follows:

$$\sum_{t=0}^{\infty}(1-\theta)^t \log C_t$$

The budget constraint involving natives' wage, $w_{n,t}$, non-human wealth A_t, average firm profit $\pi_{ave,t}$, and firms' initial sunk cost (entrepreneurial effort) f_e is as follows:

$$w_{n,t}L_t + (1+i_t)A_t + N_{d,t}\pi_{ave,t} = P_tC_t + A_{t+1} + f_eN_{e,t}$$

The solution to the optimisation of the intertemporal aggregate consumption problem yields the following equations where λ_t is the budget constraint multiplier.

$$\frac{\partial L}{\partial C_t} = 0; \quad \frac{(1-\theta)^t}{C_t} = \lambda_t P_t \tag{64}$$

$$\frac{\partial L}{\partial N_{e,t}} = 0; \quad \sum_{n=1}^{\infty} \lambda_{t+n}(1-\delta_f)^n \pi_{ave,t+n} = \lambda_t f_e \tag{65}$$

$$\frac{\partial L}{\partial A_{t+1}} = 0; \quad \lambda_{t+1}(1+i_{t+1}) = \lambda_t \tag{66}$$

Substituting equation (64) into (65) and manipulating gives the following;

$$\sum_{n=1}^{\infty}\{(1-\theta)(1-\delta_f)\}^n \frac{C_t}{C_{t+n}}\frac{\pi_{ave,t+n}}{P_{t+n}} = \frac{f_e}{P_t} \tag{67}$$

Equation (67) simplifies to a standard Euler equation involving real entry cost $(\frac{f_e}{P_t})$ and real average profits $(\frac{\pi_{ave,t+n}}{P_{t+n}})$:

$$\frac{f_e}{P_t} = (1-\theta)(1-\delta_f)E_t\frac{C_t}{C_{t+1}}\{\frac{\pi_{ave,t+1}}{P_{t+1}} + \frac{f_e}{P_{t+1}}\} \tag{68}$$

Simplifying equations (65) and (66) results in a standard Euler equation for non-human wealth (asset):

$$1 = (1-\theta)(1+r_{t+1})E_t\frac{C_t}{C_{t+1}} \tag{69}$$

In the above we make use of the relationship between the nominal interest rate and real interest rate: $(1+i_{t+1}) = \frac{P_{t+1}}{P_t}(1+r_{t+1})$.

6.4 Productivities draw and distribution

The firms' entries into the production sector is characterised by an unknown productivity draw exante. We specify the cumulative density function G(z) of the productivity draws to be a Pareto distribution with lower bound $z_{min}$and shape parameter κ where $\kappa > \sigma - 1$. The cumulative distribution function is as follows:

$$G(z) = 1 - \left(\frac{z_{min}}{z}\right)^{\kappa} \tag{70}$$

The imposition of $\kappa > \sigma - 1$ ensures that the variance of the firms' size is finite. The implication of κ is that it represents the dispersion of the firms. The higher the value for κ, the more clustered the productivities towards z_{min}, in other words the less dispersion in the distribution.

As in Merlitz and Ghironi 2009, it is useful to specify an average productivity, $\bar{z}$ in relation to the cutoff productivity level of the sector, z^*:

$$\bar{z} = \left\{\frac{1}{1-G(z^*)}\int_{z^*}^{\infty} z^{(\sigma-1)}\, dG(z)\right\}^{\frac{1}{(\sigma-1)}} \tag{71}$$

The rationale for this is that we can easily abstract the average profit, $\pi_{ave,t}$ and average revenue, R, of the firms in the production sector with a representative firm with $\bar{z}$ as the productivity level. For example:

$$\pi_{ave,t} = \pi(\bar{z}) \,; \mathrm{R} = \mathrm{R}(\bar{z})$$

The pareto distribution allows $\bar{z}$ to be simplified:

$$\bar{z} = \left\{\frac{1}{1-G(z^*)}\int_{z^*}^{\infty} z^{(\sigma-1)}\, dG(z)\right\}^{\frac{1}{(\sigma-1)}}$$

$$= \left\{\left(\frac{z^*}{z_{min}}\right)^{\kappa}\int_{z^*}^{\infty} \kappa z_{min}{}^{\kappa} z^{(\sigma-\kappa-2)}\, dz\right\}^{\frac{1}{(\sigma-1)}}$$

$$= \left(\frac{\kappa}{\kappa-\sigma+1}\right)^{\frac{1}{\sigma-1}} z^*$$

$$= \vartheta z^* \tag{72}$$

Where $\vartheta = \left(\frac{\kappa}{\kappa-\sigma+1}\right)^{\frac{1}{\sigma-1}}$

6.5 Production cost and markups

In this section, we derive the production cost and price markups in the production sector taking into account that productivity differences in firms results in different marginal costs for each firms.

From the production function given by equation (62), we simplify the equation using a labour index, L(i) which is a mix of local labour and foreign worker labour which is employed by a representative firm producing variety (i):

$$L(i) = (L_n(i)^{\frac{\varepsilon-1}{\varepsilon}} + L_i(i)^{\frac{\varepsilon-1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}} \tag{73}$$

The production function can then be written as:

$$F_i(K(i), L(i)) = z_i K(i)^{\alpha} L(i)^{1-\alpha} \tag{74}$$

We also define a wage index as the following:

$$w = (w_n^{1-\varepsilon} + (1+\tau_l)^{1-\varepsilon} w_i^{1-\varepsilon})^{\frac{1}{1-\varepsilon}} \tag{75}$$

In the above, $w_n$is the native wage rate and $w_i$is the foreign worker wage rate with τ_l the government levy on foreign workers placed on the firms. The demand for the native worker from a representative firm producing variety (i) therefore is as follows:

$$L_n(i) = \frac{w_n^{-\varepsilon}}{w^{-\varepsilon}} L(i) \; ; L_i(i) = \frac{(1+\tau_l)^{-\varepsilon} w_i^{-\varepsilon}}{w^{-\varepsilon}} L(i) \tag{76}$$

Here the interpretation is straightforward, the higher the native wages, the lower the demand for natives and the higher the demand for foreign workers. The introduction of the levy control makes the foreign workers more expensive and therefore the demand for foreign workers drops. Against the substitution effect when the levy is increased is the rise in the wage index which reduces the number of workers that can be hired as some firms exit and also reduces L(i).

With the technology function in equation (74), we can check that the golden rule still applies despite the heterogenity in firm productivity:

$$\frac{\partial \pi(i)}{\partial K(i)} = r; \quad r = p(i) z_i \alpha k^{\alpha-1}$$

$$\frac{\partial \pi(i)}{\partial L(i)} = w; \quad w = p(i)(z_i k^{\alpha} - \alpha z_i k^{\alpha})$$

Combining the two equations gives us the golden rule:

$$\frac{rK}{wL} = \frac{\alpha}{1-\alpha} \tag{77}$$

Using the property of the CRS function where the marginal cost equals the average cost as well as the golden rule, we obtain the marginal cost for a representative firm producing variety i as follows:

$$MC_i = \left(\frac{1}{1-\alpha}\right)^{1-\alpha} (\frac{1}{\alpha})^{\alpha} \frac{1}{z_i} w^{1-\alpha} r^{\alpha} \tag{78}$$

In a monopolistic competition, the price and profits are subject to the familiar markup as follows:

$$p(i) = \frac{\sigma}{\sigma - 1} MC_i$$

$$= \frac{\sigma}{\sigma-1} \left(\frac{1}{1-\alpha}\right)^{1-\alpha} (\frac{1}{\alpha})^{\alpha} \frac{1}{z_i} w^{1-\alpha} r^{\alpha} \tag{79}$$

$$\pi(i) = \frac{1}{\sigma} \frac{p(i)^{1-\sigma} C_w}{P^{-\sigma}}$$

$$= \frac{C_w}{\sigma P^{-\sigma}} \{\frac{\sigma}{\sigma-1} \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \frac{1}{z_i} w^{1-\alpha} r^{\alpha}\}^{1-\sigma} \tag{80}$$

Using the simplifications that we have done in section 6.4, we can show that the average profit is given as follows:

$$\pi_{ave} = \frac{C_w}{\sigma P^{-\sigma}} \{\frac{\sigma}{\sigma-1} \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \frac{1}{\bar{z}} w^{1-\alpha} r^{\alpha}\}^{1-\sigma} \tag{81}$$

6.6 Steady State Analysis and Equilibrium

A difficulty in the model is that of changes in Q_t which is affected by current account movements. To study the effects of wage levies, we assume a constant nominal exchange rate settings absent of any current account shocks and analyse the effects of capital flow affecting the nominal exchange rate descriptively separately.

Steady state condition for firm entries are as using equation (68):

$$\mathrm{f_e}\frac{(1-(1-\theta)(1-\delta_\mathrm{f}))}{(1-\theta)(1-\delta_\mathrm{f})}=\pi_{\mathrm{ave,t}} \tag{82}$$

Substituting for $\pi_{\mathrm{ave,t}}$ using equation 12 gives:

$$\mathrm{f_e}\frac{\big(1-(1-\theta)(1-\delta_\mathrm{f})\big)}{(1-\theta)(1-\delta_\mathrm{f})}=\frac{C_w}{\sigma P^{-\sigma}}\left\{\frac{\sigma}{\sigma-1}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}\frac{1}{\bar{z}}w^{1-\alpha}r^{\alpha}\right\}^{1-\sigma}$$

$$\bar{z}=\left(\frac{C_w}{\sigma P^{-\sigma}}\frac{(1-\theta)(1-\delta_\mathrm{f})}{\big(1-(1-\theta)(1-\delta_\mathrm{f})\big)\mathrm{f_e}}\right)^{\frac{1}{1-\sigma}}\left\{\frac{\sigma}{\sigma-1}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}r^{\alpha}\right\}w^{1-\alpha}$$

$$=Hw^{1-\alpha} \tag{83}$$

where $H=\left(\frac{C_w}{\sigma P^{-\sigma}}\frac{(1-\theta)(1-\delta_\mathrm{f})}{(1-(1-\theta)(1-\delta_\mathrm{f}))\mathrm{f_e}}\right)^{\frac{1}{1-\sigma}}\left\{\frac{\sigma}{\sigma-1}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}r^{\alpha}\right\}$

From the above equation, we note that with a fixed entry cost, as wage index increase, average productivity of the production sector will also increase. This can be easily seen as the cut off productivity has to rise in tandem with wage increases. The extent of wage rise impact on average productivity is higher with a more labour intensive industry. Also an increase in entry cost while holding wage index constant also causes the average productivity to rise as the cut off productivity rises.

Matching the supply and demand for a representative firm, we can work out the labour demand from a representative firm as follows:

$$z_iL(i)f(k(i))=\frac{C_w}{P^{-\sigma}}\left(\frac{\sigma}{\sigma-1}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}\frac{1}{z_i}w^{1-\alpha}r^{\alpha}\right)^{-\sigma} \tag{85}$$

Using the golden rule:

$\frac{r}{w}k = \frac{\alpha}{(1-\alpha)} => k^{\alpha} = (\frac{w\alpha}{r(1-\alpha)})^{\alpha}$

where r is pinned down by world interest rate.

$$L(i) = \frac{C_w}{P^{-\sigma}}\left(\frac{\sigma}{\sigma-1}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}\frac{1}{z_i}w^{1-\alpha}r^{\alpha}\right)^{-\sigma}(\frac{1}{z_i})(\frac{r(1-\alpha)}{w\alpha})^{\alpha} \tag{86}$$

Aggregate labour demand is as given by integrating with respect to G(z) and given the inherent mass of entrepreneur firms, M :

$$L = M\int_{z^*}^{\infty}\frac{C_w}{P^{-\sigma}}\left(\frac{\sigma}{\sigma-1}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}w^{1-\alpha}r^{\alpha}\right)^{-\sigma}(\frac{r(1-\alpha)}{w\alpha})^{\alpha}z^{\sigma-1}\partial G(z)$$

$$= M\int_{z^*}^{\infty}\frac{C_w}{P^{-\sigma}}\left(\frac{\sigma}{\sigma-1}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}w^{1-\alpha}r^{\alpha}\right)^{-\sigma}\kappa {z_{\min}}^{\kappa}z^{(\sigma-2-\kappa)}(\frac{r(1-\alpha)}{w\alpha})^{\alpha}\partial(z)$$

$$= M\frac{C_w}{P^{-\sigma}}\left(\frac{\sigma}{\sigma-1}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}w^{1-\alpha}r^{\alpha}\right)^{-\sigma}(\frac{r(1-\alpha)}{w\alpha})^{\alpha}\kappa {z_{\min}}^{\kappa}\int_{z^*}^{\infty}z^{(\sigma-2-\kappa)}\partial(z)$$

$$= M\frac{C_w}{P^{-\sigma}}\left(\frac{\sigma}{\sigma-1}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}r^{\alpha}\right)^{-\sigma}(\frac{r(1-\alpha)}{\alpha})^{\alpha}\kappa {z_{\min}}^{\kappa}w^{-(1-\alpha)\sigma-\alpha}\frac{z^{*(\sigma-1-\kappa)}}{\kappa-\sigma+1}$$

Substituting using free entry condition for firms for z*:

$$L = M\frac{C_w}{P^{-\sigma}}\left(\frac{\sigma}{\sigma-1}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}r^{\alpha}\right)^{-\sigma}(\frac{r(1-\alpha)}{\alpha})^{\alpha}\kappa {z_{\min}}^{\kappa}\frac{w^{(-1-\kappa)(1-\alpha)-\alpha}}{\kappa-\sigma+1}(\frac{\vartheta}{H})^{\kappa-\sigma+1} \tag{87}$$

We see that L will decrease with certainty on an increase in foreign workers' levy which increases the wage index. The more labour intensive industry ie higher(1- α) will suffer a higher reduction in the demand for labour and the more clustered the firms towards the lower end of the productivity, the higher the reduction in demand for labour to a wage index increase.

Substituting equation 76 for labour demand of natives into equation 87 gives:

$$L_n = \frac{w_n^{-\varepsilon}}{w^{-\varepsilon}} M \frac{C_w}{P^{-\sigma}} \left(\frac{\sigma}{\sigma-1}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha} r^{\alpha}\right)^{-\sigma} \left(\frac{\mathrm{r}(1-\alpha)}{\alpha}\right)^{\alpha} \kappa \mathrm{z_{min}}^{\kappa} \frac{\mathrm{w}^{(-1-\kappa)(1-\alpha)-\alpha}}{\kappa-\sigma+1} \left(\frac{\vartheta}{\mathrm{H}}\right)^{\kappa-\sigma+1}$$

$$= w_n^{-\varepsilon} M \frac{C_w}{P^{-\sigma}} \left(\frac{\sigma}{\sigma-1}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha} r^{\alpha}\right)^{-\sigma} \left(\frac{\mathrm{r}(1-\alpha)}{\alpha}\right)^{\alpha} \kappa \mathrm{z_{min}}^{\kappa} \frac{\mathrm{w}^{(-1-\kappa)(1-\alpha)-\alpha+\varepsilon}}{\kappa-\sigma+1} \left(\frac{\vartheta}{\mathrm{H}}\right)^{\kappa-\sigma+1} \quad (88)$$

To analyse the effect, we could either hold $w_n$to be constant by introducing a homogenous good into the home economy and make labour mobile between the sectors or we could do labour clearing by equating L_n to the population of home. The effects between the two are synonymous, either L_n increase or w_n increase. As performing labour clearing involves a non-linear equation which is more difficult to analyse, we perform the analysis holding w_n constant. Log linearizing the equation gives:

$$\widehat{L_n} = \{(-1-\kappa)(1-\alpha) - \alpha + \varepsilon\}\left\{\frac{(1+\tau_l)^{1-\varepsilon} w_i^{1-\varepsilon}}{w_n^{1-\varepsilon} + (1+\tau_l)^{1-\varepsilon} w_i^{1-\varepsilon}}\right\} \frac{\tau_l}{1+\tau_l} \widehat{\tau_l} \quad (89)$$

Where $\widehat{L_n}$ *and* $\widehat{\tau_l}$ are the percentage changes in L_n and τ_l respectively. While the scale of the impact of tax levy changes is dependent on the proportion of the tax level and the weightage of the foreign worker levy on the overall wage index, the direction of change is from $(-1-\kappa)(1-\alpha) - \alpha + \varepsilon$. It is not difficult to see that the last term is from the substitution effect of firms switching to local labour. The higher the substitutability between local and natives, the more labour is switched to natives. The shrinkage in overall labour is given by the first 2 terms. The higher κ, the more clustering of the local firms towards the lower productivity level and the higher the reduction in labour demand from the wage increase. The more labour intensive the sector, the higher the reduction in labour demand from the wage index. It is interesting to observe that the pricing power of the firms , σ

is not a factor. This is because of the small economy effect in which the home economy's production produces a small composition of the varieties and therefore does not have an impact on the price index P.

An increase in the tax levies would cause a demand increase in locals (or a wage rise in the locals) when the firms are more dispersed towards the higher end of productivity levels and where the industry is less labour intensive and when foreign and natives are highly substitutable. A decrease in tax levies would cause the demand for locals to drop in the same scenario. However when the scenario is the converse, an increase in tax levies would result in a decrease in demand for locals and a decrease in tax levies would result in an increase in demand for locals.

We have earlier abstracted the effects of currency rate changes Q_t which is very much affected by current account changes. We will give a qualitative treatment in this section. We have shown that a tax levy increases the wage index facing the firms resulting in a contraction in the number of firms. This could be seen as having the effect of less asset being set aside domestically for start ups investment. The outflow of capital will have a dampening effect on the contraction by reducing the relative price as Q_t depreciates. This would dampen the reduction in demand for locals if the substitution effect is lesser and would further increase the demand for locals if the substitution effect for labour is the dominating effect. In the case of tax levies being reduced, the investment in the startups is front loaded by the locals borrowing internationally. This causes Q_t to appreciate and makes the relative price of local products more expensive relatively and dampens the expansion of the firms and jobs. Therefore, job creation for locals is dampened where the substitution effect is lesser and job losses for locals are increased where the substitution effect dominates.

7. Conclusions

We started with the modelling of the households and introduced a migration rate into a growth model. The immigrants are assumed to be fully integrated into the population with the same demographic characteristics of birth rate and death rate. We used an example of an economy with constant population size to study the effects of a demographic change of a lower mortality rate and birth rate and examine the effects of the increased lifetime earnings versus the efffect of a lowering of the propensity to spend while holding wages and interest rate constant. There are 2 steady state equilibria that will arise. On one hand, the economy with higher patience than the return rate of capita is shown to increase capital accumulation and consumption. On the other hand the economy with lower patience than the return rate of capita will end up with a lower capital accumulation because the savings increase is dominated by the increase in spending due to the increased expected lifetime earnings. The observations are in the context that income is exogenously fixed and do not change with age.

In section 3, we apply the household dynamics to a small open single good economy. We derive the immigration surplus of tax receipts while the government holds per capita fiscal debt constant. There are two saddle path equilibria that emerges with immigration depending on whether the household time discount rate is higher or lower than the return on capital. We evaluate the context of the government funding the government spending with the immigration surplus. A net creditor economy ie where the household time discount rate is lower than the return of capital will end up with lower net asset holdings (which also means lower net foreign asset holding per capita in a single good economy as capital stock per capital is fixed). Per capita consumption also falls. The converse is true for the net creditor economy. We then proceeded to evaluate the impact of transfers of the immigration tax surplus to the households on a per period basis. The transfers dampen that the lowering of net foreign asset holding and consumption per capita for the net creditor economy. But the lowering of net foreign asset holding and consumption per capita still dominates. For the net debtor economy, the transfers dampens the increase but the results is still an increase of net foreign asset holding and consumption per capita.

In section 4, we extended the analysis to an economy with a tradable and non-tradable economy. The resulting changes in consumption per capita as well as government expenditure have an impact on the size of the non-tradable sector. Consumption is of the form such that households spend a fixed proportion of the total consumption expenditure on the non-tradable good. The non-tradable sector is the less capital intensive sector in comparison with the tradable sector. The capital stock of the economy decreases with a larger proportion of the population engaged in the production of the non-tradable good. We find that the capital stock of the economy decreases when per capita consumption increases and expenditure by the government in the non-tradable sector increases. For the case of the net creditor economy, the increased immigration rate causes the relative proportion of the non-tradable sector to decrease and the per capita capital stock in the economy rises. The net foreign asset holding's decrease is accentuated as the net asset holding has decreased due to the increased immigration rate. On the other hand, the presence of government spending on the non-tradable sector offsets the decline due to the decline in per capita consumption. The resulting effect is ambiguous. In the case of a net debtor economy, the increase in consumption per capita increases the relatve size of the non-tradable sector. The presence of government spending on the non-tradable good further increases the non-tradable sector. The overall effect is that capital stock per capita decreases and combined with the increase in net asset holding per capita, the net foreign asset holding per capita increases unambiguously.

In section 5, we asked the question if endogenizing labour supply has an effect on the results done earlier. Introducing household preference for leisure into the model, we found a result that is familiar from Hoon 2010 that household labour supply is not affected by changes in payroll taxes. We note that asset accumulation increases with payroll taxes subsidy and consequently consumption by the same proportion as the payroll changes leaving labour supply constant. The equilibria condition consists of 2 equilibria when the immigration rate is increased. The condition of a net creditor economy results a steady state equilibria where the results is an increase in per capita labour supply as consumption and asset holding per capita decreases. This has an effect of increasing the tax receipt and reducing the per capita debt. The condition of a net debtor economy results in a decrease in per capita labour supply as consumption and asset holding per capita increases, therefore a reduced tax receipt and higher per capita debt.

In section 6, we studied the case of a small open economy which uses foreign workers tax levies on companies. We used a small open economy model as in Ghironi 2000 but introduced firm productivity endogenity into the model. We are able to show that the effect of the tax levies depend on whether the substitution effect of labour dominates or the firms' entries and exits due to labour costs changes dominates. The latter depends on the dispersion of the firms' productivities and the labour intensivity of the sector as an increase in the overall cost of wages reduces the profit margin.

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